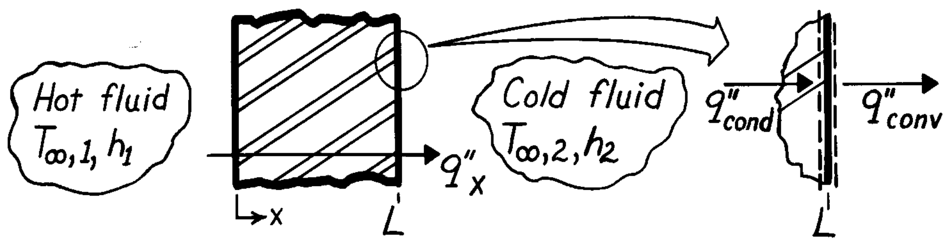
PROBLEM 3.1

KNOWN: One-dimensional, plane wall separating hot and cold fluids at $T_{\infty,1}$ and $T_{\infty,2}$, respectively.

FIND: Temperature distribution, T(x), and heat flux, q''_x, in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k and L.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation, (5) No generation.

ANALYSIS: For the foregoing conditions, the general solution to the heat diffusion equation is of the form, Equation 3.2,

$$T(x) = C_1 x + C_2. \qquad (1)$$

The constants of integration, C_1 and C_2, are determined by using surface energy balance conditions at x = 0 and x = L, Equation 2.23, and as illustrated above,

$$-k\frac{dT}{dt}\bigg]_{x=0} = h_1\left[T_{\infty,1} - T(0)\right] \qquad -k\frac{dT}{dx}\bigg]_{x=L} = h_2\left[T(L) - T_{\infty,2}\right]. \qquad (2,3)$$

For the BC at x = 0, Equation (2), use Equation (1) to find

$$-k(C_1 + 0) = h_1\left[T_{\infty,1} - (C_1 \cdot 0 + C_2)\right] \qquad (4)$$

and for the BC at x = L to find

$$-k(C_1 + 0) = h_2\left[(C_1 L + C_2) - T_{\infty,2}\right]. \qquad (5)$$

Multiply Eq. (4) by h_2 and Eq. (5) by h_1, and add the equations to obtain C_1. Then substitute C_1 into Eq. (4) to obtain C_2. The results are

$$C_1 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{k\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} \qquad C_2 = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{h_1\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]} + T_{\infty,1}$$

$$T(x) = -\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}\left[\frac{x}{k} + \frac{1}{h_1}\right] + T_{\infty,1}.$$ <

From Fourier's law, the heat flux is a constant and of the form

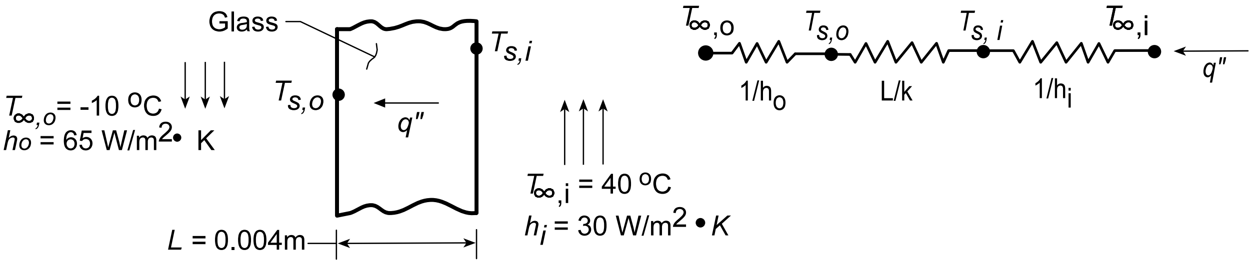
$$q''_x = -k\frac{dT}{dx} = -k\,C_1 = +\frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\left[\frac{1}{h_1} + \frac{1}{h_2} + \frac{L}{k}\right]}.$$ <

PROBLEM 3.2

KNOWN: Temperatures and convection coefficients associated with air at the inner and outer surfaces of a rear window.

FIND: (a) Inner and outer window surface temperatures, $T_{s,i}$ and $T_{s,o}$, and (b) $T_{s,i}$ and $T_{s,o}$ as a function of the outside air temperature $T_{\infty,o}$ and for selected values of outer convection coefficient, h_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation effects, (4) Constant properties.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The heat flux may be obtained from Eqs. 3.11 and 3.12,

$$q'' = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}} = \frac{40^\circ C - \left(-10^\circ C\right)}{\frac{1}{65\,W/m^2 \cdot K} + \frac{0.004\,m}{1.4\,W/m \cdot K} + \frac{1}{30\,W/m^2 \cdot K}}$$

$$q'' = \frac{50^\circ C}{\left(0.0154 + 0.0029 + 0.0333\right) m^2 \cdot K/W} = 968\,W/m^2 .$$

Hence, with $q'' = h_i\left(T_{\infty,i} - T_{\infty,o}\right)$, the inner surface temperature is

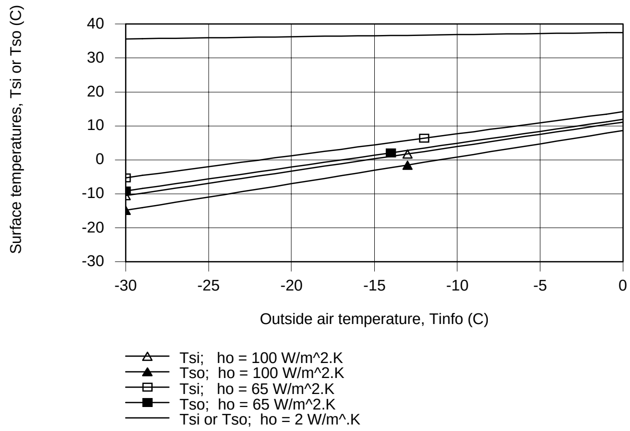
$$T_{s,i} = T_{\infty,i} - \frac{q''}{h_i} = 40^\circ C - \frac{968\,W/m^2}{30\,W/m^2 \cdot K} = 7.7^\circ C$$ <

Similarly for the outer surface temperature with $q'' = h_o\left(T_{s,o} - T_{\infty,o}\right)$ find

$$T_{s,o} = T_{\infty,o} - \frac{q''}{h_o} = -10^\circ C - \frac{968\,W/m^2}{65\,W/m^2 \cdot K} = 4.9^\circ C$$ <

(b) Using the same analysis, $T_{s,i}$ and $T_{s,o}$ have been computed and plotted as a function of the outside air temperature, $T_{\infty,o}$, for outer convection coefficients of h_o = 2, 65, and 100 W/m^2·K. As expected, $T_{s,i}$ and $T_{s,o}$ are linear with changes in the outside air temperature. The difference between $T_{s,i}$ and $T_{s,o}$ increases with increasing convection coefficient, since the heat flux through the window likewise increases. This difference is larger at lower outside air temperatures for the same reason. Note that with h_o = 2 W/m^2·K, $T_{s,i}$ - $T_{s,o}$, is too small to show on the plot.

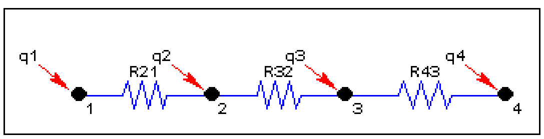
Continued …..



COMMENTS: (1) The largest resistance is that associated with convection at the inner surface. The values of $T_{s,i}$ and $T_{s,o}$ could be increased by increasing the value of h_i.

(2) The *IHT Thermal Resistance Network Model* was used to create a model of the window and generate the above plot. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:




```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 =  Tinfo          // Outside air temperature, C
//q1 =               // Heat rate, W
T2 =  Tso            // Outer surface temperature, C
q2 =  0              // Heat rate, W; node 2, no external heat source
T3 =  Tsi            // Inner surface temperature, C
q3 =  0              // Heat rate, W; node 2, no external heat source
T4 = Tinfi           // Inside air temperature, C
//q4 =               // Heat rate, W
```

// Thermal Resistances:

```
R21 =  1 / ( ho * As )        // Convection thermal resistance, K/W; outer surface
R32 =  L / ( k * As )         // Conduction thermal resistance, K/W; glass
R43 =  1 / ( hi * As )        // Convection thermal resistance, K/W; inner surface
```

// Other Assigned Variables:

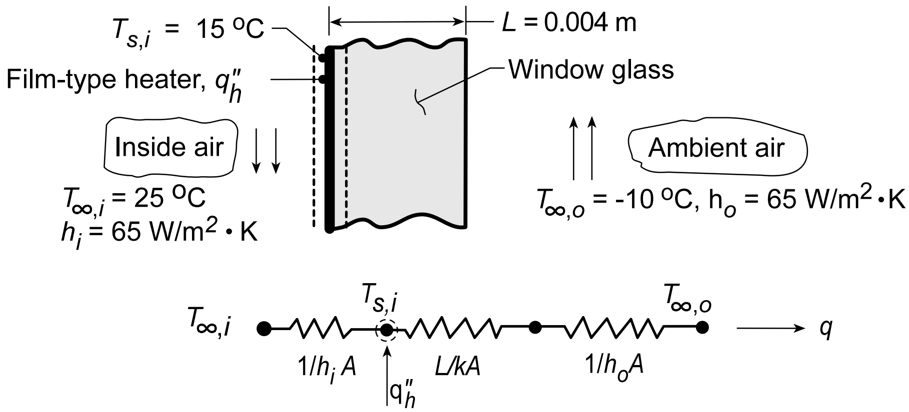
```
Tinfo = -10          // Outside air temperature, C
ho = 65              // Convection coefficient, W/m^2.K; outer surface
L = 0.004            // Thickness, m; glass
k = 1.4              // Thermal conductivity, W/m.K; glass
Tinfi = 40           // Inside air temperature, C
hi = 30              // Convection coefficient, W/m^2.K; inner surface
As = 1               // Cross-sectional area, m^2; unit area
```

PROBLEM 3.3

KNOWN: Desired inner surface temperature of rear window with prescribed inside and outside air conditions.

FIND: (a) Heater power per unit area required to maintain the desired temperature, and (b) Compute and plot the electrical power requirement as a function of $T_{\infty,o}$ for the range $-30 \le T_{\infty,o} \le 0°C$ with h_o of 2, 20, 65 and 100 W/m^2·K. Comment on heater operation needs for low h_o. If $h \sim V^n$, where V is the vehicle speed and n is a positive exponent, how does the vehicle speed affect the need for heater operation?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Uniform heater flux, q''_h, (4) Constant properties, (5) Negligible radiation effects, (6) Negligible film resistance.

PROPERTIES: *Table A-3*, Glass (300 K): k = 1.4 W/m·K.

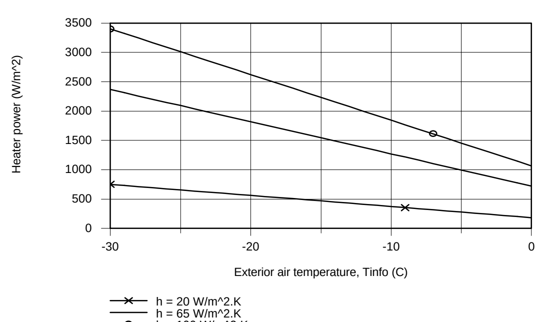
ANALYSIS: (a) From an energy balance at the inner surface and the thermal circuit, it follows that for a unit surface area,

$$\frac{T_{\infty,i} - T_{s,i}}{1/h_i} + q''_h = \frac{T_{s,i} - T_{\infty,o}}{L/k + 1/h_o}$$

$$q''_h = \frac{T_{s,i} - T_{\infty,o}}{L/k + 1/h_o} - \frac{T_{\infty,i} - T_{s,i}}{1/h_i} = \frac{15^\circ C - \left(-10^\circ C\right)}{\dfrac{0.004\,m}{1.4\,W/m \cdot K} + \dfrac{1}{65\,W/m^2 \cdot K}} - \frac{25^\circ C - 15^\circ C}{\dfrac{1}{10\,W/m^2 \cdot K}}$$

$$q''_h = (1370 - 100)\,W/m^2 = 1270\,W/m^2$$ <

(b) The heater electrical power requirement as a function of the exterior air temperature for different exterior convection coefficients is shown in the plot. When h_o = 2 W/m^2·K, the heater is unecessary, since the glass is maintained at 15°C by the interior air. If $h \sim V^n$, we conclude that, with higher vehicle speeds, the exterior convection will increase, requiring increased heat power to maintain the 15°C condition.



COMMENTS: With q''_h = 0, the inner surface temperature with $T_{\infty,o}$ = -10°C would be given by

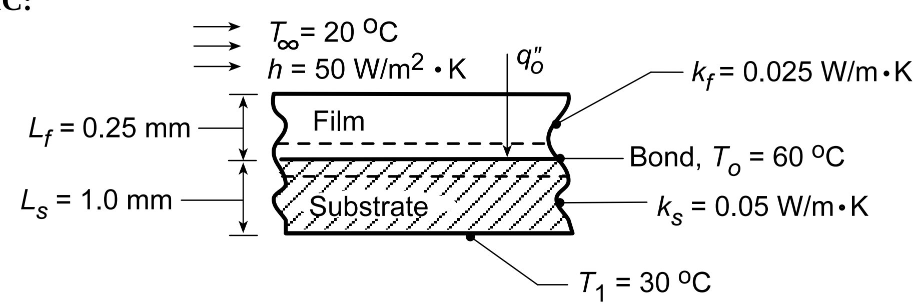
$$\frac{T_{\infty,i} - T_{s,i}}{T_{\infty,i} - T_{\infty,o}} = \frac{1/h_i}{1/h_i + L/k + 1/h_o} = \frac{0.10}{0.118} = 0.846, \qquad \text{or} \qquad T_{s,i} = 25^\circ C - 0.846\left(35^\circ C\right) = -4.6^\circ C .$$

PROBLEM 3.4

KNOWN: Curing of a transparent film by radiant heating with substrate and film surface subjected to known thermal conditions.

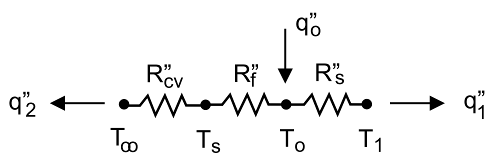
FIND: (a) Thermal circuit for this situation, (b) Radiant heat flux, q_o'' (W/m^2), to maintain bond at curing temperature, T_o, (c) Compute and plot q_o'' as a function of the film thickness for $0 \le L_f \le 1$ mm, and (d) If the film is not transparent, determine q_o'' required to achieve bonding; plot results as a function of L_f.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) All the radiant heat flux q_o'' is absorbed at the bond, (4) Negligible contact resistance.

ANALYSIS: (a) The thermal circuit for this situation is shown at the right. Note that terms are written on a per unit area basis.



(b) Using this circuit and performing an energy balance on the film-substrate interface,

$$q_o'' = q_1'' + q_2'' \qquad q_o'' = \frac{T_o - T_\infty}{R_{cv}'' + R_f''} + \frac{T_o - T_1}{R_s''}$$

where the thermal resistances are

$$R_{cv}'' = 1/h = 1/50\,W/m^2 \cdot K = 0.020\,m^2 \cdot K/W$$

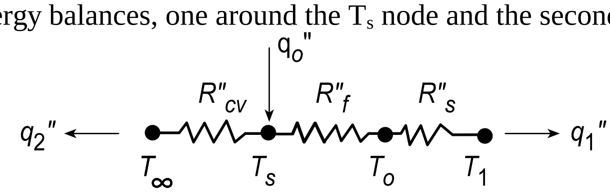
$$R_f'' = L_f/k_f = 0.00025\,m/0.025\,W/m \cdot K = 0.010\,m^2 \cdot K/W$$

$$R_s'' = L_s/k_s = 0.001\,m/0.05\,W/m \cdot K = 0.020\,m^2 \cdot K/W$$

$$q_o'' = \frac{(60-20)^\circ C}{[0.020+0.010]m^2 \cdot K/W} + \frac{(60-30)^\circ C}{0.020\,m^2 \cdot K/W} = (133+1500)\,W/m^2 = 2833\,W/m^2 \qquad <$$

(c) For the transparent film, the radiant flux required to achieve bonding as a function of film thickness L_f is shown in the plot below.

(d) If the film is opaque (not transparent), the thermal circuit is shown below. In order to find q_o'', it is necessary to write two energy balances, one around the T_s node and the second about the T_o node.

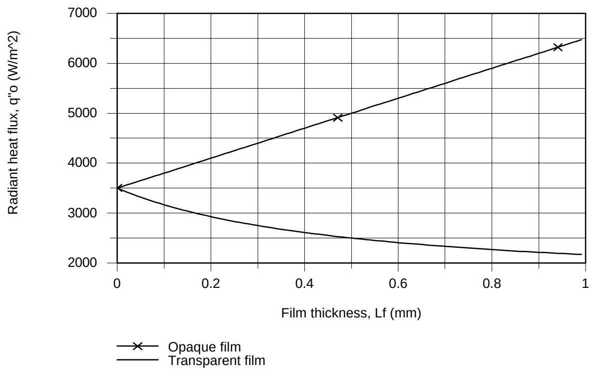


.

The results of the analyses are plotted below.

Continued...

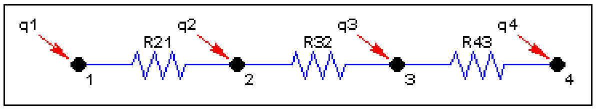
PROBLEM 3.4 (Cont.)



COMMENTS: (1) When the film is transparent, the radiant flux is absorbed on the bond. The flux required decreases with increasing film thickness. Physically, how do you explain this? Why is the relationship not linear?

(2) When the film is opaque, the radiant flux is absorbed on the surface, and the flux required increases with increasing thickness of the film. Physically, how do you explain this? Why is the relationship linear?

(3) The IHT Thermal Resistance Network Model was used to create a model of the film-substrate system and generate the above plot. The Workspace is shown below.



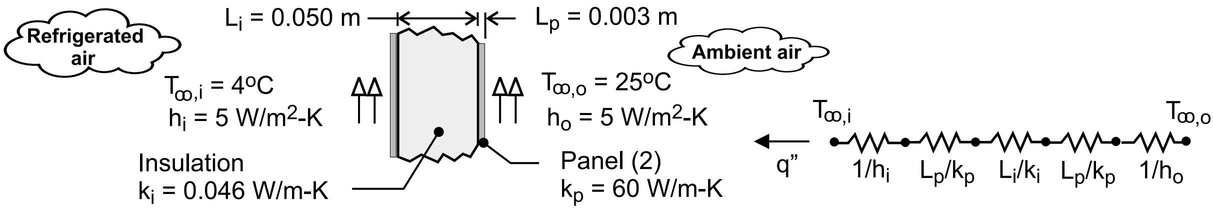

```
// Thermal Resistance Network
Model:
// The Network:

// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32
q43 = (T4 - T3) / R43

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 + q43 = 0
q4 - q43 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal points
at which there is no external source of heat. */
T1 = Tinf            // Ambient air temperature, C
//q1 =               // Heat rate, W; film side
T2 = Ts              // Film surface temperature, C
q2 = 0               // Radiant flux, W/m^2; zero for part (a)
T3 = To              // Bond temperature, C
q3 = qo              // Radiant flux, W/m^2; part (a)
T4 = Tsub            // Substrate temperature, C
//q4 =               // Heat rate, W; substrate side

// Thermal Resistances:
R21 = 1 / ( h * As )          // Convection resistance, K/W
R32 = Lf / (kf * As)          // Conduction resistance, K/W; film
R43 = Ls / (ks * As)          // Conduction resistance, K/W; substrate

// Other Assigned Variables:
Tinf = 20            // Ambient air temperature, C
h = 50               // Convection coefficient, W/m^2.K
Lf = 0.00025         // Thickness, m; film
kf = 0.025           // Thermal conductivity, W/m.K; film
To = 60              // Cure temperature, C
Ls = 0.001           // Thickness, m; substrate
ks = 0.05            // Thermal conductivity, W/m.K; substrate
Tsub = 30            // Substrate temperature, C
As = 1               // Cross-sectional area, m^2; unit area
```

PROBLEM 3.5

KNOWN: Thicknesses and thermal conductivities of refrigerator wall materials. Inner and outer air temperatures and convection coefficients.

FIND: Heat gain per surface area.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Negligible contact resistance, (4) Negligible radiation, (5) Constant properties.

ANALYSIS: From the thermal circuit, the heat gain per unit surface area is

$$q'' = \frac{T_{\infty,o} - T_{\infty,i}}{(1/h_i) + (L_p/k_p) + (L_i/k_i) + (L_p/k_p) + (1/h_o)}$$

$$q'' = \frac{(25-4)^\circ C}{2\left(1/5\,W/m^2\cdot K\right) + 2\left(0.003m/60\,W/m\cdot K\right) + \left(0.050m/0.046\,W/m\cdot K\right)}$$

$$q'' = \frac{21^\circ C}{(0.4 + 0.0001 + 1.087)\,m^2\cdot K/W} = 14.1\ W/m^2 \qquad <$$

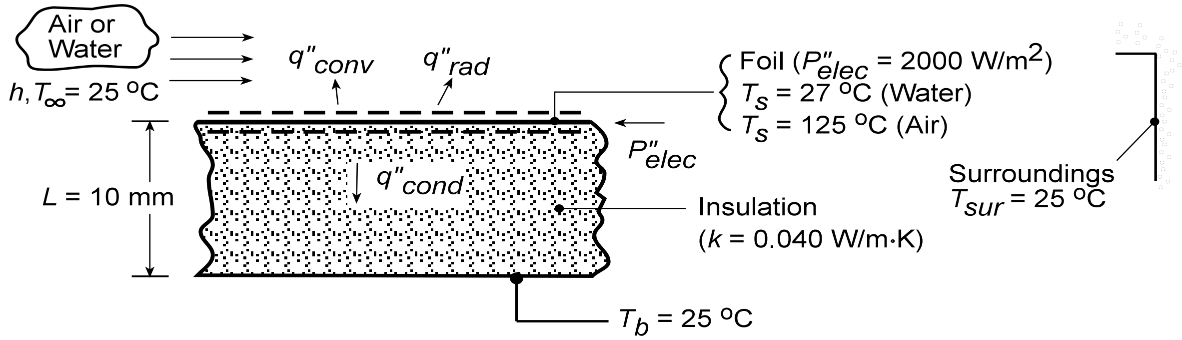
COMMENTS: Although the contribution of the panels to the total thermal resistance is negligible, that due to convection is not inconsequential and is comparable to the thermal resistance of the insulation.

PROBLEM 3.6

KNOWN: Design and operating conditions of a heat flux gage.

FIND: (a) Convection coefficient for water flow (T_s = 27°C) and error associated with neglecting conduction in the insulation, (b) Convection coefficient for air flow (T_s = 125°C) and error associated with neglecting conduction and radiation, (c) Effect of convection coefficient on error associated with neglecting conduction for T_s = 27°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant k.

ANALYSIS: (a) The electric power dissipation is balanced by convection to the water and conduction through the insulation. An energy balance applied to a control surface about the foil therefore yields

$$P''_{elec} = q''_{conv} + q''_{cond} = h\left(T_s - T_\infty\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - k\left(T_s - T_b\right)/L}{T_s - T_\infty} = \frac{2000\,\mathrm{W/m^2} - 0.04\,\mathrm{W/m\cdot K}\left(2\,\mathrm{K}\right)/0.01\,\mathrm{m}}{2\,\mathrm{K}}$$

$$h = \frac{\left(2000 - 8\right)\mathrm{W/m^2}}{2\,\mathrm{K}} = 996\,\mathrm{W/m^2\cdot K}$$ <

If conduction is neglected, a value of h = 1000 $\mathrm{W/m^2\cdot K}$ is obtained, with an attendant error of (1000 - 996)/996 = 0.40%

(b) In air, energy may also be transferred from the foil surface by radiation, and the energy balance yields

$$P''_{elec} = q''_{conv} + q''_{rad} + q''_{cond} = h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) + k\left(T_s - T_b\right)/L$$

Hence,

$$h = \frac{P''_{elec} - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) - k\left(T_s - T_\infty\right)/L}{T_s - T_\infty}$$

$$= \frac{2000\,\mathrm{W/m^2} - 0.15\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(398^4 - 298^4\right)\mathrm{K^4} - 0.04\,\mathrm{W/m\cdot K}\,(100\,\mathrm{K})/0.01\,\mathrm{m}}{100\,\mathrm{K}}$$

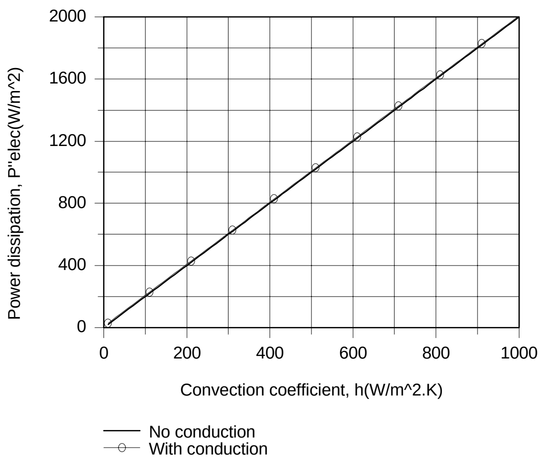
$$= \frac{\left(2000 - 146 - 400\right)\mathrm{W/m^2}}{100\,\mathrm{K}} = 14.5\,\mathrm{W/m^2\cdot K}$$ <

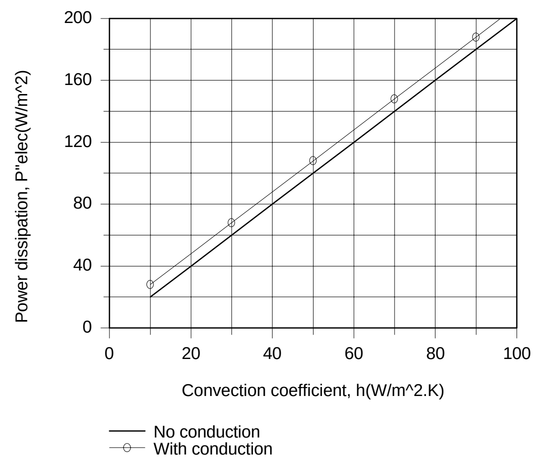
Continued...

PROBLEM 3.6 (Cont.)

If conduction, radiation, or conduction and radiation are neglected, the corresponding values of h and the percentage errors are 18.5 $W/m^2 \cdot K$ (27.6%), 16 $W/m^2 \cdot K$ (10.3%), and 20 $W/m^2 \cdot K$ (37.9%).

(c) For a fixed value of T_s = 27°C, the conduction loss remains at q''_{cond} = 8 W/m^2, which is also the fixed difference between P''_{elec} and q''_{conv}. Although this difference is not clearly shown in the plot for $10 \le h \le 1000$ $W/m^2 \cdot K$, it is revealed in the subplot for $10 \le 100$ $W/m^2 \cdot K$.





Errors associated with neglecting conduction decrease with increasing h from values which are significant for small h (h < 100 $W/m^2 \cdot K$) to values which are negligible for large h.

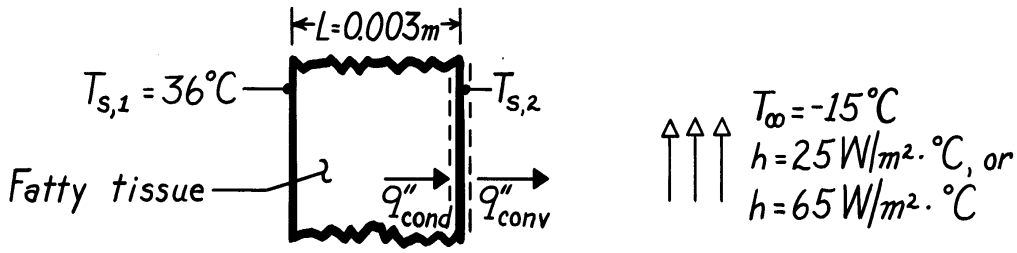
COMMENTS: In liquids (large h), it is an excellent approximation to neglect conduction and assume that all of the dissipated power is transferred to the fluid.

PROBLEM 3.7

KNOWN: A layer of fatty tissue with fixed inside temperature can experience different outside convection conditions.

FIND: (a) Ratio of heat loss for different convection conditions, (b) Outer surface temperature for different convection conditions, and (c) Temperature of still air which achieves same cooling as moving air (*wind chill* effect).

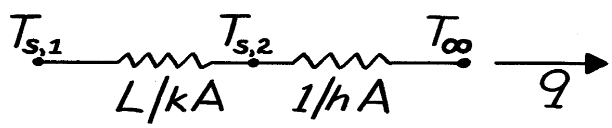
SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) Homogeneous medium with constant properties, (4) No internal heat generation (metabolic effects are negligible), (5) Negligible radiation effects.

PROPERTIES: *Table A-3*, Tissue, fat layer: k = 0.2 W/m·K.

ANALYSIS: The thermal circuit for this situation is



Hence, the heat rate is

$$q = \frac{T_{s,1} - T_\infty}{R_{tot}} = \frac{T_{s,1} - T_\infty}{L/kA + 1/hA}.$$

Therefore,

$$\frac{q''_{calm}}{q''_{windy}} = \frac{\left[\frac{L}{k} + \frac{1}{h}\right]_{windy}}{\left[\frac{L}{k} + \frac{1}{h}\right]_{calm}}.$$

Applying a surface energy balance to the outer surface, it also follows that

$$q''_{cond} = q''_{conv}.$$

Continued …..

PROBLEM 3.7 (Cont.)

Hence,

$$\frac{k}{L}\left(T_{s,1}-T_{s,2}\right)=h\left(T_{s,2}-T_{\infty}\right)$$

$$T_{s,2}=\frac{T_{\infty}+\frac{k}{hL}T_{s,1}}{1+\frac{k}{hL}}.$$

To determine the wind chill effect, we must determine the heat loss for the windy day and use it to evaluate the hypothetical ambient air temperature, T_{∞}', which would provide the same heat loss on a calm day, Hence,

$$q''=\frac{T_{s,1}-T_{\infty}}{\left[\frac{L}{k}+\frac{1}{h}\right]_{windy}}=\frac{T_{s,1}-T_{\infty}'}{\left[\frac{L}{k}+\frac{1}{h}\right]_{calm}}$$

From these relations, we can now find the results sought:

(a)

$$\frac{q''_{calm}}{q''_{windy}}=\frac{\frac{0.003\ \text{m}}{0.2\ \text{W/m}\cdot\text{K}}+\frac{1}{65\ \text{W/m}^2\cdot\text{K}}}{\frac{0.003\ \text{m}}{0.2\ \text{W/m}\cdot\text{K}}+\frac{1}{25\ \text{W/m}^2\cdot\text{K}}}=\frac{0.015+0.0154}{0.015+0.04}$$

$$\frac{q''_{calm}}{q''_{windy}}=0.553$$

<

(b)

$$T_{s,2}\Big]_{calm}=\frac{-15^{\circ}\text{C}+\frac{0.2\ \text{W/m}\cdot\text{K}}{\left(25\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\ \text{m}\right)}36^{\circ}\text{C}}{1+\frac{0.2\ \text{W/m}\cdot\text{K}}{\left(25\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\ \text{m}\right)}}=22.1^{\circ}\text{C}$$

<

$$T_{s,2}\Big]_{windy}=\frac{-15^{\circ}\text{C}+\frac{0.2\ \text{W/m}\cdot\text{K}}{\left(65\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}36^{\circ}\text{C}}{1+\frac{0.2\ \text{W/m}\cdot\text{K}}{\left(65\ \text{W/m}^2\cdot\text{K}\right)\left(0.003\text{m}\right)}}=10.8^{\circ}\text{C}$$

<

(c)

$$T_{\infty}'=36^{\circ}\text{C}-\left(36+15\right)^{\circ}\text{C}\frac{\left(0.003/0.2+1/25\right)}{\left(0.003/0.2+1/65\right)}=-56.3^{\circ}\text{C}$$

<

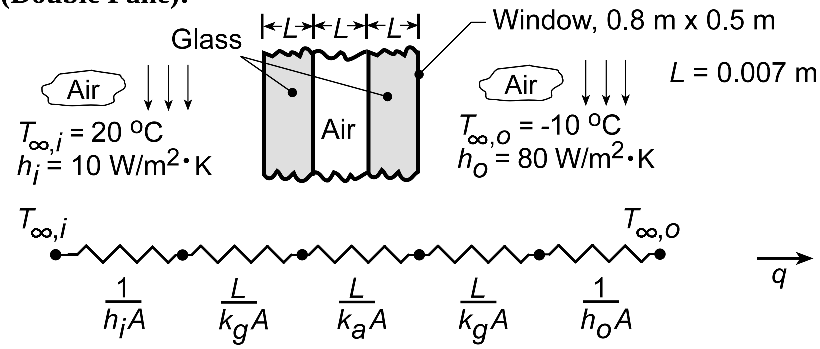
COMMENTS: The wind chill effect is equivalent to a decrease of $T_{s,2}$ by 11.3°C and increase in the heat loss by a factor of $(0.553)^{-1}$ = 1.81.

PROBLEM 3.8

KNOWN: Dimensions of a thermopane window. Room and ambient air conditions.

FIND: (a) Heat loss through window, (b) Effect of variation in outside convection coefficient for double and triple pane construction.

SCHEMATIC (Double Pane):



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible radiation effects, (5) Air between glass is stagnant.

PROPERTIES: *Table A-3*, Glass (300 K): k_g = 1.4 W/m·K; *Table A-4*, Air (T = 278 K): k_a = 0.0245 W/m·K.

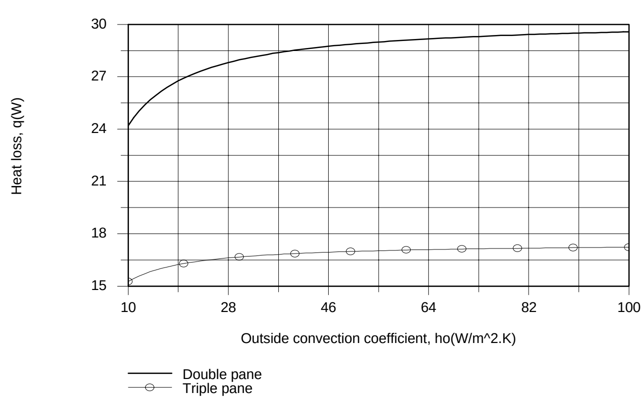
ANALYSIS: (a) From the thermal circuit, the heat loss is

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\frac{1}{A}\left(\frac{1}{h_i} + \frac{L}{k_g} + \frac{L}{k_a} + \frac{L}{k_g} + \frac{1}{h_o}\right)}$$

$$q = \frac{20^\circ C - \left(-10^\circ C\right)}{\left(\frac{1}{0.4\,m^2}\right)\left(\frac{1}{10\,W/m^2\cdot K} + \frac{0.007\,m}{1.4\,W/m\cdot K} + \frac{0.007\,m}{0.0245\,W/m\cdot K} + \frac{0.007\,m}{1.4\,W/m\cdot K} + \frac{1}{80\,W/m^2\cdot K}\right)}$$

$$q = \frac{30^\circ C}{(0.25 + 0.0125 + 0.715 + 0.0125 + 0.03125)\,K/W} = \frac{30^\circ C}{1.021\,K/W} = 29.4\ W$$ <

(b) For the triple pane window, the additional pane and airspace increase the total resistance from 1.021 K/W to 1.749 K/W, thereby reducing the heat loss from 29.4 to 17.2 W. The effect of h_o on the heat loss is plotted as follows.



Continued...

PROBLEM 3.8 (Cont.)

Changes in h_o influence the heat loss at small values of h_o, for which the outside convection resistance is not negligible relative to the total resistance. However, the resistance becomes negligible with increasing h_o, particularly for the triple pane window, and changes in h_o have little effect on the heat loss.

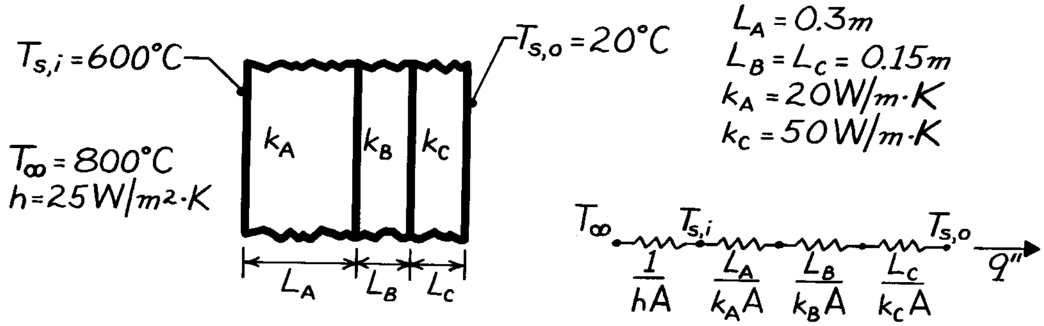
COMMENTS: The largest contribution to the thermal resistance is due to conduction across the enclosed air. Note that this air could be in motion due to free convection currents. If the corresponding convection coefficient exceeded 3.5 $W/m^2 \cdot K$, the thermal resistance would be less than that predicted by assuming conduction across stagnant air.

PROBLEM 3.9

KNOWN: Thicknesses of three materials which form a composite wall and thermal conductivities of two of the materials. Inner and outer surface temperatures of the composite; also, temperature and convection coefficient associated with adjoining gas.

FIND: Value of unknown thermal conductivity, k_B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation effects.

ANALYSIS: Referring to the thermal circuit, the heat flux may be expressed as

$$q'' = \frac{T_{s,i} - T_{s,o}}{\frac{L_A}{k_A} + \frac{L_B}{k_B} + \frac{L_C}{k_C}} = \frac{(600-20)^\circ \text{C}}{\frac{0.3\text{ m}}{20\text{ W/m}\cdot\text{K}} + \frac{0.15\text{ m}}{k_B} + \frac{0.15\text{ m}}{50\text{ W/m}\cdot\text{K}}}$$

$$q'' = \frac{580}{0.018 + 0.15/k_B}\text{ W/m}^2. \qquad (1)$$

The heat flux may be obtained from

$$q'' = h\left(T_\infty - T_{s,i}\right) = 25\text{ W/m}^2\cdot\text{K}\,(800\text{-}600)^\circ\text{C} \qquad (2)$$

$$q'' = 5000\text{ W/m}^2.$$

Substituting for the heat flux from Eq. (2) into Eq. (1), find

$$\frac{0.15}{k_B} = \frac{580}{q''} - 0.018 = \frac{580}{5000} - 0.018 = 0.098$$

$$k_B = 1.53\text{ W/m}\cdot\text{K}. \qquad <$$

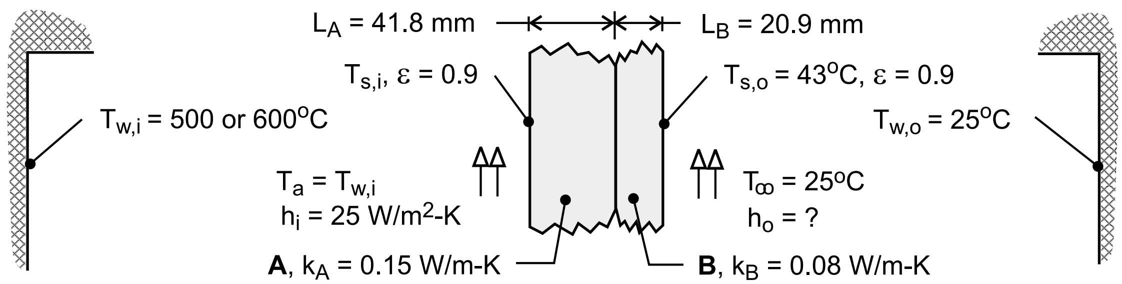
COMMENTS: Radiation effects are likely to have a significant influence on the net heat flux at the inner surface of the oven.

PROBLEM 3.10

KNOWN: Properties and dimensions of a composite oven window providing an outer surface safe-to-touch temperature $T_{s,o}$ = 43°C with outer convection coefficient h_o = 30 W/m^2·K and ε = 0.9 when the oven wall air temperatures are $T_w = T_a$ = 400°C. See Example 3.1.

FIND: Values of the outer convection coefficient h_o required to maintain the safe-to-touch condition when the oven wall-air temperature is raised to 500°C or 600°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in window with no contact resistance and constant properties, (3) Negligible absorption in window material, (4) Radiation exchange processes are between small surface and large isothermal surroundings.

ANALYSIS: From the analysis in the Ex. 3.1 Comment 2, the surface energy balances at the inner and outer surfaces are used to determine the required value of h_o when $T_{s,o}$ = 43°C and $T_{w,i} = T_a$ = 500 or 600°C.

$$\varepsilon\sigma\left(T_{w,i}^4 - T_{s,i}^4\right) + h_i\left(T_a - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\left(L_A / k_A\right) + \left(L_B / k_B\right)}$$

$$\frac{T_{s,i} - T_{s,o}}{\left(L_A / k_A\right) + \left(L_B / k_B\right)} = \varepsilon\sigma\left(T_{s,o}^4 - T_{w,o}^4\right) + h_o\left(T_{s,o} - T_\infty\right)$$

Using these relations in IHT, the following results were calculated:

$T_{w,i}$, T_s(°C)	$T_{s,i}$(°C)	h_o(W/m^2·K)
400	392	30
500	493	40.4
600	594	50.7

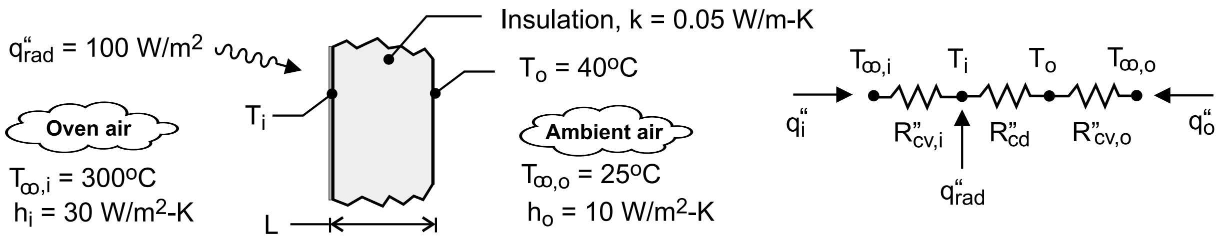
COMMENTS: Note that the window inner surface temperature is closer to the oven air-wall temperature as the outer convection coefficient increases. Why is this so?

PROBLEM 3.11

KNOWN: Drying oven wall having material with known thermal conductivity sandwiched between thin metal sheets. Radiation and convection conditions prescribed on inner surface; convection conditions on outer surface.

FIND: (a) Thermal circuit representing wall and processes and (b) Insulation thickness required to maintain outer wall surface at $T_o = 40°C$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Thermal resistance of metal sheets negligible.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) Perform energy balances on the i- and o- nodes finding

$$\frac{T_{\infty,i} - T_i}{R''_{cv,i}} + \frac{T_o - T_i}{R''_{cd}} + q''_{rad} = 0 \qquad (1)$$

$$\frac{T_i - T_o}{R''_{cd}} + \frac{T_{\infty,o} - T_o}{R''_{cv,o}} = 0 \qquad (2)$$

where the thermal resistances are

$$R''_{cv,i} = 1/h_i = 0.0333\ m^2 \cdot K/W \qquad (3)$$

$$R''_{cd} = L/k = L/0.05\ m^2 \cdot K/W \qquad (4)$$

$$R''_{cv,o} = 1/h_o = 0.0100\ m^2 \cdot K/W \qquad (5)$$

Substituting numerical values, and solving Eqs. (1) and (2) simultaneously, find

$$L = 86\ mm$$ <

COMMENTS: (1) The temperature at the inner surface can be found from an energy balance on the i-node using the value found for L.

$$\frac{T_{\infty,i} - T_i}{R''_{cv,o}} + \frac{T_{\infty,o} - T_i}{R''_{cd} + R''_{cv,i}} + q''_{rad} = 0 \qquad T_i = 298.3°C$$

It follows that T_i is close to $T_{\infty,i}$ since the wall represents the dominant resistance of the system.

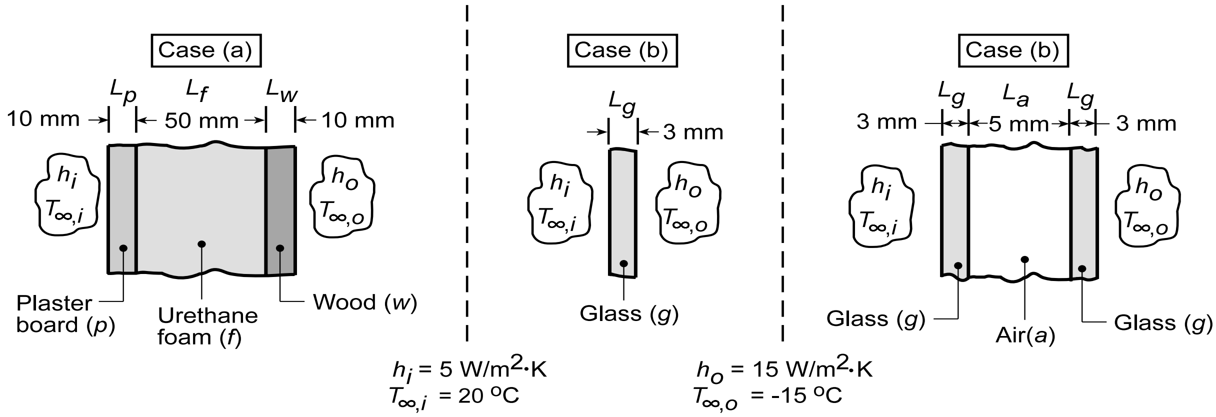
(2) Verify that $q''_i = 50\ W/m^2$ and $q''_o = 150\ W/m^2$. Is the overall energy balance on the system satisfied?

PROBLEM 3.12

KNOWN: Configurations of exterior wall. Inner and outer surface conditions.

FIND: Heating load for each of the three cases.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: (T = 300 K): *Table A.3*: plaster board, k_p = 0.17 W/m·K; urethane, k_f = 0.026 W/m·K; wood, k_w = 0.12 W/m·K; glass, k_g = 1.4 W/m·K. *Table A.4*: air, k_a = 0.0263 W/m·K.

ANALYSIS: (a) The heat loss may be obtained by dividing the overall temperature difference by the total thermal resistance. For the composite wall of unit surface area, A = 1 m^2,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[(1/h_i) + (L_p/k_p) + (L_f/k_f) + (L_w/k_w) + (1/h_o)\right]/A}$$

$$q = \frac{20^\circ C - (-15^\circ C)}{\left[(0.2 + 0.059 + 1.92 + 0.083 + 0.067)\,m^2 \cdot K/W\right]/1\,m^2}$$

$$q = \frac{35^\circ C}{2.33\,K/W} = 15.0\,W$$ <

(b) For the single pane of glass,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[(1/h_i) + (L_g/k_g) + (1/h_o)\right]/A}$$

$$q = \frac{35^\circ C}{\left[(0.2 + 0.002 + 0.067)\,m^2 \cdot K/W\right]/1\,m^2} = \frac{35^\circ C}{0.269\,K/W} = 130.3\,W$$ <

(c) For the double pane window,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{\left[(1/h_i) + 2(L_g/k_g) + (L_a/k_a) + (1/h_o)\right]/A}$$

$$q = \frac{35^\circ C}{\left[(0.2 + 0.004 + 0.190 + 0.067)\,m^2 \cdot K/W\right]/1\,m^2} = \frac{35^\circ C}{0.461\,K/W} = 75.9\,W$$ <

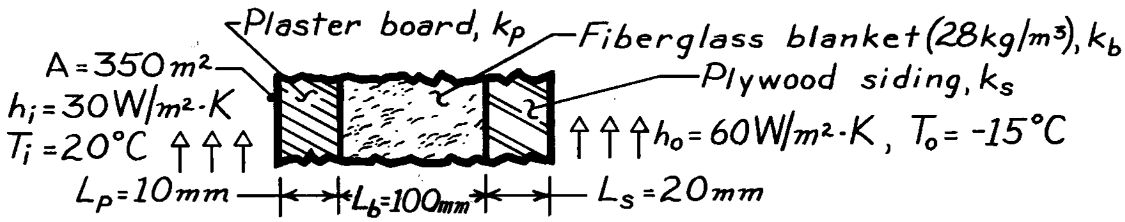
COMMENTS: The composite wall is clearly superior from the standpoint of reducing heat loss, and the dominant contribution to its total thermal resistance (82%) is associated with the foam insulation. Even with double pane construction, heat loss through the window is significantly larger than that for the composite wall.

PROBLEM 3.13

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: (a) Expression for thermal resistance of house wall, R_{tot}; (b) Total heat loss, q(W); (c) Effect on heat loss due to increase in outside heat transfer convection coefficient, h_o; and (d) Controlling resistance for heat loss from house.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Negligible contact resistance.

PROPERTIES: *Table A-3*, $\left(\bar{T} = (T_i + T_o)/2 = (20-15)^\circ C/2 = 2.5^\circ C \approx 300K\right)$: Fiberglass blanket, 28 kg/m^3, k_b = 0.038 W/m·K; Plywood siding, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: (a) The expression for the total thermal resistance of the house wall follows from Eq. 3.18.

$$R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}. \qquad <$$

(b) The total heat loss through the house wall is

$$q = \Delta T / R_{tot} = (T_i - T_o)/R_{tot}.$$

Substituting numerical values, find

$$R_{tot} = \frac{1}{30 W/m^2 \cdot K \times 350 m^2} + \frac{0.01m}{0.17 W/m \cdot K \times 350 m^2} + \frac{0.10m}{0.038 W/m \cdot K \times 350 m^2} + \frac{0.02m}{0.12 W/m \cdot K \times 350 m^2} + \frac{1}{60 W/m^2 \cdot K \times 350 m^2}$$

$$R_{tot} = [9.52 + 16.8 + 752 + 47.6 + 4.76] \times 10^{-5}\ ^\circ C/W = 831 \times 10^{-5}\ ^\circ C/W$$

The heat loss is then,

$$q = [20-(-15)]^\circ C / 831 \times 10^{-5}\ ^\circ C/W = 4.21\ kW. \qquad <$$

(c) If h_o changes from 60 to 300 W/m^2·K, $R_o = 1/h_oA$ changes from 4.76 × 10^{-5} °C/W to 0.95 × 10^{-5} °C/W. This reduces R_{tot} to 826 × 10^{-5} °C/W, which is a 0.5% decrease and hence a 0.5% increase in q.

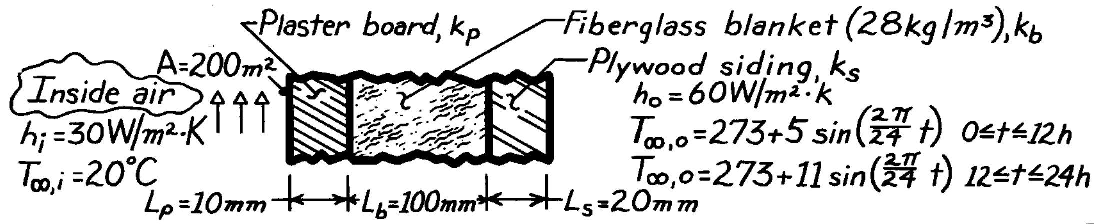
(d) From the expression for R_{tot} in part (b), note that the insulation resistance, L_b/k_bA, is 752/830 ≈ 90% of the total resistance. Hence, this material layer controls the resistance of the wall. From part (c) note that a 5-fold decrease in the outer convection resistance due to an increase in the wind velocity has a negligible effect on the heat loss.

PROBLEM 3.14

KNOWN: Composite wall of a house with prescribed convection processes at inner and outer surfaces.

FIND: Daily heat loss for prescribed diurnal variation in ambient air temperature.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction (negligible change in wall thermal energy storage over 24h period), (2) Negligible contact resistance.

PROPERTIES: *Table A-3*, T ≈ 300 K: Fiberglass blanket (28 kg/m^3), k_b = 0.038 W/m·K; Plywood, k_s = 0.12 W/m·K; Plasterboard, k_p = 0.17 W/m·K.

ANALYSIS: The heat loss may be approximated as $Q=\int_0^{24h}\frac{T_{\infty,i}-T_{\infty,o}}{R_{tot}}dt$ where

$$R_{tot}=\frac{1}{A}\left[\frac{1}{h_i}+\frac{L_p}{k_p}+\frac{L_b}{k_b}+\frac{L_s}{k_s}+\frac{1}{h_o}\right]$$

$$R_{tot}=\frac{1}{200\text{m}^2}\left[\frac{1}{30\ \text{W/m}^2\cdot\text{K}}+\frac{0.01\text{m}}{0.17\ \text{W/m}\cdot\text{K}}+\frac{0.1\text{m}}{0.038\ \text{W/m}\cdot\text{K}}+\frac{0.02\text{m}}{0.12\ \text{W/m}\cdot\text{K}}+\frac{1}{60\ \text{W/m}^2\cdot\text{K}}\right]$$

$$R_{tot}=0.01454\ \text{K/W}.$$

Hence the heat rate is

$$Q=\frac{1}{R_{tot}}\left\{\int_0^{12h}\left[293-\left[273+5\sin\frac{2\pi}{24}t\right]\right]dt+\int_{12}^{24h}\left[293-\left[273+11\sin\frac{2\pi}{24}t\right]\right]dt\right\}$$

$$Q=68.8\frac{\text{W}}{\text{K}}\left\{\left[20t+5\left[\frac{24}{2\pi}\right]\cos\frac{2\pi t}{24}\right]\Bigg|_0^{12}+\left[20t+11\left[\frac{24}{2\pi}\right]\cos\frac{2\pi t}{24}\right]\Bigg|_{12}^{24}\right\}\text{K}\cdot\text{h}$$

$$Q=68.8\left\{\left[240+\frac{60}{\pi}(-1-1)\right]+\left[480-240+\frac{132}{\pi}(1+1)\right]\right\}\text{W}\cdot\text{h}$$

$$Q=68.8\ \{480\text{-}38.2\text{+}84.03\}\ \text{W}\cdot\text{h}$$

$$Q=36.18\ \text{kW}\cdot\text{h}=1.302\times10^8\text{J}.$$ <

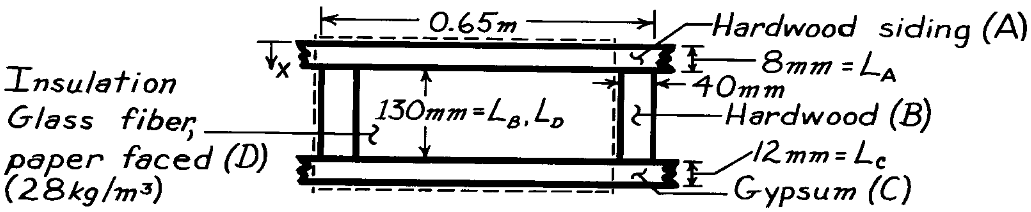
COMMENTS: From knowledge of the fuel cost, the total daily heating bill could be determined. For example, at a cost of 0.10$/kW·h, the heating bill would be $3.62/day.

PROBLEM 3.15

KNOWN: Dimensions and materials associated with a composite wall (2.5m × 6.5m, 10 studs each 2.5m high).

FIND: Wall thermal resistance.

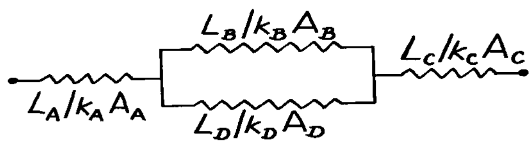
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of composite depends only on x (surfaces normal to x are isothermal), (3) Constant properties, (4) Negligible contact resistance.

PROPERTIES: *Table A-3* (T ≈ 300K): Hardwood siding, k_A = 0.094 W/m·K; Hardwood, k_B = 0.16 W/m·K; Gypsum, k_C = 0.17 W/m·K; Insulation (glass fiber paper faced, 28 kg/m^3), k_D = 0.038 W/m·K.

ANALYSIS: Using the isothermal surface assumption, the thermal circuit associated with a single unit (enclosed by dashed lines) of the wall is



$$\left(L_A / k_A A_A\right) = \frac{0.008\text{m}}{0.094\ \text{W/m}\cdot\text{K}\ \left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0524\ \text{K/W}$$

$$\left(L_B / k_B A_B\right) = \frac{0.13\text{m}}{0.16\ \text{W/m}\cdot\text{K}\ \left(0.04\text{m}\times 2.5\text{m}\right)} = 8.125\ \text{K/W}$$

$$\left(L_D / k_D A_D\right) = \frac{0.13\text{m}}{0.038\ \text{W/m}\cdot\text{K}\left(0.61\text{m}\times 2.5\text{m}\right)} = 2.243\ \text{K/W}$$

$$\left(L_C / k_C A_C\right) = \frac{0.012\text{m}}{0.17\ \text{W/m}\cdot\text{K}\left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0434\ \text{K/W}.$$

The equivalent resistance of the core is

$$R_{eq} = \left(1/R_B + 1/R_D\right)^{-1} = \left(1/8.125 + 1/2.243\right)^{-1} = 1.758\ \text{K/W}$$

and the total unit resistance is

$$R_{tot,1} = R_A + R_{eq} + R_C = 1.854\ \text{K/W}.$$

With 10 such units in parallel, the total wall resistance is

$$R_{tot} = \left(10\times 1/R_{tot,1}\right)^{-1} = 0.1854\ \text{K/W}.$$ <

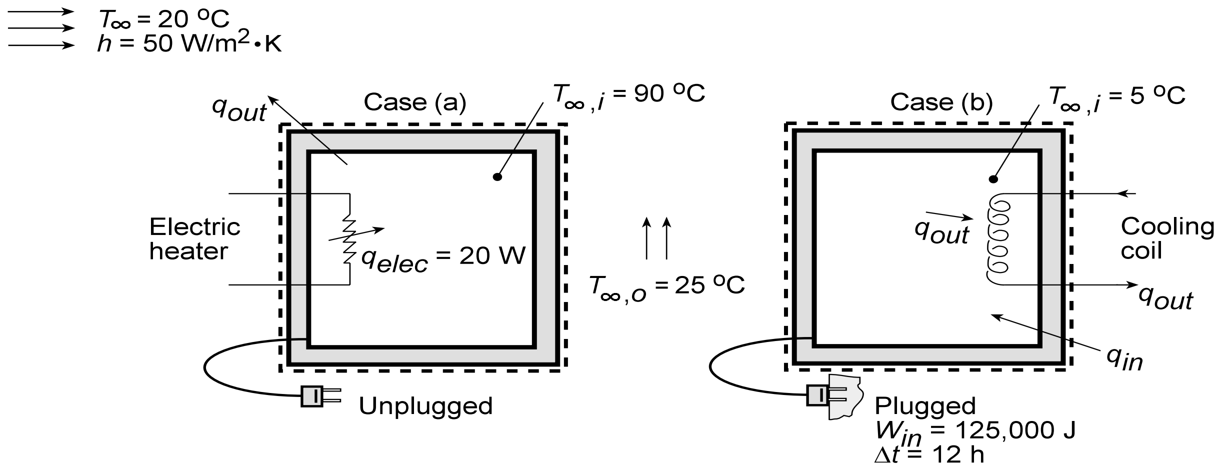
COMMENTS: If surfaces parallel to the heat flow direction are assumed adiabatic, the thermal circuit and the value of R_{tot} will differ.

PROBLEM 3.16

KNOWN: Conditions associated with maintaining heated and cooled conditions within a refrigerator compartment.

FIND: Coefficient of performance (COP).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state operating conditions, (2) Negligible radiation, (3) Compartment completely sealed from ambient air.

ANALYSIS: The Case (a) experiment is performed to determine the overall thermal resistance to heat transfer between the interior of the refrigerator and the ambient air. Applying an energy balance to a control surface about the refrigerator, it follows from Eq. 1.11a that, at any instant,

$$\dot{E}_g - \dot{E}_{out} = 0$$

Hence,

$$q_{elec} - q_{out} = 0$$

where $q_{out} = \left(T_{\infty,i} - T_{\infty,o}\right)/R_t$. It follows that

$$R_t = \frac{T_{\infty,i} - T_{\infty,o}}{q_{elec}} = \frac{(90-25)^\circ\,C}{20\,W} = 3.25^\circ C/W$$

For Case (b), heat transfer from the ambient air to the compartment (the heat load) is balanced by heat transfer to the refrigerant ($q_{in} = q_{out}$). Hence, the thermal energy transferred from the refrigerator over the 12 hour period is

$$Q_{out} = q_{out}\Delta t = q_{in}\Delta t = \frac{T_{\infty,i} - T_{\infty,o}}{R_t}\Delta t$$

$$Q_{out} = \frac{(25-5)^\circ\,C}{3.25^\circ\,C/W}\left(12\,h \times 3600\,s/h\right) = 266,000\,J$$

The coefficient of performance (COP) is therefore

$$COP = \frac{Q_{out}}{W_{in}} = \frac{266,000}{125,000} = 2.13$$ <

COMMENTS: The ideal (Carnot) COP is

$$COP)_{ideal} = \frac{T_c}{T_h - T_c} = \frac{278\,K}{(298-278)\,K} = 13.9$$

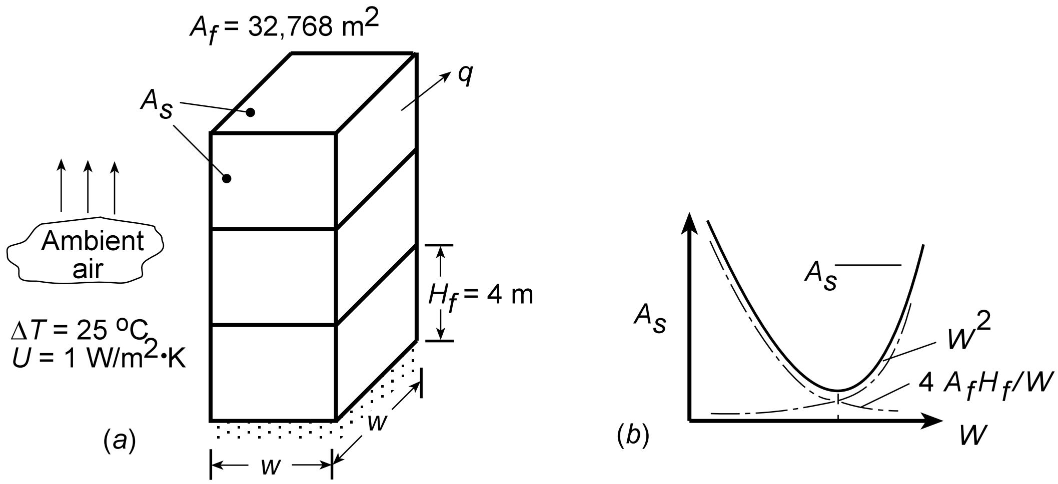
and the system is operating well below its peak possible performance.

PROBLEM 3.17

KNOWN: Total floor space and vertical distance between floors for a square, flat roof building.

FIND: (a) Expression for width of building which minimizes heat loss, (b) Width and number of floors which minimize heat loss for a prescribed floor space and distance between floors. Corresponding heat loss, percent heat loss reduction from 2 floors.

SCHEMATIC:



ASSUMPTIONS: Negligible heat loss to ground.

ANALYSIS: (a) To minimize the heat loss q, the exterior surface area, A_s, must be minimized. From Fig. (a)

$$A_s = W^2 + 4WH = W^2 + 4WN_f H_f$$

where

$$N_f = A_f / W^2$$

Hence,

$$A_s = W^2 + 4WA_f\ H_f / W^2 = W^2 + 4A_f\ H_f / W$$

The optimum value of W corresponds to

$$\frac{dA_s}{dW} = 2W - \frac{4A_f H_f}{W^2} = 0$$

or

$$W_{op} = (2A_f H_f)^{1/3} \qquad <$$

The competing effects of W on the areas of the roof and sidewalls, and hence the basis for an optimum, is shown schematically in Fig. (b).

(b) For A_f = 32,768 m^2 and H_f = 4 m,

$$W_{op} = \left(2 \times 32,768\,m^2 \times 4\,m\right)^{1/3} = 64\,m \qquad <$$

Continued …..

PROBLEM 3.17 (Cont.)

Hence,

$$N_f = \frac{A_f}{W^2} = \frac{32,768\,\text{m}^2}{(64\,\text{m})^2} = 8$$ <

and

$$q = UA_s\Delta T = 1\,\text{W/m}^2\cdot\text{K}\left[(64\,\text{m})^2 + \frac{4\times 32,768\,\text{m}^2\times 4\,\text{m}}{64\,\text{m}}\right]25^\circ\text{C} = 307,200\,\text{W}$$ <

For $N_f = 2$,

$$W = (A_f/N_f)^{1/2} = (32,768\ \text{m}^2/2)^{1/2} = 128\ \text{m}$$

$$q = 1\,\text{W/m}^2\cdot\text{K}\left[(128\,\text{m})^2 + \frac{4\times 32,768\,\text{m}^2\times 4\,\text{m}}{128\,\text{m}}\right]25^\circ\text{C} = 512,000\,\text{W}$$

% reduction in q = (512,000 - 307,200)/512,000 = 40% <

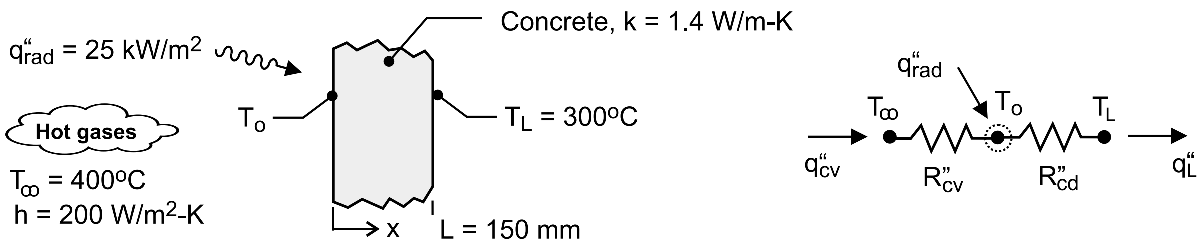
COMMENTS: Even the minimum heat loss is excessive and could be reduced by reducing U.

PROBLEM 3.18

KNOWN: Concrete wall of 150 mm thickness experiences a flash-over fire with prescribed radiant flux and hot-gas convection on the fire-side of the wall. Exterior surface condition is 300°C, typical ignition temperature for most household and office materials.

FIND: (a) Thermal circuit representing wall and processes and (b) Temperature at the fire-side of the wall; comment on whether wall is likely to experience structural collapse for these conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in wall, (3) Constant properties.

PROPERTIES: *Table A-3*, Concrete (stone mix, 300 K): k = 1.4 W/m·K.

ANALYSIS: (a) The thermal cirucit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

(b) To determine the fire-side wall surface temperatures, perform an energy balance on the o-node.

$$\frac{T_\infty - T_o}{R''_{cv}} + q''_{rad} = \frac{T_L - T_o}{R''_{cd}}$$

where the thermal resistances are

$$R''_{cv} = 1/h_i = 1/200\ \mathrm{W/m^2 \cdot K} = 0.00500\ \mathrm{m^2 \cdot K/W}$$

$$R''_{cd} = L/k = 0.150\ \mathrm{m}/1.4\,\mathrm{W/m \cdot K} = 0.107\ \mathrm{m^2 \cdot K/W}$$

Substituting numerical values,

$$\frac{(400 - T_o)\mathrm{K}}{0.005\ \mathrm{m^2 \cdot K/W}} + 25,000\ \mathrm{W/m^2}\ \frac{(300 - T_o)\mathrm{K}}{0.107\ \mathrm{m^2 \cdot K/W}} = 0$$

$$T_o = 515°\mathrm{C}$$ <

COMMENTS: (1) The fire-side wall surface temperature is within the 350 to 600°C range for which explosive spalling could occur. It is likely the wall will experience structural collapse for these conditions.

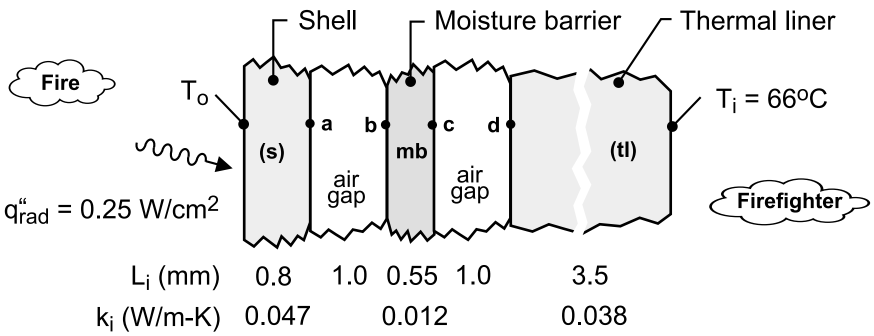
(2) This steady-state condition is an extreme condition, as the wall may fail before near steady-state conditions can be met.

PROBLEM 3.19

KNOWN: Representative dimensions and thermal conductivities for the layers of fire-fighter's protective clothing, a turnout coat.

FIND: (a) Thermal circuit representing the turnout coat; tabulate thermal resistances of the layers and processes; and (b) For a prescribed radiant heat flux on the fire-side surface and temperature of T_i =.60°C at the inner surface, calculate the fire-side surface temperature, T_o.

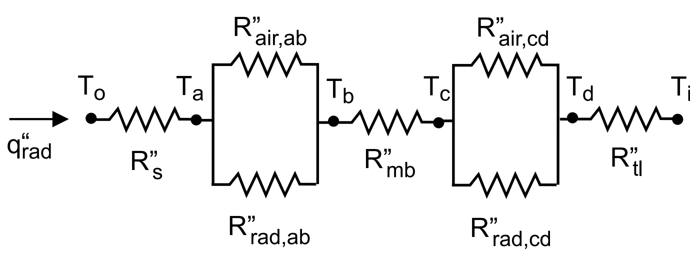
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through the layers, (3) Heat is transferred by conduction and radiation exchange across the stagnant air gaps, (3) Constant properties.

PROPERTIES: *Table A-4*, Air (470 K, 1 atm): $k_{ab} = k_{cd} = 0.0387$ W/m·K.

ANALYSIS: (a) The thermal circuit is shown with labels for the temperatures and thermal resistances.



The conduction thermal resistances have the form $R''_{cd} = L/k$ while the radiation thermal resistances across the air gaps have the form

$$R''_{rad} = \frac{1}{h_{rad}} = \frac{1}{4\sigma T_{avg}^3}$$

The linearized radiation coefficient follows from Eqs. 1.8 and 1.9 with ε = 1 where T_{avg} represents the average temperature of the surfaces comprising the gap

$$h_{rad} = \sigma\left(T_1 + T_2\right)\left(T_1^2 + T_2^2\right) \approx 4\sigma T_{avg}^3$$

For the radiation thermal resistances tabulated below, we used T_{avg} = 470 K.

Continued …..

	Shell (s)	Air gap (a-b)	Barrier (mb)	Air gap (c-d)	Liner (tl)	*Total* (tot)
$R''_{cd}\left(m^2 \cdot K/W\right)$	0.01702	0.0259	0.04583	0.0259	0.00921	--
$R''_{rad}\left(m^2 \cdot K/W\right)$	--	0.04264	--	0.04264	--	--
$R''_{gap}\left(m^2 \cdot K/W\right)$	--	0.01611	--	0.01611	--	--
R''_{total}	--	--	--	--	--	0.1043

From the thermal circuit, the resistance across the gap for the conduction and radiation processes is

$$\frac{1}{R''_{gap}} = \frac{1}{R''_{cd}} + \frac{1}{R''_{rad}}$$

and the total thermal resistance of the turn coat is

$$R''_{tot} = R''_{cd,s} + R''_{gap,a-b} + R''_{cd,mb} + R''_{gap,c-d} + R''_{cd,tl}$$

(b) If the heat flux through the coat is 0.25 W/cm^2, the fire-side surface temperature T_o can be calculated from the rate equation written in terms of the overall thermal resistance.

$$q'' = \left(T_o - T_i\right)/R''_{tot}$$

$$T_o = 66°C + 0.25\ W/cm^2 \times \left(10^2 cm/m\right)^2 \times 0.1043\ m^2 \cdot K/W$$

$$T_o = 327°C$$

COMMENTS: (1) From the tabulated results, note that the thermal resistance of the moisture barrier (mb) is nearly 3 times larger than that for the shell or air gap layers, and 4.5 times larger than the thermal liner layer.

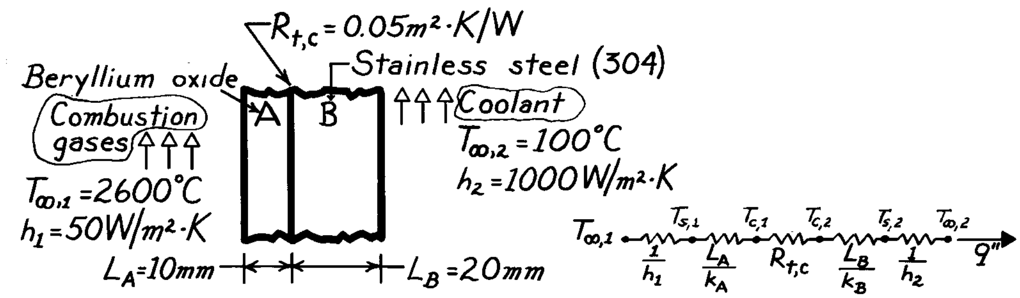
(2) The air gap conduction and radiation resistances were calculated based upon the average temperature of 470 K. This value was determined by setting $T_{avg} = (T_o + T_i)/2$ and solving the equation set using *IHT* with $k_{air} = k_{air}(T_{avg})$.

PROBLEM 3.20

KNOWN: Materials and dimensions of a composite wall separating a combustion gas from a liquid coolant.

FIND: (a) Heat loss per unit area, and (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties, (4) Negligible radiation effects.

PROPERTIES: *Table A-1*, St. St. (304) $(\bar{T} \approx 1000K)$: k = 25.4 W/m·K; *Table A-2*, Beryllium Oxide (T ≈ 1500K): k = 21.5 W/m·K.

ANALYSIS: (a) The desired heat flux may be expressed as

$$q'' = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1} + \frac{L_A}{k_A} + R_{t,c} + \frac{L_B}{k_B} + \frac{1}{h_2}} = \frac{(2600-100)^\circ C}{\left[\frac{1}{50} + \frac{0.01}{21.5} + 0.05 + \frac{0.02}{25.4} + \frac{1}{1000}\right] \frac{m^2.K}{W}}$$

$$q'' = 34,600 \ W/m^2.$$ <

(b) The composite surface temperatures may be obtained by applying appropriate rate equations. From the fact that $q'' = h_1 \left(T_{\infty,1} - T_{s,1}\right)$, it follows that

$$T_{s,1} = T_{\infty,1} - \frac{q''}{h_1} = 2600^\circ C - \frac{34,600 \ W/m^2}{50 \ W/m^2 \cdot K} 1908^\circ C.$$

With $q'' = (k_A / L_A)(T_{s,1} - T_{c,1})$, it also follows that

$$T_{c,1} = T_{s,1} - \frac{L_A q''}{k_A} = 1908^\circ C - \frac{0.01m \times 34,600 \ W/m^2}{21.5 \ W/m \cdot K} = 1892^\circ C.$$

Similarly, with $q'' = \left(T_{c,1} - T_{c,2}\right) / R_{t,c}$

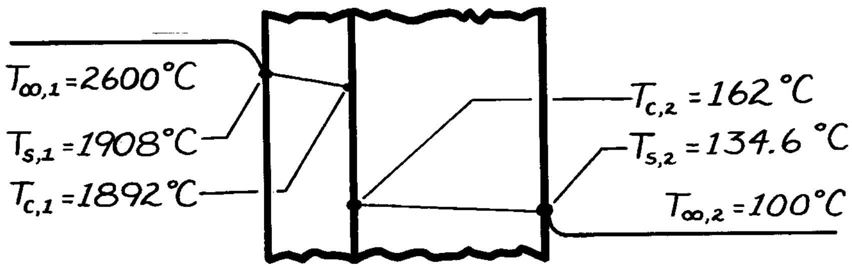
$$T_{c,2} = T_{c,1} - R_{t,c} q'' = 1892^\circ C - 0.05 \frac{m^2 \cdot K}{W} \times 34,600 \frac{W}{m^2} = 162^\circ C$$

Continued …..

and with $q''=(k_B/L_B)(T_{c,2}-T_{s,2})$,

$$T_{s,2}=T_{c,2}-\frac{L_B q''}{k_B}=162^\circ C-\frac{0.02m\times 34,600\ W/m^2}{25.4\ W/m\cdot K}=134.6^\circ C.$$

The temperature distribution is therefore of the following form:



<

COMMENTS: (1) The calculations may be checked by recomputing q'' from

$$q''=h_2\left(T_{s,2}-T_{\infty,2}\right)=1000W/m^2\cdot K\left(134.6\text{-}100\right)^\circ C=34,600W/m^2$$

(2) The initial *estimates* of the mean material temperatures are in error, particularly for the stainless steel. For improved accuracy the calculations should be repeated using k values corresponding to T ≈ 1900°C for the oxide and T ≈ 115°C for the steel.

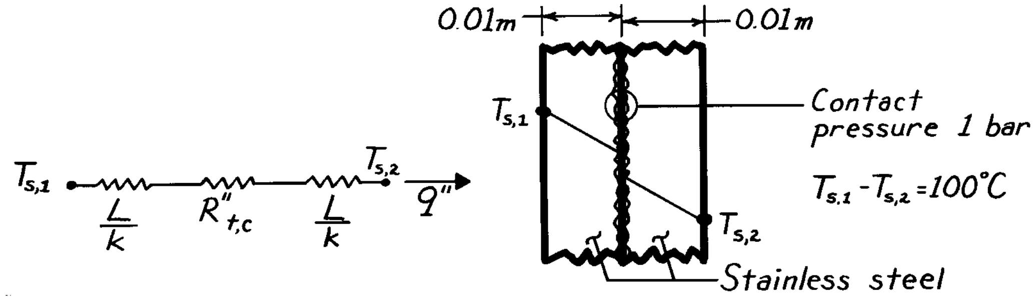
(3) The major contributions to the total resistance are made by the combustion gas boundary layer and the contact, where the temperature drops are largest.

PROBLEM 3.21

KNOWN: Thickness, overall temperature difference, and pressure for two stainless steel plates.

FIND: (a) Heat flux and (b) Contact plane temperature drop.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Constant properties.

PROPERTIES: *Table A-1*, Stainless Steel (T ≈ 400K): k = 16.6 W/m·K.

ANALYSIS: (a) With $R''_{t,c} \approx 15\times10^{-4}\ \mathrm{m^2\cdot K/W}$ from Table 3.1 and

$$\frac{L}{k} = \frac{0.01\mathrm{m}}{16.6\ \mathrm{W/m\cdot K}} = 6.02\times10^{-4}\mathrm{m^2\cdot K/W},$$

it follows that

$$R''_{tot} = 2(L/k) + R''_{t,c} \approx 27\times10^{-4}\mathrm{m^2\cdot K/W};$$

hence

$$q'' = \frac{\Delta T}{R''_{tot}} = \frac{100^\circ\mathrm{C}}{27\times10^{-4}\mathrm{m^2\cdot K/W}} = 3.70\times10^4\,\mathrm{W/m^2}. \quad <$$

(b) From the thermal circuit,

$$\frac{\Delta T_c}{T_{s,1} - T_{s,2}} = \frac{R''_{t,c}}{R''_{tot}} = \frac{15\times10^{-4}\mathrm{m^2\cdot K/W}}{27\times10^{-4}\mathrm{m^2\cdot K/W}} = 0.556.$$

Hence,

$$\Delta T_c = 0.556\left(T_{s,1} - T_{s,2}\right) = 0.556\left(100^\circ\mathrm{C}\right) = 55.6^\circ\mathrm{C}. \quad <$$

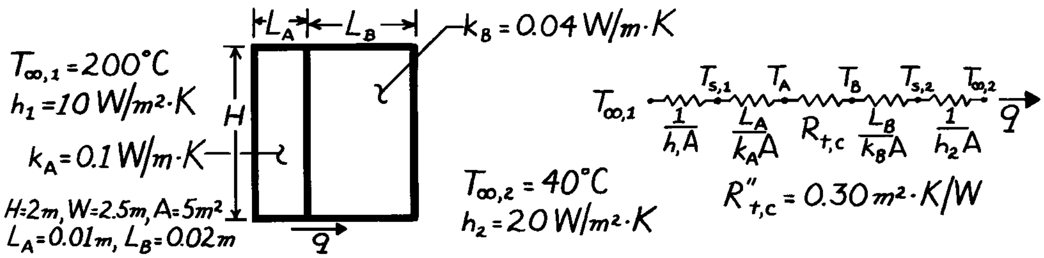
COMMENTS: The contact resistance is significant relative to the conduction resistances. The value of $R''_{t,c}$ would diminish, however, with increasing pressure.

PROBLEM 3.22

KNOWN: Temperatures and convection coefficients associated with fluids at inner and outer surfaces of a composite wall. Contact resistance, dimensions, and thermal conductivities associated with wall materials.

FIND: (a) Rate of heat transfer through the wall, (b) Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible radiation, (4) Constant properties.

ANALYSIS: (a) Calculate the total resistance to find the heat rate,

$$R_{tot} = \frac{1}{h_1A} + \frac{L_A}{k_AA} + R_{t,c} + \frac{L_B}{k_BA} + \frac{1}{h_2A}$$

$$R_{tot} = \left[\frac{1}{10\times5} + \frac{0.01}{0.1\times5} + \frac{0.3}{5} + \frac{0.02}{0.04\times5} + \frac{1}{20\times5}\right]\frac{K}{W}$$

$$R_{tot} = [0.02 + 0.02 + 0.06 + 0.10 + 0.01]\frac{K}{W} = 0.21\frac{K}{W}$$

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} = \frac{(200-40)^\circ C}{0.21\ K/W} = 762\ W.$$ <

(b) It follows that

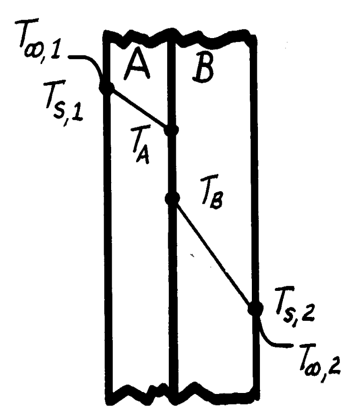
$$T_{s,1} = T_{\infty,1} - \frac{q}{h_1A} = 200^\circ C - \frac{762\ W}{50\ W/K} = 184.8^\circ C$$

$$T_A = T_{s,1} - \frac{qL_A}{k_AA} = 184.8^\circ C - \frac{762 W \times 0.01 m}{0.1 \frac{W}{m\cdot K} \times 5 m^2} = 169.6^\circ C$$

$$T_B = T_A - qR_{t,c} = 169.6^\circ C - 762 W \times 0.06 \frac{K}{W} = 123.8^\circ C$$

$$T_{s,2} = T_B - \frac{qL_B}{k_BA} = 123.8^\circ C - \frac{762 W \times 0.02 m}{0.04 \frac{W}{m\cdot K} \times 5 m^2} = 47.6^\circ C$$

$$T_{\infty,2} = T_{s,2} - \frac{q}{h_2A} = 47.6^\circ C - \frac{762 W}{100 W/K} = 40^\circ C$$

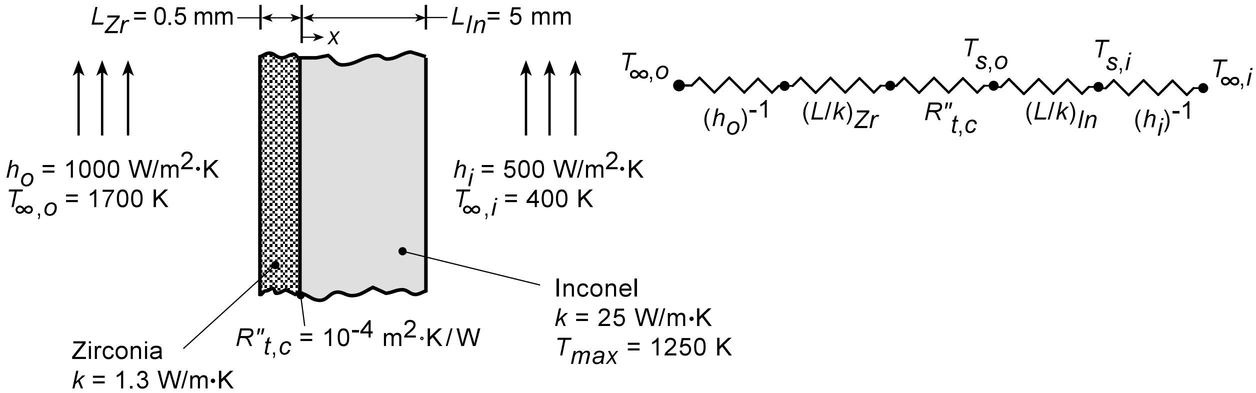


PROBLEM 3.23

KNOWN: Outer and inner surface convection conditions associated with zirconia-coated, Inconel turbine blade. Thicknesses, thermal conductivities, and interfacial resistance of the blade materials. Maximum allowable temperature of Inconel.

FIND: Whether blade operates below maximum temperature. Temperature distribution in blade, with and without the TBC.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in a composite plane wall, (2) Constant properties, (3) Negligible radiation.

ANALYSIS: For a unit area, the total thermal resistance with the TBC is

$$R''_{tot,w} = h_o^{-1} + (L/k)_{Zr} + R''_{t,c} + (L/k)_{In} + h_i^{-1}$$

$$R''_{tot,w} = \left(10^{-3} + 3.85\times10^{-4} + 10^{-4} + 2\times10^{-4} + 2\times10^{-3}\right)\text{m}^2\cdot\text{K/W} = 3.69\times10^{-3}\,\text{m}^2\cdot\text{K/W}$$

With a heat flux of

$$q''_w = \frac{T_{\infty,o} - T_{\infty,i}}{R''_{tot,w}} = \frac{1300\,\text{K}}{3.69\times10^{-3}\,\text{m}^2\cdot\text{K/W}} = 3.52\times10^5\,\text{W/m}^2$$

the inner and outer surface temperatures of the Inconel are

$$T_{s,i(w)} = T_{\infty,i} + (q''_w/h_i) = 400\,\text{K} + \left(3.52\times10^5\,\text{W/m}^2 / 500\,\text{W/m}^2\cdot\text{K}\right) = 1104\,\text{K}$$

$$T_{s,o(w)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right]q''_w = 400\,\text{K} + \left(2\times10^{-3} + 2\times10^{-4}\right)\text{m}^2\cdot\text{K/W}\left(3.52\times10^5\,\text{W/m}^2\right) = 1174\,\text{K}$$

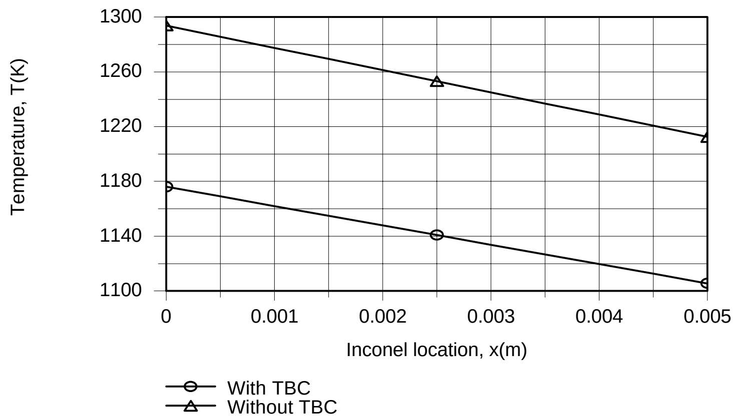
Without the TBC, $R''_{tot,wo} = h_o^{-1} + (L/k)_{In} + h_i^{-1} = 3.20\times10^{-3}\,\text{m}^2\cdot\text{K/W}$, and $q''_{wo} = \left(T_{\infty,o} - T_{\infty,i}\right)/R''_{tot,wo}$ = (1300 K)/3.20×10^{-3} $\text{m}^2\cdot\text{K/W}$ = 4.06×10^5 W/m^2. The inner and outer surface temperatures of the Inconel are then

$$T_{s,i(wo)} = T_{\infty,i} + (q''_{wo}/h_i) = 400\,\text{K} + \left(4.06\times10^5\,\text{W/m}^2 / 500\,\text{W/m}^2\cdot\text{K}\right) = 1212\,\text{K}$$

$$T_{s,o(wo)} = T_{\infty,i} + \left[(1/h_i) + (L/k)_{In}\right]q''_{wo} = 400\,\text{K} + \left(2\times10^{-3} + 2\times10^{-4}\right)\text{m}^2\cdot\text{K/W}\left(4.06\times10^5\,\text{W/m}^2\right) = 1293\,\text{K}$$

Continued...

PROBLEM 3.23 (Cont.)



Use of the TBC facilitates operation of the Inconel below T_{max} = 1250 K.

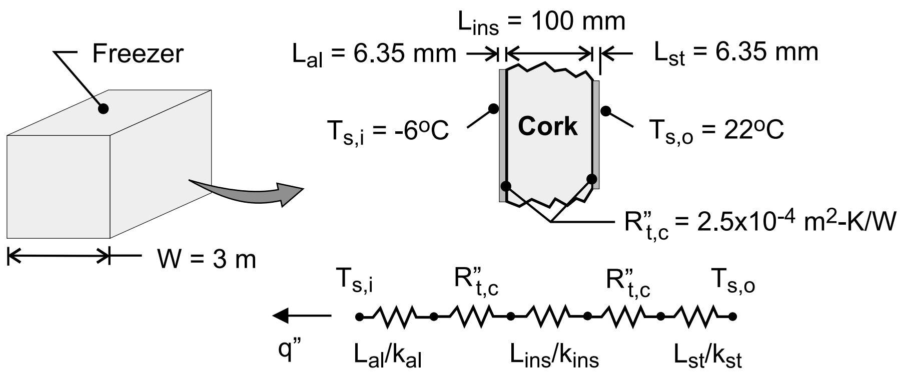
COMMENTS: Since the durability of the TBC decreases with increasing temperature, which increases with increasing thickness, limits to the thickness are associated with reliability considerations.

PROBLEM 3.24

KNOWN: Size and surface temperatures of a cubical freezer. Materials, thicknesses and interface resistances of freezer wall.

FIND: Cooling load.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum 2024 (~267K): k_{al} = 173 W/m·K. *Table A-1*, Carbon steel AISI 1010 (~295K): k_{st} = 64 W/m·K. *Table A-3* (~300K): k_{ins} = 0.039 W/m·K.

ANALYSIS: For a unit wall surface area, the total thermal resistance of the composite wall is

$$R''_{tot} = \frac{L_{al}}{k_{al}} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + R''_{t,c} + \frac{L_{st}}{k_{st}}$$

$$R''_{tot} = \frac{0.00635\text{m}}{173\ \text{W/m}\cdot\text{K}} + 2.5\times10^{-4}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.100\text{m}}{0.039\ \text{W/m}\cdot\text{K}} + 2.5\times10^{-4}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} + \frac{0.00635\text{m}}{64\ \text{W/m}\cdot\text{K}}$$

$$R''_{tot} = \left(3.7\times10^{-5} + 2.5\times10^{-4} + 2.56 + 2.5\times10^{-4} + 9.9\times10^{-5}\right)\text{m}^2\cdot\text{K/W}$$

Hence, the heat flux is

$$q'' = \frac{T_{s,o} - T_{s,i}}{R''_{tot}} = \frac{\left[22-(-6)\right]^\circ\text{C}}{2.56\ \text{m}^2\cdot\text{K/W}} = 10.9\ \frac{\text{W}}{\text{m}^2}$$

and the cooling load is

$$q = A_s\, q'' = 6\,W^2\, q'' = 54\text{m}^2 \times 10.9\ \text{W/m}^2 = 590\,\text{W}$$ <

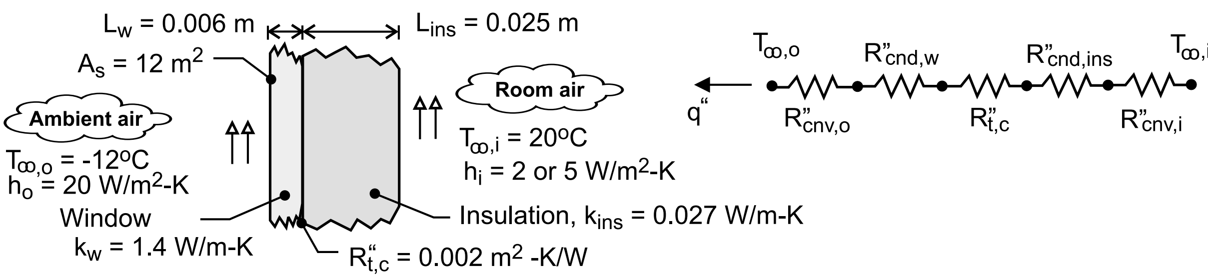
COMMENTS: Thermal resistances associated with the cladding and the adhesive joints are negligible compared to that of the insulation.

PROBLEM 3.25

KNOWN: Thicknesses and thermal conductivity of window glass and insulation. Contact resistance. Environmental temperatures and convection coefficients. Furnace efficiency and fuel cost.

FIND: (a) Reduction in heat loss associated with the insulation, (b) Heat losses for prescribed conditions, (c) Savings in fuel costs for 12 hour period.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Constant properties.

ANALYSIS: (a) The percentage reduction in heat loss is

$$R_q = \frac{q''_{wo} - q''_{with}}{q''_{wo}} \times 100\% = \left(1 - \frac{q''_{with}}{q''_{wo}}\right) \times 100\% = \left(1 - \frac{R''_{tot,wo}}{R''_{tot,with}}\right) \times 100\%$$

where the total thermal resistances without and with the insulation, respectively, are

$$R''_{tot,wo} = R''_{cnv,o} + R''_{cnd,w} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + \frac{1}{h_i}$$

$$R''_{tot,wo} = (0.050 + 0.004 + 0.200)\, m^2 \cdot K/W = 0.254\ m^2 \cdot K/W$$

$$R''_{tot,with} = R''_{cnv,o} + R''_{cnd,w} + R''_{t,c} + R''_{cnd,ins} + R''_{cnv,i} = \frac{1}{h_o} + \frac{L_w}{k_w} + R''_{t,c} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h_i}$$

$$R''_{tot,with} = (0.050 + 0.004 + 0.002 + 0.926 + 0.500)\, m^2 \cdot K/W = 1.482\ m^2 \cdot K/W$$

$$R_q = (1 - 0.254/1.482) \times 100\% = 82.9\%$$ <

(b) With $A_s = 12\ m^2$, the heat losses without and with the insulation are

$$q_{wo} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,wo} = 12\,m^2 \times 32°C/0.254\,m^2 \cdot K/W = 1512\,W$$ <

$$q_{with} = A_s\left(T_{\infty,i} - T_{\infty,o}\right)/R''_{tot,with} = 12\,m^2 \times 32°C/1.482\,m^2 \cdot K/W = 259\ W$$ <

(c) With the windows covered for 12 hours per day, the daily savings are

$$S = \frac{(q_{wo} - q_{with})}{\eta_f} \Delta t\ C_g \times 10^{-6}\ MJ/J = \frac{(1512 - 259)\,W}{0.8} 12h \times 3600\,s/h \times \$0.01/MJ \times 10^{-6}\,MJ/J = \$0.677$$

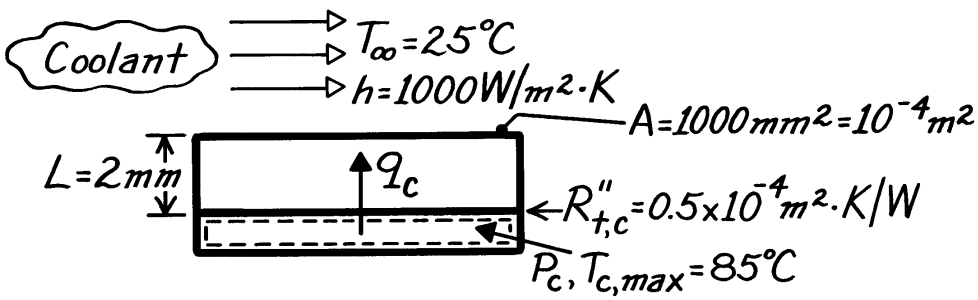
COMMENTS: (1) The savings may be insufficient to justify the cost of the insulation, as well as the daily tedium of applying and removing the insulation. However, the losses are significant and unacceptable. The owner of the building should install double pane windows. (2) The dominant contributions to the total thermal resistance are made by the insulation and convection at the inner surface.

PROBLEM 3.26

KNOWN: Surface area and maximum temperature of a chip. Thickness of aluminum cover and chip/cover contact resistance. Fluid convection conditions.

FIND: Maximum chip power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Negligible heat loss from sides and bottom, (4) Chip is isothermal.

PROPERTIES: *Table A.1*, Aluminum (T ≈ 325 K): k = 238 W/m·K.

ANALYSIS: For a control surface about the chip, conservation of energy yields

$$\dot{E}_g - \dot{E}_{out} = 0$$

or

$$P_c - \frac{(T_c - T_\infty)A}{\left[(L/k) + R''_{t,c} + (1/h)\right]} = 0$$

$$P_{c,max} = \frac{(85-25)^\circ C\left(10^{-4} m^2\right)}{\left[(0.002/238) + 0.5\times10^{-4} + (1/1000)\right] m^2 \cdot K/W}$$

$$P_{c,max} = \frac{60\times10^{-4}\ {}^\circ C \cdot m^2}{\left(8.4\times10^{-6} + 0.5\times10^{-4} + 10^{-3}\right) m^2 \cdot K/W}$$

$$P_{c,max} = 5.7\ W.$$ <

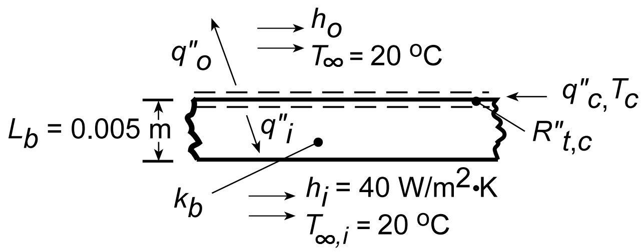
COMMENTS: The dominant resistance is that due to convection $\left(R_{conv} > R_{t,c} >> R_{cond}\right)$.

PROBLEM 3.27

KNOWN: Operating conditions for a board mounted chip.

FIND: (a) Equivalent thermal circuit, (b) Chip temperature, (c) Maximum allowable heat dissipation for dielectric liquid (h_o = 1000 W/m^2·K) and air (h_o = 100 W/m^2·K). Effect of changes in circuit board temperature and contact resistance.

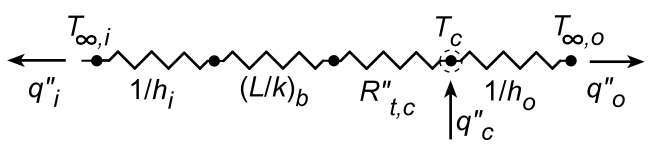
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible chip thermal resistance, (4) Negligible radiation, (5) Constant properties.

PROPERTIES: *Table A-3*, Aluminum oxide (polycrystalline, 358 K): k_b = 32.4 W/m·K.

ANALYSIS: (a)



(b) Applying conservation of energy to a control surface about the chip $\left(\dot{E}_{in} - \dot{E}_{out} = 0\right)$,

$$q''_c - q''_i - q''_o = 0$$

$$q''_c = \frac{T_c - T_{\infty,i}}{1/h_i + (L/k)_b + R''_{t,c}} + \frac{T_c - T_{\infty,o}}{1/h_o}$$

With $q''_c = 3\times10^4\ \text{W/m}^2$, h_o = 1000 W/m^2·K, k_b = 1 W/m·K and $R''_{t,c} = 10^{-4}\ \text{m}^2\cdot\text{K/W}$,

$$3\times10^4\ \text{W/m}^2 = \frac{T_c - 20^\circ\text{C}}{\left(1/40 + 0.005/1 + 10^{-4}\right)\text{m}^2\cdot\text{K/W}} + \frac{T_c - 20^\circ\text{C}}{(1/1000)\,\text{m}^2\cdot\text{K/W}}$$

$$3\times10^4\ \text{W/m}^2 = \left(33.2T_c - 664 + 1000T_c - 20{,}000\right)\text{W/m}^2\cdot\text{K}$$

$$1003T_c = 50{,}664$$

$$T_c = 49^\circ\text{C}.$$ <

(c) For T_c = 85°C and h_o = 1000 W/m^2·K, the foregoing energy balance yields

$$q''_c = 67{,}160\ \text{W/m}^2$$ <

with q''_o = 65,000 W/m^2 and q''_i = 2160 W/m^2. Replacing the dielectric with air (h_o = 100 W/m^2·K), the following results are obtained for different combinations of k_b and $R''_{t,c}$.

Continued...

PROBLEM 3.27 (Cont.)

k_b (W/m·K)	$R''_{t,c}$ (m²·K/W)	q''_i (W/m²)	q''_o (W/m²)	q''_c (W/m²)
1	10^{-4}	2159	6500	8659
32.4	10^{-4}	2574	6500	9074
1	10^{-5}	2166	6500	8666
32.4	10^{-5}	2583	6500	9083

<

COMMENTS: 1. For the conditions of part (b), the total internal resistance is 0.0301 m²·K/W, while the outer resistance is 0.001 m²·K/W. Hence

$$\frac{q''_o}{q''_i} = \frac{\left(T_c - T_{\infty,o}\right)/R''_o}{\left(T_c - T_{\infty,i}\right)/R''_i} = \frac{0.0301}{0.001} = 30\,.$$

and only approximately 3% of the heat is dissipated through the board.

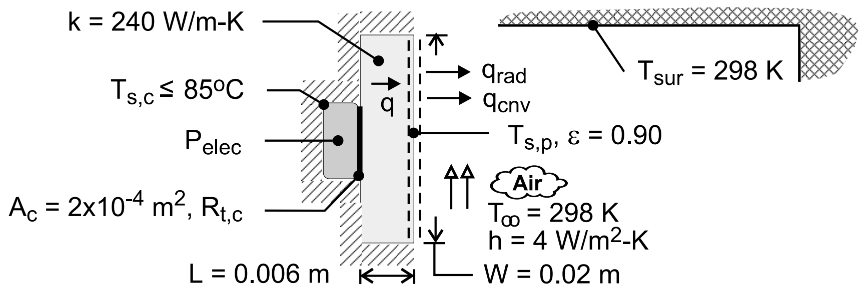
2. With h_o = 100 W/m²·K, the outer resistance increases to 0.01 m²·K/W, in which case $q''_o/q''_i = R''_i/R''_o$ = 0.0301/0.01 = 3.1 and now almost 25% of the heat is dissipated through the board. Hence, although measures to reduce R''_i would have a negligible effect on q''_c for the liquid coolant, some improvement may be gained for air-cooled conditions. As shown in the table of part (b), use of an aluminum oxide board increase q''_i by 19% (from 2159 to 2574 W/m²) by reducing R''_i from 0.0301 to 0.0253 m²·K/W. Because the initial contact resistance ($R''_{t,c} = 10^{-4}\,m^2 \cdot K/W$) is already much less than R''_i, any reduction in its value would have a negligible effect on q''_i. The largest gain would be realized by increasing h_i, since the inside convection resistance makes the dominant contribution to the total internal resistance.

PROBLEM 3.28

KNOWN: Dimensions, thermal conductivity and emissivity of base plate. Temperature and convection coefficient of adjoining air. Temperature of surroundings. Maximum allowable temperature of transistor case. Case-plate interface conditions.

FIND: (a) Maximum allowable power dissipation for an air-filled interface, (b) Effect of convection coefficient on maximum allowable power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible heat transfer from the enclosure, to the surroundings. (3) One-dimensional conduction in the base plate, (4) Radiation exchange at surface of base plate is with large surroundings, (5) Constant thermal conductivity.

PROPERTIES: Aluminum-aluminum interface, air-filled, 10 μm roughness, 10^5 N/m^2 contact pressure (Table 3.1): $R''_{t,c} = 2.75 \times 10^{-4}\ \text{m}^2 \cdot \text{K/W}$.

ANALYSIS: (a) With all of the heat dissipation transferred through the base plate,

$$P_{elec} = q = \frac{T_{s,c} - T_\infty}{R_{tot}} \tag{1}$$

where $R_{tot} = R_{t,c} + R_{cnd} + \left[(1/R_{cnv}) + (1/R_{rad})\right]^{-1}$

$$R_{tot} = \frac{R''_{t,c}}{A_c} + \frac{L}{kW^2} + \frac{1}{W^2}\left(\frac{1}{h + h_r}\right) \tag{2}$$

and $$h_r = \varepsilon\sigma\left(T_{s,p} + T_{sur}\right)\left(T_{s,p}^2 + T_{sur}^2\right) \tag{3}$$

To obtain $T_{s,p}$, the following energy balance must be performed on the plate surface,

$$q = \frac{T_{s,c} - T_{s,p}}{R_{t,c} + R_{cnd}} = q_{cnv} + q_{rad} = hW^2\left(T_{s,p} - T_\infty\right) + h_r\, W^2\left(T_{s,p} - T_{sur}\right) \tag{4}$$

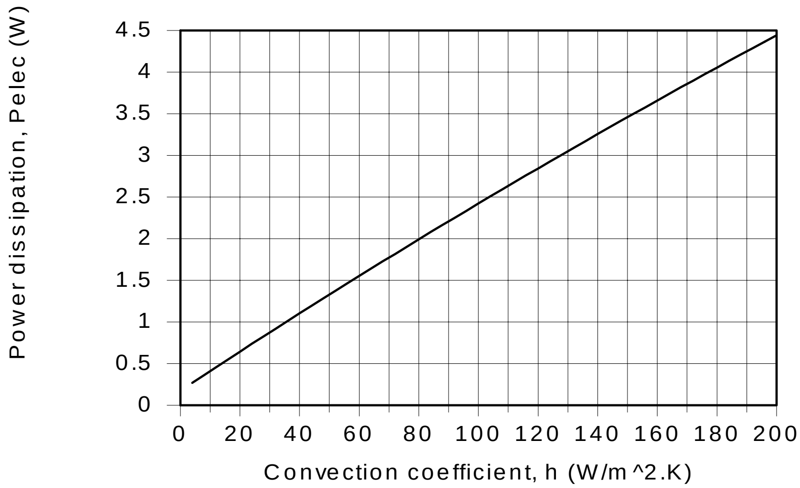
With $R_{t,c} = 2.75 \times 10^{-4}\ \text{m}^2\cdot\text{K/W}/2\times10^{-4}\ \text{m}^2 = 1.375$ K/W, $R_{cnd} = 0.006\ \text{m}/(240\ \text{W/m}\cdot\text{K} \times 4 \times 10^{-4}\ \text{m}^2) = 0.0625$ K/W, and the prescribed values of h, W, $T_\infty = T_{sur}$ and ε, Eq. (4) yields a surface temperature of $T_{s,p} = 357.6$ K = 84.6°C and a power dissipation of

Continued …..

$$P_{elec} = q = 0.268 \text{ W} \qquad <$$

The convection and radiation resistances are R_{cnv} = 625 m·K/W and R_{rad} = 345 m·K/W, where h_r = 7.25 W/m^2·K.

(b) With the major contribution to the total resistance made by convection, significant benefit may be derived by increasing the value of h.



For h = 200 W/m^2·K, R_{cnv} = 12.5 m·K/W and $T_{s,p}$ = 351.6 K, yielding R_{rad} = 355 m·K/W. The effect of radiation is then negligible.

COMMENTS: (1) The plate conduction resistance is negligible, and even for h = 200 W/m^2·K, the contact resistance is small relative to the convection resistance. However, $R_{t,c}$ could be rendered negligible by using indium foil, instead of an air gap, at the interface. From Table 3.1, $R''_{t,c} = 0.07 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$, in which case $R_{t,c}$ = 0.035 m·K/W.

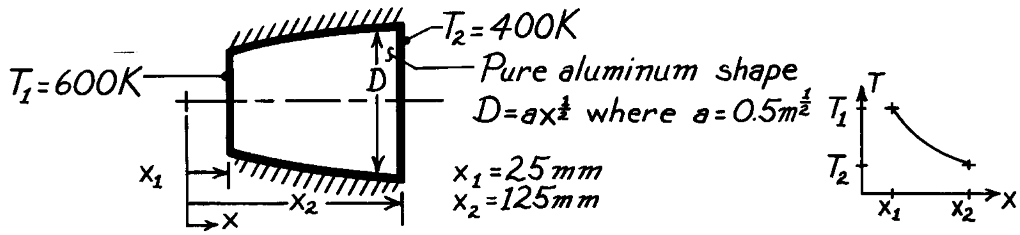
(2) Because $A_c < W^2$, heat transfer by conduction in the plate is actually two-dimensional, rendering the conduction resistance even smaller.

PROBLEM 3.29

KNOWN: Conduction in a conical section with prescribed diameter, D, as a function of x in the form $D = ax^{1/2}$.

FIND: (a) Temperature distribution, T(x), (b) Heat transfer rate, q_x.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) No internal heat generation, (4) Constant properties.

PROPERTIES: *Table A-2*, Pure Aluminum (500K): k= 236 W/m·K.

ANALYSIS: (a) Based upon the assumptions, and following the same methodology of Example 3.3, q_x is a constant independent of x. Accordingly,

$$q_x = -kA\frac{dT}{dx} = -k\left[\pi\left(ax^{1/2}\right)^2/4\right]\frac{dT}{dx} \qquad (1)$$

using $A = \pi D^2/4$ where $D = ax^{1/2}$. Separating variables and identifying limits,

$$\frac{4q_x}{\pi a^2 k}\int_{x_1}^{x}\frac{dx}{x} = -\int_{T_1}^{T} dT. \qquad (2)$$

Integrating and solving for T(x) and then for T_2,

$$T(x) = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x}{x_1} \qquad T_2 = T_1 - \frac{4q_x}{\pi a^2 k}\ln\frac{x_2}{x_1}. \qquad (3,4)$$

Solving Eq. (4) for q_x and then substituting into Eq. (3) gives the results,

$$q_x = -\frac{\pi}{4}a^2 k\left(T_1 - T_2\right)/1n\ \left(x_1/x_2\right) \qquad (5)$$

$$T(x) = T_1 + \left(T_1 - T_2\right)\frac{\ln\ (x/x_1)}{\ln\ \left(x_1/x_2\right)}. \qquad <$$

From Eq. (1) note that (dT/dx)·x = Constant. It follows that T(x) has the distribution shown above.

(b) The heat rate follows from Eq. (5),

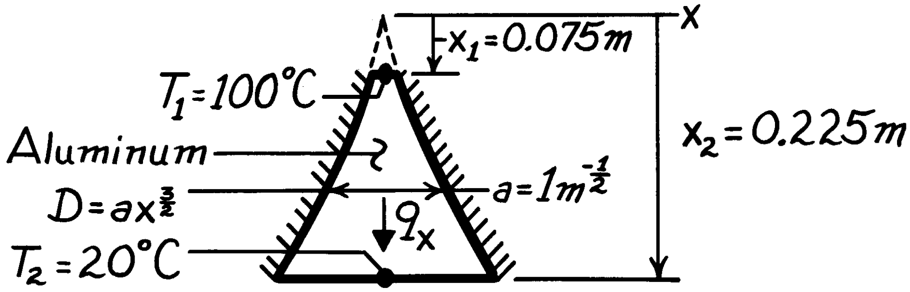
$$q_x = \frac{\pi}{4}\times 0.5^2 m \times 236\frac{W}{m\cdot K}\left(600-400\right)K/\ln\frac{25}{125} = 5.76kW. \qquad <$$

PROBLEM 3.30

KNOWN: Geometry and surface conditions of a truncated solid cone.

FIND: (a) Temperature distribution, (b) Rate of heat transfer across the cone.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x, (3) Constant properties.

PROPERTIES: *Table A-1*, Aluminum (333K): k = 238 W/m·K.

ANALYSIS: (a) From Fourier's law, Eq. (2.1), with $A=\pi D^2/4=\left(\pi a^2/4\right)x^3$, it follows that

$$\frac{4q_x dx}{\pi a^2 x^3} = -k dT.$$

Hence, since q_x is independent of x,

$$\frac{4q_x}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^3} = -k\int_{T_1}^{T} dT$$

or

$$\frac{4q_x}{\pi a^2}\left[-\frac{1}{2x^2}\right]\Bigg|_{x_1}^{x} = -k\left(T-T_1\right).$$

Hence

$$T = T_1 + \frac{2q_x}{\pi a^2 k}\left[\frac{1}{x^2}-\frac{1}{x_1^2}\right].$$ <

(b) From the foregoing expression, it also follows that

$$q_x = \frac{\pi a^2 k}{2}\,\frac{T_2-T_1}{\left[1/x_2^2-1/x_1^2\right]}$$

$$q_x = \frac{\pi\left(1m^{-1}\right)238\ W/m\cdot K}{2}\times\frac{(20-100)^\circ C}{\left[(0.225)^{-2}-(0.075)^{-2}\right]m^{-2}}$$

$$q_x = 189\ W.$$ <

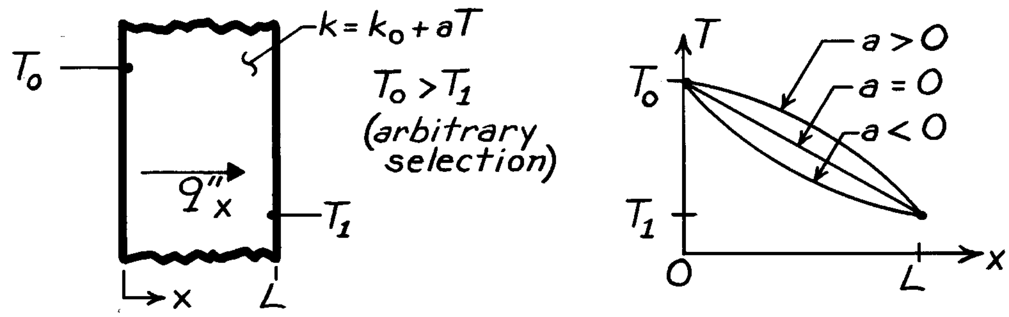
COMMENTS: The foregoing results are approximate due to use of a one-dimensional model in treating what is inherently a two-dimensional problem.

PROBLEM 3.31

KNOWN: Temperature dependence of the thermal conductivity, k.

FIND: Heat flux and form of temperature distribution for a plane wall.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through a plane wall, (2) Steady-state conditions, (3) No internal heat generation.

ANALYSIS: For the assumed conditions, q_x and A(x) are constant and Eq. 3.21 gives

$$q''_x \int_0^L dx = -\int_{T_o}^{T_1} (k_o + aT) dT$$

$$q''_x = \frac{1}{L}\left[k_o (T_o - T_1) + \frac{a}{2}\left(T_o^2 - T_1^2\right)\right].$$

From Fourier's law,

$$q''_x = -(k_o + aT)\ dT/dx.$$

Hence, since the product of (k_o+aT) and dT/dx) is constant, decreasing T with increasing x implies,

a > 0: decreasing (k_o+aT) and increasing |dT/dx| with increasing x

a = 0: $k = k_o$ => constant (dT/dx)

a < 0: increasing (k_o+aT) and decreasing |dT/dx| with increasing x.

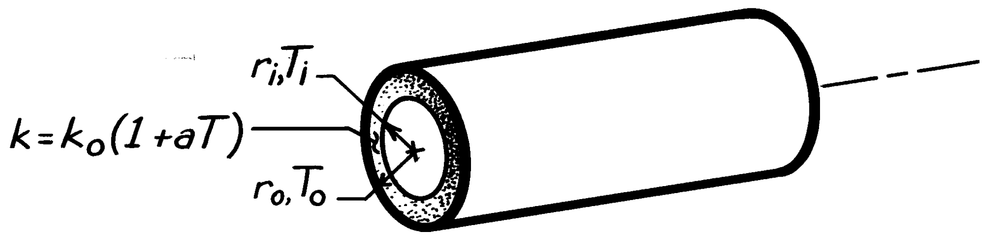
The temperature distributions appear as shown in the above sketch.

PROBLEM 3.32

KNOWN: Temperature dependence of tube wall thermal conductivity.

FIND: Expressions for heat transfer per unit length and tube wall thermal (conduction) resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal heat generation.

ANALYSIS: From Eq. 3.24, the appropriate form of Fourier's law is

$$q_r = -kA_r \frac{dT}{dr} = -k(2\pi\ rL)\frac{dT}{dr}$$
$$q_r' = -2\pi\ kr\frac{dT}{dr}$$
$$q_r' = -2\pi\ rk_o(1+aT)\frac{dT}{dr}.$$

Separating variables,

$$-\frac{q_r'}{2\pi}\frac{dr}{r} = k_o(1+aT)dT$$

and integrating across the wall, find

$$-\frac{q_r'}{2\pi}\int_{r_i}^{r_o}\frac{dr}{r} = k_o\int_{T_i}^{T_o}(1+aT)dT$$
$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[T + \frac{aT^2}{2}\right]\Bigg|_{T_i}^{T_o}$$
$$-\frac{q_r'}{2\pi}\ln\frac{r_o}{r_i} = k_o\left[(T_o - T_i) + \frac{a}{2}\left(T_o^2 - T_i^2\right)\right]$$

$$q_r' = -2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]\frac{(T_o - T_i)}{\ln(r_o/r_i)}.$$ <

It follows that the overall thermal resistance per unit length is

$$R_t' = \frac{\Delta T}{q_r'} = \frac{\ln(r_o/r_i)}{2\pi k_o\left[1 + \frac{a}{2}(T_o + T_i)\right]}.$$ <

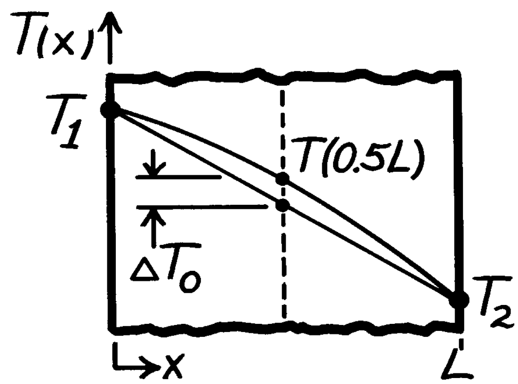
COMMENTS: Note the necessity of the stated assumptions to treating q_r' as independent of r.

PROBLEM 3.33

KNOWN: Steady-state temperature distribution of convex shape for material with $k = k_o(1 + \alpha T)$ where α is a constant and the mid-point temperature is ΔT_o higher than expected for a linear temperature distribution.

FIND: Relationship to evaluate α in terms of ΔT_o and T_1, T_2 (the temperatures at the boundaries).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) No internal heat generation, (4) α is positive and constant.

ANALYSIS: At any location in the wall, Fourier's law has the form

$$q''_x = -k_o(1+\alpha T)\frac{dT}{dx}. \tag{1}$$

Since q''_x is a constant, we can separate Eq. (1), identify appropriate integration limits, and integrate to obtain

$$\int_0^L q''_x dx = -\int_{T_1}^{T_2} k_o(1+\alpha T)dT \tag{2}$$

$$q''_x = -\frac{k_o}{L}\left[\left(T_2 + \frac{\alpha T_2^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right]. \tag{3}$$

We could perform the same integration, but with the upper limits at x = L/2, to obtain

$$q''_x = -\frac{2k_o}{L}\left[\left(T_{L/2} + \frac{\alpha T_{L/2}^2}{2}\right) - \left(T_1 + \frac{\alpha T_1^2}{2}\right)\right] \tag{4}$$

where

$$T_{L/2} = T(L/2) = \frac{T_1 + T_2}{2} + \Delta T_o. \tag{5}$$

Setting Eq. (3) equal to Eq. (4), substituting from Eq. (5) for $T_{L/2}$, and solving for α, it follows that

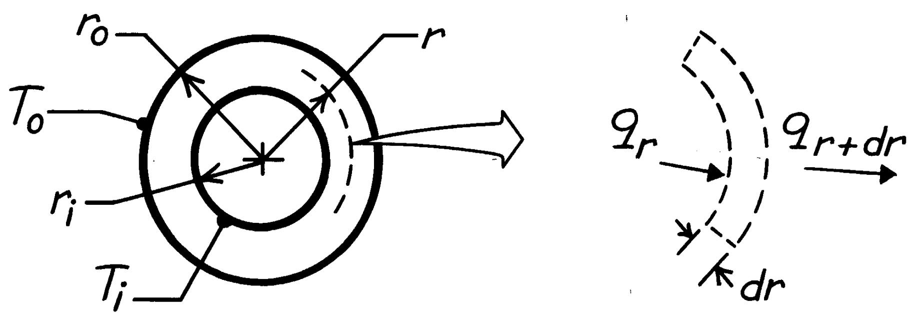
$$\alpha = \frac{2\Delta T_o}{\left(T_2^2 + T_1^2\right)/2 - \left[(T_1 + T_2)/2 + \Delta T_o\right]^2}.$$ <

PROBLEM 3.34

KNOWN: Hollow cylinder of thermal conductivity k, inner and outer radii, r_i and r_o, respectively, and length L.

FIND: Thermal resistance using the alternative conduction analysis method.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No internal volumetric generation, (4) Constant properties.

ANALYSIS: For the differential control volume, energy conservation requires that $q_r = q_{r+dr}$ for steady-state, one-dimensional conditions with no heat generation. With Fourier's law,

$$q_r = -kA\frac{dT}{dr} = -k(2\pi\ rL)\frac{dT}{dr} \qquad (1)$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer. Since q_r is constant, Eq. (1) may be separated and expressed in integral form,

$$\frac{q_r}{2\pi\ L}\int_{r_i}^{r_o}\frac{dr}{r} = -\int_{T_i}^{T_o} k(T)dT.$$

Assuming k is constant, the heat rate is

$$q_r = \frac{2\pi\ Lk(T_i - T_o)}{\ln(r_o / r_i)}.$$

Remembering that the thermal resistance is defined as

$$R_t \equiv \Delta T/q$$

it follows that for the hollow cylinder,

$$R_t = \frac{\ln(r_o / r_i)}{2\pi\ LK}. \qquad <$$

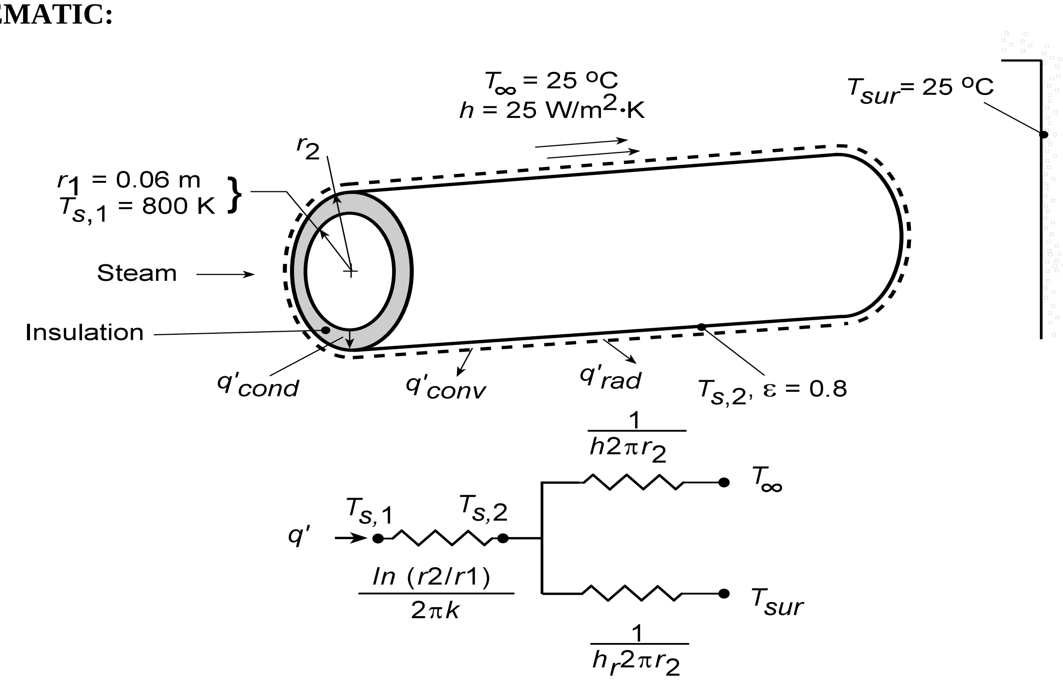
COMMENTS: Compare the *alternative* method used in this analysis with the *standard* method employed in Section 3.3.1 to obtain the same result.

PROBLEM 3.35

KNOWN: Thickness and inner surface temperature of calcium silicate insulation on a steam pipe. Convection and radiation conditions at outer surface.

FIND: (a) Heat loss per unit pipe length for prescribed insulation thickness and outer surface temperature. (b) Heat loss and radial temperature distribution as a function of insulation thickness.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

PROPERTIES: *Table A-3*, Calcium Silicate (T = 645 K): k = 0.089 W/m·K.

ANALYSIS: (a) From Eq. 3.27 with $T_{s,2}$ = 490 K, the heat rate per unit length is

$$q' = q_r/L = \frac{2\pi k\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

$$q' = \frac{2\pi\left(0.089\,W/m\cdot K\right)\left(800-490\right)K}{\ln\left(0.08\,m/0.06\,m\right)}$$

$$q' = 603\,W/m\,.$$ <

(b) Performing an energy for a control surface around the outer surface of the insulation, it follows that

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,1} - T_{s,2}}{\ln\left(r_2/r_1\right)/2\pi k} = \frac{T_{s,2} - T_\infty}{1/\left(2\pi r_2 h\right)} + \frac{T_{s,2} - T_{sur}}{1/\left(2\pi r_2 h_r\right)}$$

where $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. Solving this equation for $T_{s,2}$, the heat rate may be determined from

$$q' = 2\pi r_2\left[h\left(T_{s,2} - T_\infty\right) + h_r\left(T_{s,2} - T_{sur}\right)\right]$$

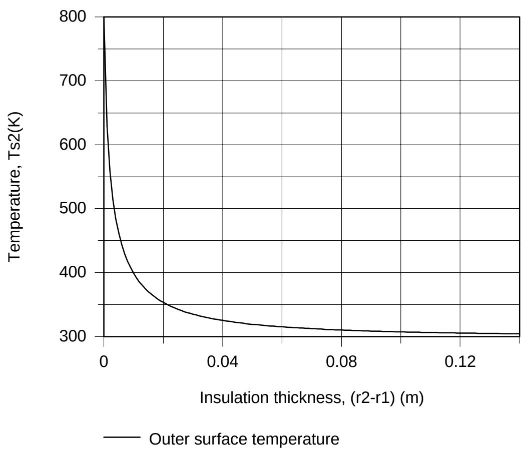
Continued...

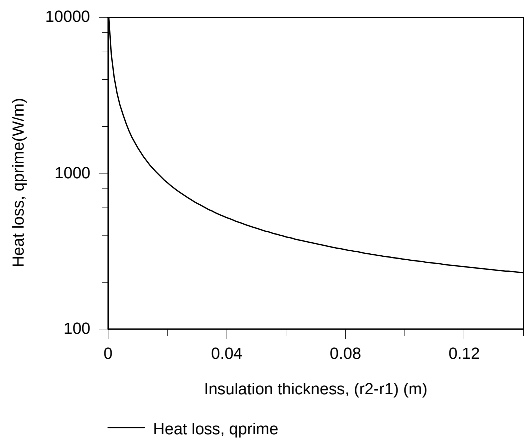
PROBLEM 3.35 (Cont.)

and from Eq. 3.26 the temperature distribution is

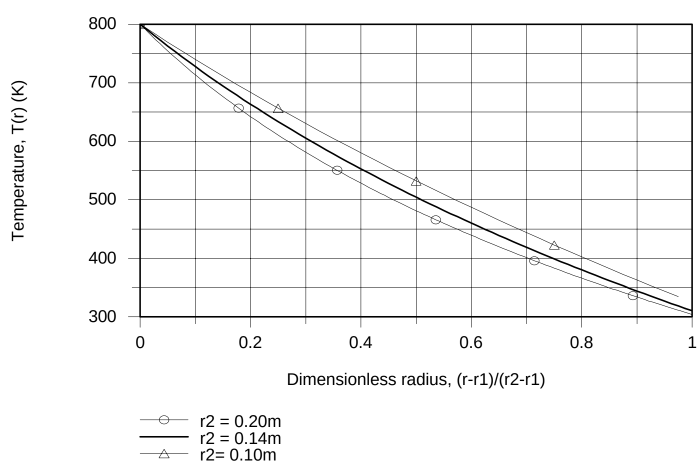
$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

As shown below, the outer surface temperature of the insulation $T_{s,2}$ and the heat loss q' decay precipitously with increasing insulation thickness from values of $T_{s,2} = T_{s,1} = 800$ K and $q' = 11,600$ W/m, respectively, at $r_2 = r_1$ (no insulation).





When plotted as a function of a dimensionless radius, $(r - r_1)/(r_2 - r_1)$, the temperature decay becomes more pronounced with increasing r_2.



Note that $T(r_2) = T_{s,2}$ increases with decreasing r_2 and a linear temperature distribution is approached as r_2 approaches r_1.

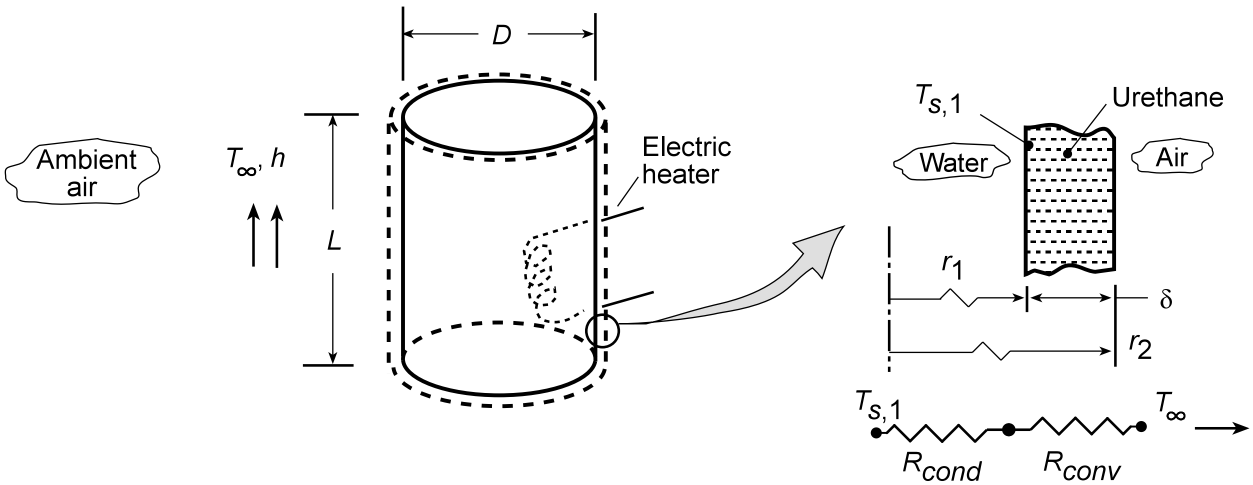
COMMENTS: An insulation layer thickness of 20 mm is sufficient to maintain the outer surface temperature and heat rate below 350 K and 1000 W/m, respectively.

PROBLEM 3.36

KNOWN: Temperature and volume of hot water heater. Nature of heater insulating material. Ambient air temperature and convection coefficient. Unit cost of electric power.

FIND: Heater dimensions and insulation thickness for which annual cost of heat loss is less than $50.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through side and end walls, (2) Conduction resistance dominated by insulation, (3) Inner surface temperature is approximately that of the water ($T_{s,1}$ = 55°C), (4) Constant properties, (5) Negligible radiation.

PROPERTIES: *Table A.3*, Urethane Foam (T = 300 K): k = 0.026 W/m·K.

ANALYSIS: To minimize heat loss, tank dimensions which minimize the total surface area, $A_{s,t}$, should be selected. With $L = 4\forall/\pi D^2$, $A_{s,t} = \pi DL + 2\left(\pi D^2/4\right) = 4\forall/D + \pi D^2/2$, and the tank diameter for which $A_{s,t}$ is an extremum is determined from the requirement

$$dA_{s,t}/dD = -4\forall/D^2 + \pi D = 0$$

It follows that

$$D = (4\forall/\pi)^{1/3} \qquad \text{and} \qquad L = (4\forall/\pi)^{1/3}$$

With $d^2A_{s,t}/dD^2 = 8\forall/D^3 + \pi > 0$, the foregoing conditions yield the desired minimum in $A_{s,t}$. Hence, for $\forall$ = 100 gal × 0.00379 m³/gal = 0.379 m³,

$$D_{op} = L_{op} = 0.784\,\text{m}$$ <

The total heat loss through the side and end walls is

$$q = \frac{T_{s,1} - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi k L_{op}} + \dfrac{1}{h 2\pi r_2 L_{op}}} + \frac{2\left(T_{s,1} - T_\infty\right)}{\dfrac{\delta}{k\left(\pi D_{op}^2/4\right)} + \dfrac{1}{h\left(\pi D_{op}^2/4\right)}}$$

We begin by estimating the heat loss associated with a 25 mm thick layer of insulation. With $r_1 = D_{op}/2$ = 0.392 m and $r_2 = r_1 + \delta$ = 0.417 m, it follows that

Continued...

$$q = \frac{(55-20)^\circ C}{\dfrac{\ln(0.417/0.392)}{2\pi(0.026\,W/m\cdot K)0.784\,m} + \dfrac{1}{(2\,W/m^2\cdot K)2\pi(0.417\,m)0.784\,m}}$$

$$+\frac{2(55-20)^\circ C}{\dfrac{0.025\,m}{(0.026\,W/m\cdot K)\pi/4(0.784\,m)^2} + \dfrac{1}{(2\,W/m^2\cdot K)\pi/4(0.784\,m)^2}}$$

$$q = \frac{35^\circ C}{(0.483+0.243)\,K/W} + \frac{2(35^\circ C)}{(1.992+1.036)\,K/W} = (48.2+23.1)\,W = 71.3\,W$$

The annual energy loss is therefore

$$Q_{annual} = 71.3\,W\,(365\,days)(24\,h/day)\left(10^{-3}\,kW/W\right) = 625\,kWh$$

With a unit electric power cost of $0.08/kWh, the annual cost of the heat loss is

C = ($0.08/kWh)625 kWh = $50.00

Hence, an insulation thickness of

δ = 25 mm <

will satisfy the prescribed cost requirement.

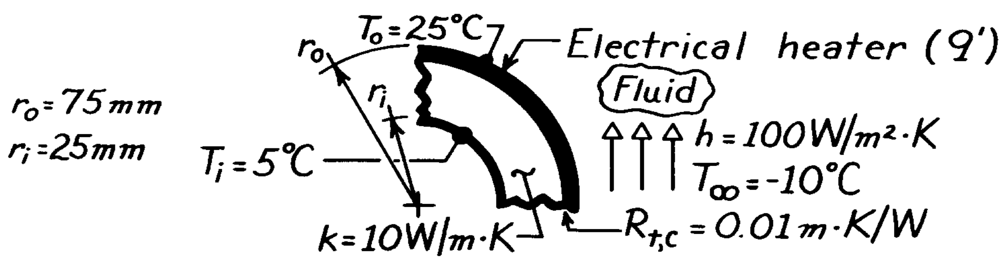
COMMENTS: Cylindrical containers of aspect ratio L/D = 1 are seldom used because of floor space constraints. Choosing L/D = 2, $\forall = \pi D^3/2$ and $D = (2\forall/\pi)^{1/3} = 0.623$ m. Hence, L = 1.245 m, r_1 = 0.312m and r_2 = 0.337 m. It follows that q = 76.1 W and C = $53.37. The 6.7% increase in the annual cost of the heat loss is small, providing little justification for using the optimal heater dimensions.

PROBLEM 3.37

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface and is exposed to a fluid of prescribed h and T_∞. Thermal contact resistance between heater and tube wall and wall inner surface temperature.

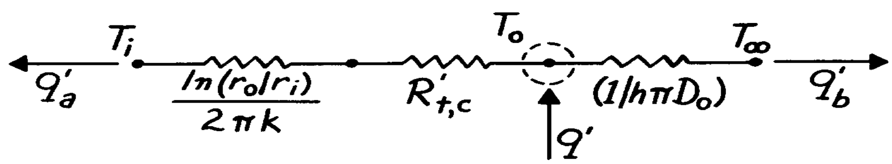
FIND: Heater power per unit length required to maintain a heater temperature of 25°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater.

ANALYSIS: The thermal circuit has the form



Applying an energy balance to a control surface about the heater,

$$q' = q'_a + q'_b$$

$$q' = \frac{T_o - T_i}{\frac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/h\pi D_o)}$$

$$q' = \frac{(25\text{-}5)^\circ\text{C}}{\frac{\ln(75\text{mm}/25\text{mm})}{2\pi\times 10\text{ W/m}\cdot\text{K}} + 0.01\frac{\text{m}\cdot\text{K}}{\text{W}}} + \frac{[25-(-10)]^\circ\text{C}}{\left[1/\left(100\text{ W/m}^2\cdot\text{K}\times\pi\times 0.15\text{m}\right)\right]}$$

$$q' = (728 + 1649)\text{ W/m}$$

$$q' = 2377\text{ W/m.}$$ <

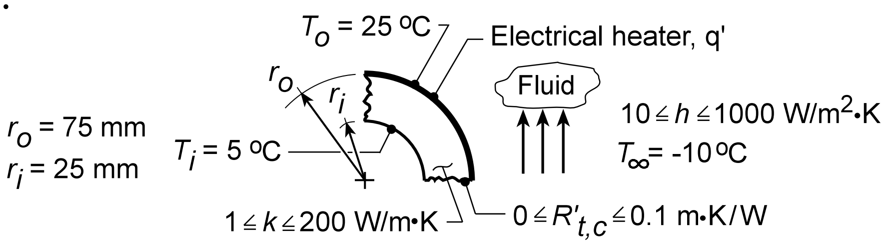
COMMENTS: The conduction, contact and convection resistances are 0.0175, 0.01 and 0.021 m ·K/W, respectively,

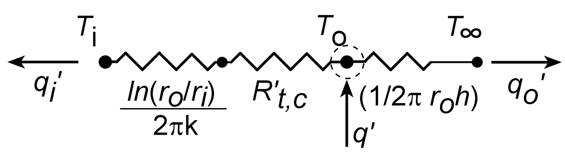
PROBLEM 3.38

KNOWN: Inner and outer radii of a tube wall which is heated electrically at its outer surface. Inner and outer wall temperatures. Temperature of fluid adjoining outer wall.

FIND: Effect of wall thermal conductivity, thermal contact resistance, and convection coefficient on total heater power and heat rates to outer fluid and inner surface.

SCHEMATIC:





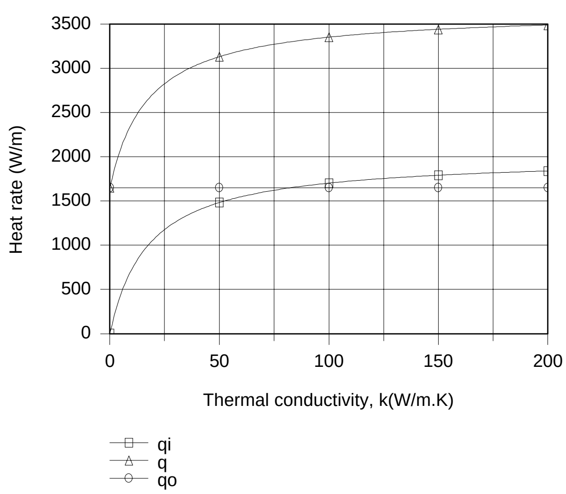
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across heater, (5) Negligible radiation.

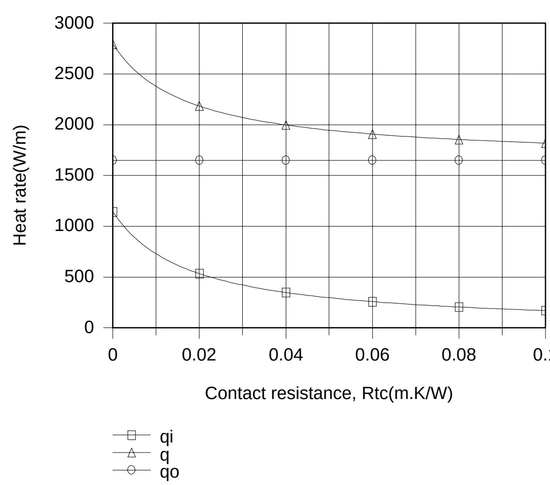
ANALYSIS: Applying an energy balance to a control surface about the heater,

$$q' = q'_i + q'_o$$

$$q' = \frac{T_o - T_i}{\dfrac{\ln(r_o/r_i)}{2\pi k} + R'_{t,c}} + \frac{T_o - T_\infty}{(1/2\pi r_o h)}$$

Selecting nominal values of k = 10 W/m·K, $R'_{t,c}$ = 0.01 m·K/W and h = 100 W/m^2·K, the following parametric variations are obtained





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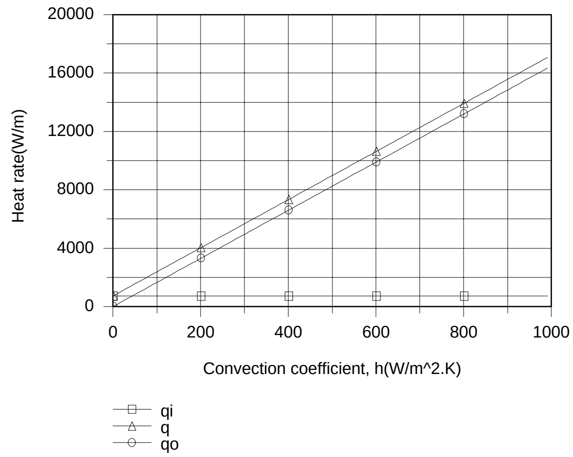
PROBLEM 3.38 (Cont.)



For a prescribed value of h, q'_o is fixed, while q'_i, and hence q', increase and decrease, respectively, with increasing k and $R'_{t,c}$. These trends are attributable to the effects of k and $R'_{t,c}$ on the total (conduction plus contact) resistance separating the heater from the inner surface. For fixed k and $R'_{t,c}$, q'_i is fixed, while q'_o, and hence q', increase with increasing h due to a reduction in the convection resistance.

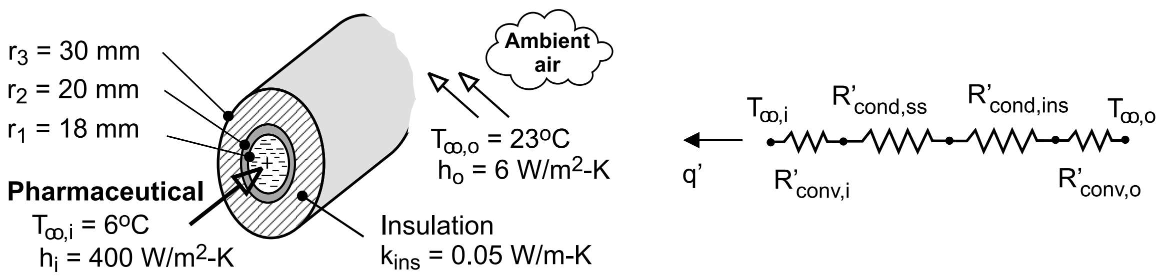
COMMENTS: For the prescribed nominal values of k, $R'_{t,c}$ and h, the electric power requirement is q' = 2377 W/m. To maintain the prescribed heater temperature, q' would increase with any changes which reduce the conduction, contact and/or convection resistances.

PROBLEM 3.39

KNOWN: Wall thickness and diameter of stainless steel tube. Inner and outer fluid temperatures and convection coefficients.

FIND: (a) Heat gain per unit length of tube, (b) Effect of adding a 10 mm thick layer of insulation to outer surface of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance between tube and insulation, (5) Negligible effect of radiation.

PROPERTIES: *Table A-1*, St. St. 304 (~280K): k_{st} = 14.4 W/m·K.

ANALYSIS: (a) Without the insulation, the total thermal resistance per unit length is

$$R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{conv,o} = \frac{1}{2\pi r_i h_i} + \frac{\ln(r_2/r_i)}{2\pi k_{st}} + \frac{1}{2\pi r_2 h_o}$$

$$R'_{tot} = \frac{1}{2\pi(0.018\text{m})400\ \text{W/m}^2\cdot\text{K}} + \frac{\ln(20/18)}{2\pi(14.4\ \text{W/m}\cdot\text{K})} + \frac{1}{2\pi(0.020\text{m})6\ \text{W/m}^2\cdot\text{K}}$$

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.33\right)\text{m}\cdot\text{K/W} = 1.35\ \text{m}\cdot\text{K/W}$$

The heat gain per unit length is then

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{(23-6)°\text{C}}{1.35\,\text{m}\cdot\text{K/W}} = 12.6\ \text{W/m}$$ <

(b) With the insulation, the total resistance per unit length is now $R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{cond,ins} + R'_{conv,o}$, where $R'_{conv,i}$ and $R'_{cond,st}$ remain the same. The thermal resistance of the insulation is

$$R'_{cond,ins} = \frac{\ln(r_3/r_2)}{2\pi k_{ins}} = \frac{\ln(30/20)}{2\pi(0.05\ \text{W/m}\cdot\text{K})} = 1.29\ \text{m}\cdot\text{K/W}$$

and the outer convection resistance is now

$$R'_{conv,o} = \frac{1}{2\pi r_3 h_o} = \frac{1}{2\pi(0.03\text{m})6\ \text{W/m}^2\cdot\text{K}} = 0.88\ \text{m}\cdot\text{K/W}$$

The total resistance is now

$$R'_{tot} = \left(0.0221 + 1.16\times10^{-3} + 1.29 + 0.88\right)\text{m}\cdot\text{K/W} = 2.20\,\text{m}\cdot\text{K/W}$$

Continued …..

and the heat gain per unit length is

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{17°C}{2.20 \text{ m} \cdot \text{K/W}} = 7.7 \text{ W/m}$$

COMMENTS: (1) The validity of assuming negligible radiation may be assessed for the worst case condition corresponding to the bare tube. Assuming a tube outer surface temperature of $T_s = T_{\infty,i} =$ 279K, large surroundings at $T_{sur} = T_{\infty,o} =$ 296K, and an emissivity of $\varepsilon = 0.7$, the heat gain due to net radiation exchange with the surroundings is $q'_{rad} = \varepsilon\sigma\left(2\pi r_2\right)\left(T_{sur}^4 - T_s^4\right) = 7.7 \text{ W/m}$. Hence, the net rate of heat transfer by radiation to the tube surface is comparable to that by convection, and the assumption of negligible radiation is inappropriate.

(2) If heat transfer from the air is by natural convection, the value of h_o with the insulation would actually be less than the value for the bare tube, thereby further reducing the heat gain. Use of the insulation would also increase the outer surface temperature, thereby reducing net radiation transfer from the surroundings.

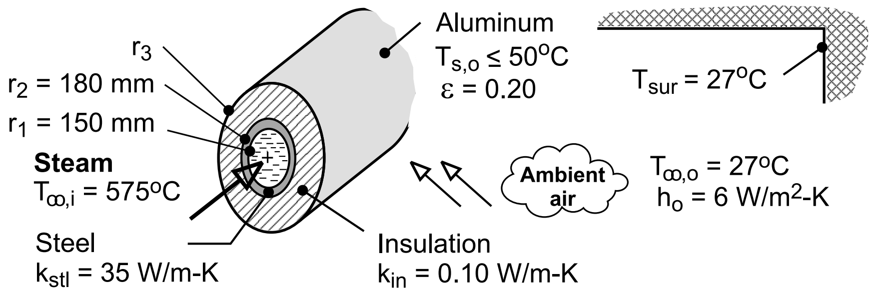
(3) The critical radius is $r_{cr} = k_{ins}/h \approx 8$ mm $< r_2$. Hence, as indicated by the calculations, heat transfer is reduced by the insulation.

PROBLEM 3.40

KNOWN: Diameter, wall thickness and thermal conductivity of steel tubes. Temperature of steam flowing through the tubes. Thermal conductivity of insulation and emissivity of aluminum sheath. Temperature of ambient air and surroundings. Convection coefficient at outer surface and maximum allowable surface temperature.

FIND: (a) Minimum required insulation thickness (r3 – r2) and corresponding heat loss per unit length, (b) Effect of insulation thickness on outer surface temperature and heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction, (3) Negligible contact resistances at the material interfaces, (4) Negligible steam side convection resistance ($T_{\infty,i} = T_{s,i}$), (5) Negligible conduction resistance for aluminum sheath, (6) Constant properties, (7) Large surroundings.

ANALYSIS: (a) To determine the insulation thickness, an energy balance must be performed at the outer surface, where $q' = q'_{conv,o} + q'_{rad}$. With $q'_{conv,o} = 2\pi r_3 h_o \left(T_{s,o} - T_{\infty,o}\right)$, $q'_{rad} = 2\pi r_3 \ \varepsilon\sigma \left(T_{s,o}^4 - T_{sur}^4\right)$, $q' = \left(T_{s,i} - T_{s,o}\right)/\left(R'_{cond,st} + R'_{cond,ins}\right)$, $R'_{cond,st} = \ell n\left(r_2/r_1\right)/2\pi k_{st}$, and $R'_{cond,ins} = \ell n\left(r_3/r_2\right)/2\pi k_{ins}$, it follows that

$$\frac{2\pi\left(T_{s,i} - T_{s,o}\right)}{\dfrac{\ell n\left(r_2/r_1\right)}{k_{st}} + \dfrac{\ell n\left(r_3/r_2\right)}{k_{ins}}} = 2\pi r_3\left[h_o\left(T_{s,o} - T_{\infty,o}\right) + \varepsilon\sigma\left(T_{s,o}^4 - T_{sur}^4\right)\right]$$

$$\frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\text{ W/m}\cdot\text{K}} + \dfrac{\ell n\left(r_3/0.18\right)}{0.10\text{ W/m}\cdot\text{K}}} = 2\pi r_3\left[6\text{ W/m}^2\cdot\text{K}\left(323 - 300\right)\text{K} + 0.20\times 5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(323^4 - 300^4\right)\text{K}^4\right]$$

A trial-and-error solution yields $r_3 = 0.394$ m = 394 mm, in which case the insulation thickness is

$$t_{ins} = r_3 - r_2 = 214\,\text{mm}$$ <

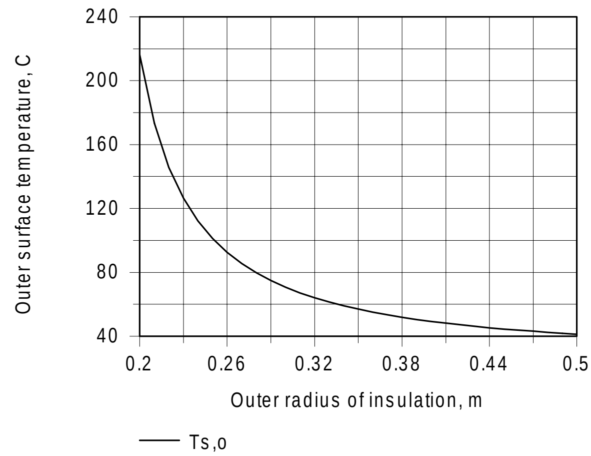
The heat rate is then

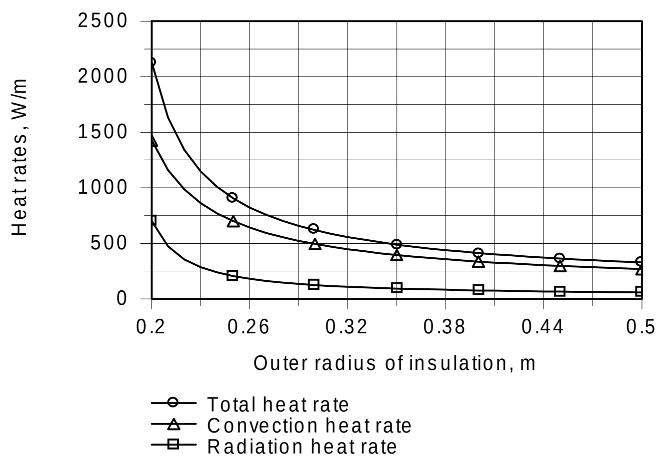
$$q' = \frac{2\pi\left(848 - 323\right)\text{K}}{\dfrac{\ell n\left(0.18/0.15\right)}{35\,\text{W/m}\cdot\text{K}} + \dfrac{\ell n\left(0.394/0.18\right)}{0.10\,\text{W/m}\cdot\text{K}}} = 420\,\text{W/m}$$ <

(b) The effects of r_3 on $T_{s,o}$ and q' have been computed and are shown below.

Conditioned …..

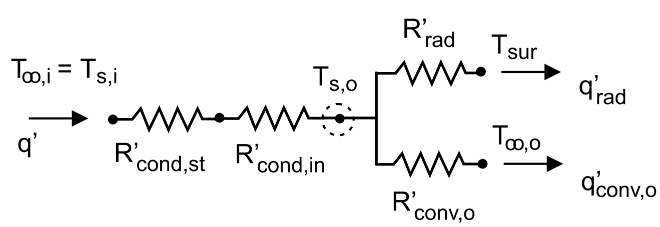
PROBLEM 3.40 (Cont.)





Beyond $r_3 \approx 0.40$m, there are rapidly diminishing benefits associated with increasing the insulation thickness.

COMMENTS: Note that the thermal resistance of the insulation is much larger than that for the tube wall. For the conditions of Part (a), the radiation coefficient is $h_r = 1.37$ W/m, and the heat loss by radiation is less than 25% of that due to natural convection $\left(q'_{rad} = 78\,\mathrm{W/m},\; q'_{conv,o} = 342\;\mathrm{W/m}\right)$.

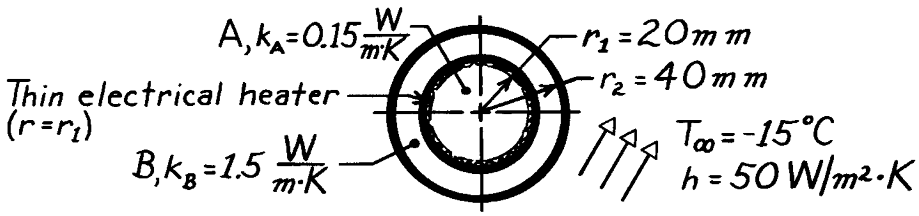


PROBLEM 3.41

KNOWN: Thin electrical heater fitted between two concentric cylinders, the outer surface of which experiences convection.

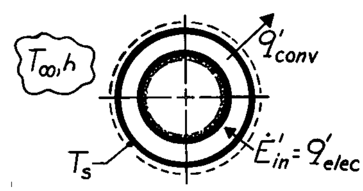
FIND: (a) Electrical power required to maintain outer surface at a specified temperature, (b) Temperature at the center

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Heater element has negligible thickness, (4) Negligible contact resistance between cylinders and heater, (5) Constant properties, (6) No generation.

ANALYSIS: (a) Perform an energy balance on the composite system to determine the power required to maintain $T(r_2) = T_s = 5°C$.



$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$+q'_{elec} - q'_{conv} = 0.$$

Using Newton's law of cooling,

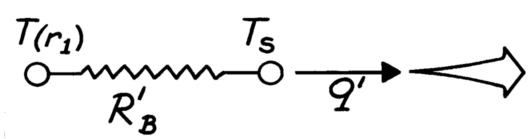
$$q'_{elec} = q'_{conv} = h \cdot 2\pi\, r_2 \left(T_s - T_\infty\right)$$

$$q'_{elec} = 50 \frac{W}{m^2 \cdot K} \times 2\pi \left(0.040 m\right) \left[5 - \left(-15\right)\right]^{\circ} C = 251 \text{ W/m}.$$ <

(b) From a control volume about Cylinder A, we recognize that the cylinder must be isothermal, that is,

$$T(0) = T(r_1).$$

Represent Cylinder B by a thermal circuit:



$$q' = \frac{T(r_1) - T_s}{R'_B}$$

For the cylinder, from Eq. 3.28,

$$R'_B = \ln r_2 / r_1 / 2\pi\, k_B$$

giving

$$T\left(r_1\right) = T_s + q' R'_B = 5^{\circ}C + 253.1 \frac{W}{m} \frac{\ln 40/20}{2\pi \times 1.5 \text{ W/m} \cdot K} = 23.5^{\circ} C$$

Hence, $T(0) = T(r_1) = 23.5°C$. <

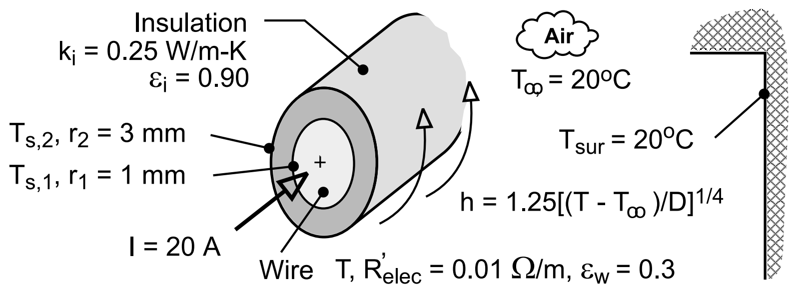
Note that k_A has no influence on the temperature T(0).

PROBLEM 3.42

KNOWN: Electric current and resistance of wire. Wire diameter and emissivity. Thickness, emissivity and thermal conductivity of coating. Temperature of ambient air and surroundings. Expression for heat transfer coefficient at surface of the wire or coating.

FIND: (a) Heat generation per unit length and volume of wire, (b) Temperature of uninsulated wire, (c) Inner and outer surface temperatures of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible contact resistance between insulation and wire, (5) Negligible radial temperature gradients in wire, (6) Large surroundings.

ANALYSIS: (a) The rates of energy generation per unit length and volume are, respectively,

$$\dot{E}'_g = I^2 R'_{elec} = (20\,A)^2 (0.01\Omega/m) = 4\ W/m \quad <$$

$$\dot{q} = \dot{E}'_g / A_c = 4\dot{E}'_g / \pi D^2 = 16\,W/m/\pi(0.002m)^2 = 1.27\times10^6\ W/m^3 \quad <$$

(b) Without the insulation, an energy balance at the surface of the wire yields

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h (T - T_\infty) + \pi D \varepsilon_w \sigma \left(T^4 - T^4_{sur}\right)$$

where $h = 1.25[(T - T_\infty)/D]^{1/4}$. Substituting,

$$4\,W/m = 1.25\pi(0.002m)^{3/4}(T-293)^{5/4} + \pi(0.002m)0.3\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T^4 - 293^4\right)K^4$$

and a trial-and-error solution yields

$$T = 331K = 58°C \quad <$$

(c) Performing an energy balance at the outer surface,

$$\dot{E}'_g = q' = q'_{conv} + q'_{rad} = \pi D h\left(T_{s,2} - T_\infty\right) + \pi D \varepsilon_i \sigma\left(T^4_{s,2} - T^4_{sur}\right)$$

$$4\,W/m = 1.25\pi(0.006m)^{3/4}\left(T_{s,2} - 293\right)^{5/4} + \pi(0.006m)0.9\times5.67\times10^{-8}\,W/m^2\cdot K^4\left(T^4_{s,2} - 293^4\right)K^4$$

and an iterative solution yields the following value of the surface temperature

$$T_{s,2} = 307.8\,K = 34.8°C \quad <$$

The inner surface temperature may then be obtained from the following expression for heat transfer by conduction in the insulation.

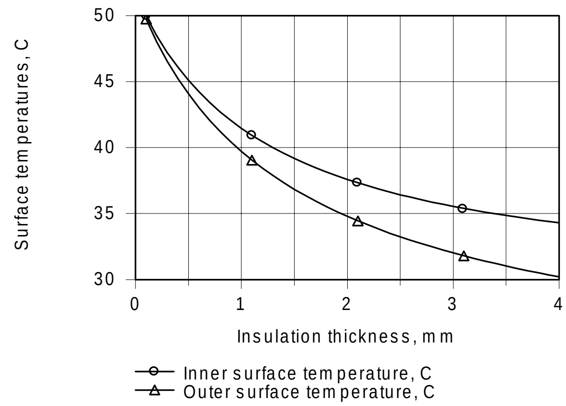
Continued …..

$$q' = \frac{T_{s,i} - T_2}{R'_{cond}} = \frac{T_{s,i} - T_{s,2}}{\ell n(r_2/r_1)/2\pi k_i}$$

$$4\,W = \frac{2\pi(0.25\,W/m\cdot K)(T_{s,i} - 307.8\,K)}{\ell n\,3}$$

$$T_{s,i} = 310.6\,K = 37.6°C$$ <

As shown below, the effect of increasing the insulation thickness is to *reduce*, not increase, the surface temperatures.



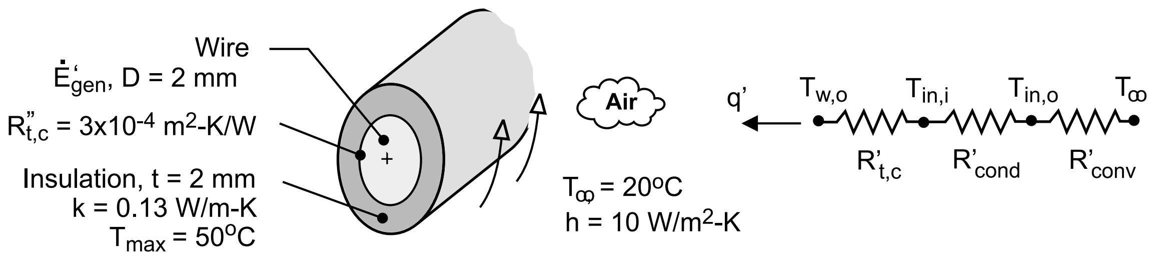
This behavior is due to a reduction in the total resistance to heat transfer with increasing r_2. Although the convection, h, and radiation, $h_r = \varepsilon\sigma(T_{s,2} + T_{sur})(T_{s,2}^2 + T_{sur}^2)$, coefficients decrease with increasing r_2, the corresponding increase in the surface area is more than sufficient to provide for a reduction in the total resistance. Even for an insulation thickness of t = 4 mm, h = h + h_r = (7.1 + 5.4) $W/m^2\cdot K$ = 12.5 $W/m^2\cdot K$, and r_{cr} = k/h = 0.25 W/m·K/12.5 $W/m^2\cdot K$ = 0.020m = 20 mm > r_2 = 5 mm. The outer radius of the insulation is therefore well below the critical radius.

PROBLEM 3.43

KNOWN: Diameter of electrical wire. Thickness and thermal conductivity of rubberized sheath. Contact resistance between sheath and wire. Convection coefficient and ambient air temperature. Maximum allowable sheath temperature.

FIND: Maximum allowable power dissipation per unit length of wire. Critical radius of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction through insulation, (3) Constant properties, (4) Negligible radiation exchange with surroundings.

ANALYSIS: The maximum insulation temperature corresponds to its inner surface and is independent of the contact resistance. From the thermal circuit, we may write

$$\dot{E}'_g = q' = \frac{T_{in,i} - T_\infty}{R'_{cond} + R'_{conv}} = \frac{T_{in,i} - T_\infty}{\left[\ell n\left(r_{in,o}/r_{in,i}\right)/2\pi k\right] + \left(1/2\pi r_{in,o} h\right)}$$

where $r_{in,i} = D/2 = 0.001\text{m}$, $r_{in,o} = r_{in,i} + t = 0.003\text{m}$, and $T_{in,i} = T_{max} = 50°\text{C}$ yields the maximum allowable power dissipation. Hence,

$$\dot{E}'_{g,max} = \frac{(50-20)°\text{C}}{\frac{\ell n 3}{2\pi \times 0.13\,\text{W/m}\cdot\text{K}} + \frac{1}{2\pi\,(0.003\text{m})10\,\text{W/m}^2\cdot\text{K}}} = \frac{30°\text{C}}{(1.35+5.31)\,\text{m}\cdot\text{K/W}} = 4.51\,\text{W/m}$$ <

The critical insulation radius is also unaffected by the contact resistance and is given by

$$r_{cr} = \frac{k}{h} = \frac{0.13\,\text{W/m}\cdot\text{K}}{10\,\text{W/m}^2\cdot\text{K}} = 0.013\text{m} = 13\,\text{mm}$$ <

Hence, $r_{in,o} < r_{cr}$ and $E'_{g,max}$ could be increased by increasing $r_{in,o}$ up to a value of 13 mm (t = 12 mm).

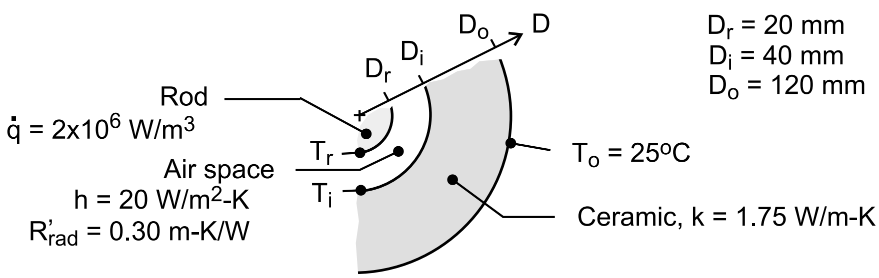
COMMENTS: The contact resistance affects the temperature of the wire, and for $q' = \dot{E}'_{g,max}$ $= 4.51\,\text{W/m}$, the outer surface temperature of the wire is $T_{w,o} = T_{in,i} + q' R'_{t,c} = 50°\text{C} + (4.51\,\text{W/m})$ $\left(3\times10^{-4}\,\text{m}^2\cdot\text{K/W}\right)/\pi\,(0.002\text{m}) = 50.2°\text{C}$. Hence, the temperature change across the contact resistance is negligible.

PROBLEM 3.44

KNOWN: Long rod experiencing uniform volumetric generation of thermal energy, $\dot{q}$, concentric with a hollow ceramic cylinder creating an enclosure filled with air. Thermal resistance per unit length due to radiation exchange between enclosure surfaces is R'_{rad}. The free convection coefficient for the enclosure surfaces is h = 20 W/m^2·K.

FIND: (a) Thermal circuit of the system that can be used to calculate the surface temperature of the rod, T_r; label all temperatures, heat rates and thermal resistances; evaluate the thermal resistances; and (b) Calculate the surface temperature of the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction through the hollow cylinder, (3) The enclosure surfaces experience free convection and radiation exchange.

ANALYSIS: (a) The thermal circuit is shown below. Note labels for the temperatures, thermal resistances and the relevant heat fluxes.

Enclosure, radiation exchange (given):

$$R'_{rad} = 0.30 \text{ m}\cdot\text{K/W}$$

Enclosure, free convection:

$$R'_{cv,rod} = \frac{1}{h\pi D_r} = \frac{1}{20 \text{ W/m}^2\cdot\text{K}\times\pi\times0.020\text{m}} = 0.80 \text{ m}\cdot\text{K/W}$$

$$R'_{cv,cer} = \frac{1}{h\pi D_i} = \frac{1}{20 \text{ W/m}^2\cdot\text{K}\times\pi\times0.040\text{m}} = 0.40 \text{ m}\cdot\text{K/W}$$

Ceramic cylinder, conduction:

$$R'_{cd} = \frac{\ell n\left(D_o/D_i\right)}{2\pi k} = \frac{\ell n\left(0.120/0.040\right)}{2\pi\times1.75 \text{ W/m}\cdot\text{K}} = 0.10 \text{ m}\cdot\text{K/W}$$

The thermal resistance between the enclosure surfaces (r-i) due to convection and radiation exchange is

$$\frac{1}{R'_{enc}} = \frac{1}{R'_{rad}} + \frac{1}{R'_{cv,rod} + R'_{cv,cer}}$$

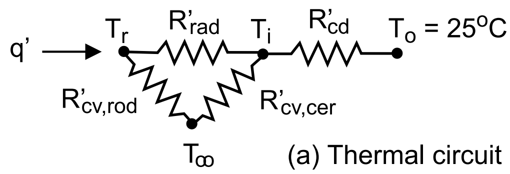
$$R'_{enc} = \left[\frac{1}{0.30} + \frac{1}{0.80+0.40}\right]^{-1} \text{m}\cdot\text{K/W} = 0.24 \text{ m}\cdot\text{K/W}$$

The total resistance between the rod surface (r) and the outer surface of the cylinder (o) is

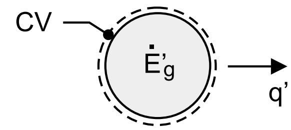
$$R'_{tot} = R'_{enc} + R'_{cd} = \left(0.24+0.1\right)\text{m}\cdot\text{K/W} = 0.34 \text{ m}\cdot\text{K/W}$$

Continued …..

PROBLEM 3.44 (Cont.)



(a) Thermal circuit



(b) Overall energy balance on rod

(b) From an energy balance on the rod (see schematic) find T_r.

$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}'_{gen} = 0$$

$$-q + \dot{q}\forall = 0$$

$$-(T_r - T_i)/R'_{tot} + \dot{q}\left(\pi D_r^2/4\right) = 0$$

$$-(T_r - 25)\text{K}/0.34\ \text{m}\cdot\text{K/W} + 2\times10^6\ \text{W/m}^3\left(\pi\times0.020\text{m}^2/4\right) = 0$$

$$T_r = 239°\text{C}$$ <

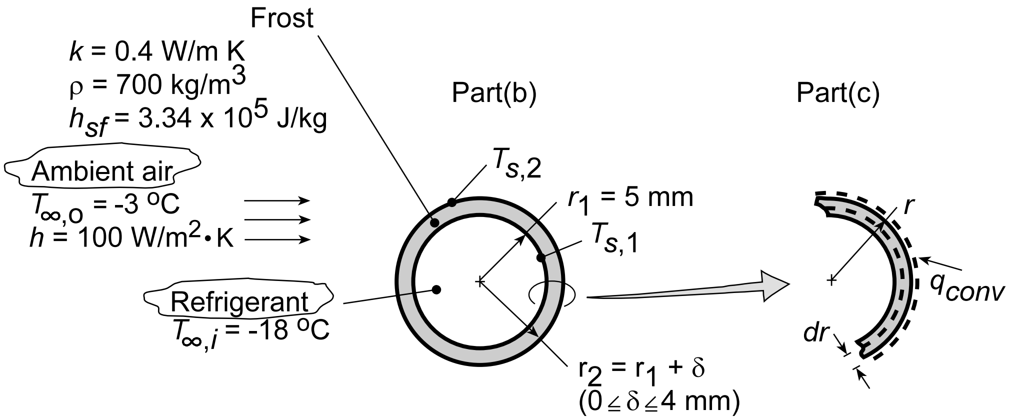
COMMENTS: In evaluating the convection resistance of the air space, it was necessary to define an average air temperature (T_∞) and consider the convection coefficients for each of the space surfaces. As you'll learn later in Chapter 9, correlations are available for directly estimating the convection coefficient (h_{enc}) for the enclosure so that $q_{cv} = h_{enc}(T_r - T_1)$.

PROBLEM 3.45

KNOWN: Tube diameter and refrigerant temperature for evaporator of a refrigerant system. Convection coefficient and temperature of outside air.

FIND: (a) Rate of heat extraction without frost formation, (b) Effect of frost formation on heat rate, (c) Time required for a 2 mm thick frost layer to melt in ambient air for which h = 2 W/m^2·K and T_∞ = 20°C.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible convection resistance for refrigerant flow $\left(T_{\infty,i} = T_{s,1}\right)$, (3) Negligible tube wall conduction resistance, (4) Negligible radiation exchange at outer surface.

ANALYSIS: (a) The cooling capacity in the defrosted condition (δ = 0) corresponds to the rate of heat extraction from the airflow. Hence,

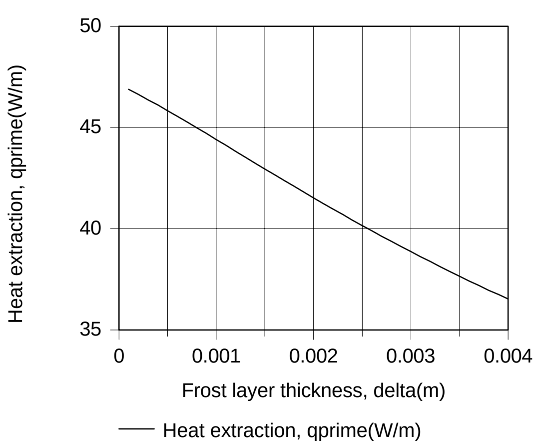
$$q' = h 2\pi r_1 \left(T_{\infty,o} - T_{s,1}\right) = 100\,\mathrm{W/m^2 \cdot K}\left(2\pi \times 0.005\,\mathrm{m}\right)\left(-3+18\right)^\circ \mathrm{C}$$

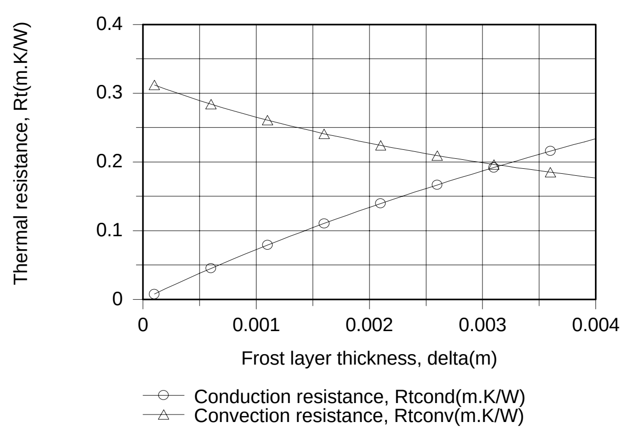
$$q' = 47.1\,\mathrm{W/m}$$ <

(b) With the frost layer, there is an additional (conduction) resistance to heat transfer, and the extraction rate is

$$q' = \frac{T_{\infty,o} - T_{s,1}}{R'_{conv} + R'_{cond}} = \frac{T_{\infty,o} - T_{s,1}}{1/\left(h 2\pi r_2\right) + \ln\left(r_2/r_1\right)/2\pi k}$$

For $5 \le r_2 \le 9$ mm and k = 0.4 W/m·K, this expression yields





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PROBLEM 3.45 (Cont.)

The heat extraction, and hence the performance of the evaporator coil, decreases with increasing frost layer thickness due to an increase in the total resistance to heat transfer. Although the convection resistance decreases with increasing δ, the reduction is exceeded by the increase in the conduction resistance.

(c) The time t_m required to melt a 2 mm thick frost layer may be determined by applying an energy balance, Eq. 1.11b, over the differential time interval dt and to a differential control volume extending inward from the surface of the layer.

$$\dot{E}_{in}dt = dE_{st} = dU_{lat}$$

$$h(2\pi rL)(T_{\infty,o} - T_f)dt = -h_{sf}\,\rho d\forall = -h_{sf}\,\rho(2\pi rL)dr$$

$$h(T_{\infty,o} - T_f)\int_0^{t_m} dt = -\rho h_{sf}\int_{r_2}^{r_1} dr$$

$$t_m = \frac{\rho h_{sf}(r_2 - r_1)}{h(T_{\infty,o} - T_f)} = \frac{700\,\text{kg/m}^3\left(3.34\times10^5\,\text{J/kg}\right)(0.002\,\text{m})}{2\,\text{W/m}^2\cdot\text{K}(20-0)^\circ\text{C}}$$

$$t_m = 11,690\,\text{s} = 3.25\,\text{h}$$ <

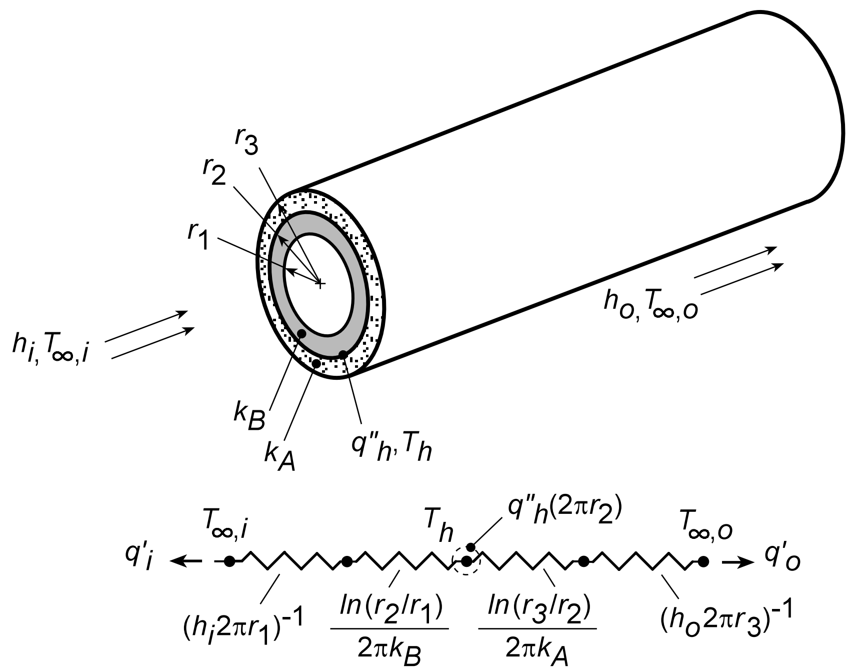
COMMENTS: The tube radius r_1 exceeds the critical radius r_{cr} = k/h = 0.4 W/m·K/100 W/m^2·K = 0.004 m, in which case any frost formation will reduce the performance of the coil.

PROBLEM 3.46

KNOWN: Conditions associated with a composite wall and a thin electric heater.

FIND: (a) Equivalent thermal circuit, (b) Expression for heater temperature, (c) Ratio of outer and inner heat flows and conditions for which ratio is minimized.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant properties, (3) Isothermal heater, (4) Negligible contact resistance(s).

ANALYSIS: (a) On the basis of a unit axial length, the circuit, thermal resistances, and heat rates are as shown in the schematic.

(b) Performing an energy balance for the heater, $\dot{E}_{in} = \dot{E}_{out}$, it follows that

$$q''_h(2\pi r_2) = q'_i + q'_o = \frac{T_h - T_{\infty,i}}{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}} + \frac{T_h - T_{\infty,o}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}} \quad <$$

(c) From the circuit,

$$\frac{q'_o}{q'_i} = \frac{(T_h - T_{\infty,o})}{(T_h - T_{\infty,i})} \times \frac{(h_i 2\pi r_1)^{-1} + \frac{\ln(r_2/r_1)}{2\pi k_B}}{(h_o 2\pi r_3)^{-1} + \frac{\ln(r_3/r_2)}{2\pi k_A}} \quad <$$

To reduce q'_o/q'_i, one could increase k_B, h_i, and r_3/r_2, while reducing k_A, h_o and r_2/r_1.

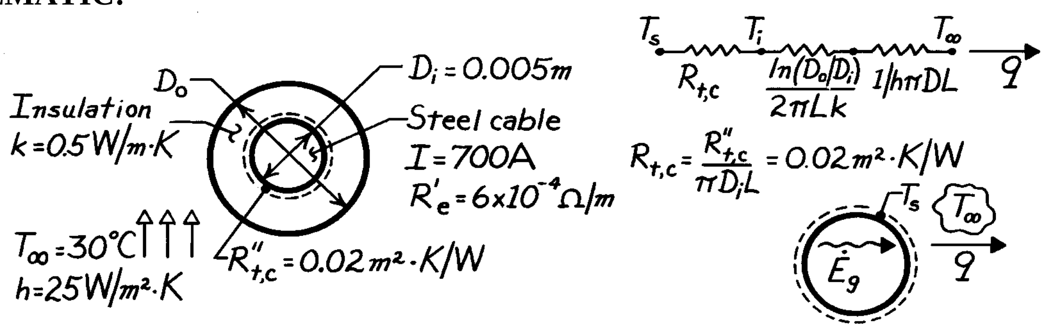
COMMENTS: Contact resistances between the heater and materials A and B could be important.

PROBLEM 3.47

KNOWN: Electric current flow, resistance, diameter and environmental conditions associated with a cable.

FIND: (a) Surface temperature of bare cable, (b) Cable surface and insulation temperatures for a thin coating of insulation, (c) Insulation thickness which provides the lowest value of the maximum insulation temperature. Corresponding value of this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties.

ANALYSIS: (a) The rate at which heat is transferred to the surroundings is fixed by the rate of heat generation in the cable. Performing an energy balance for a control surface about the cable, it follows that $\dot{E}_g = q$ or, for the bare cable, $I^2 R'_e L = h(\pi D_i L)(T_s - T_\infty)$. With $q' = I^2 R'_e = (700\text{A})^2 \left(6\times10^{-4}\,\Omega/\text{m}\right) = 294\text{ W/m}$, it follows that

$$T_s = T_\infty + \frac{q'}{h\pi D_i} = 30^\circ\text{C} + \frac{294\text{ W/m}}{\left(25\text{ W/m}^2\cdot\text{K}\right)\pi(0.005\text{m})}$$

$$T_s = 778.7^\circ\text{C}.$$ <

(b) With a thin coating of insulation, there exist contact and convection resistances to heat transfer from the cable. The heat transfer rate is determined by heating within the cable, however, and therefore remains the same.

$$q = \frac{T_s - T_\infty}{R_{t,c} + \dfrac{1}{h\pi D_i L}} = \frac{T_s - T_\infty}{\dfrac{R''_{t,c}}{\pi D_i L} + \dfrac{1}{h\pi D_i L}}$$

$$q' = \frac{\pi D_i (T_s - T_\infty)}{R''_{t,c} + 1/h}$$

and solving for the surface temperature, find

$$T_s = \frac{q'}{\pi D_i}\left[R''_{t,c} + \frac{1}{h}\right] + T_\infty = \frac{294\text{ W/m}}{\pi(0.005\text{m})}\left[0.02\frac{\text{m}^2\cdot\text{K}}{\text{W}} + 0.04\frac{\text{m}^2\cdot\text{K}}{\text{W}}\right] + 30^\circ\text{C}$$

$$T_s = 1153^\circ\text{C}.$$ <

Continued …..

The insulation temperature is then obtained from

$$q=\frac{T_s - T_i}{R_{t,c}}$$

or

$$T_i = T_s - qR_{t,c} = 1153^\circ C - q\frac{R''_{t,c}}{\pi\ D_i L} = 1153^\circ C - \frac{294\frac{W}{m}\times 0.02\frac{m^2\cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 778.7^\circ C.$$ <

(c) The maximum insulation temperature could be reduced by reducing the resistance to heat transfer from the outer surface of the insulation. Such a reduction is possible if $D_i < D_{cr}$. From Example 3.4,

$$r_{cr} = \frac{k}{h} = \frac{0.5\ W/m\cdot K}{25\ W/m^2\cdot K} = 0.02m.$$

Hence, D_{cr} = 0.04m > D_i = 0.005m. To minimize the maximum temperature, which exists at the inner surface of the insulation, add insulation in the amount

$$t=\frac{D_o - D_i}{2} = \frac{D_{cr} - D_i}{2} = \frac{(0.04 - 0.005)m}{2}$$

t = 0.0175m. <

The cable surface temperature may then be obtained from

$$q'=\frac{T_s - T_\infty}{\frac{R''_{t,c}}{\pi\ D_i} + \frac{\ln(D_{cr}/D_i)}{2\pi\ k} + \frac{1}{h\pi\ D_{cr}}} = \frac{T_s - 30^\circ C}{\frac{0.02\ m^2\cdot K/W}{\pi(0.005m)} + \frac{\ln(0.04/0.005)}{2\pi(0.5\ W/m\cdot K)} + \frac{1}{25\frac{W}{m^2\cdot K}\pi(0.04m)}}$$

Hence,

$$294\frac{W}{m} = \frac{T_s - 30^\circ C}{(1.27+0.66+0.32)m\cdot K/W} = \frac{T_s - 30^\circ C}{2.25\ m\cdot K/W}$$

$$T_s = 692.5^\circ C$$

Recognizing that q = (T_s - T_i)/$R_{t,c}$, find

$$T_i = T_s - qR_{t,c} = T_s - q\frac{R''_{t,c}}{\pi\ D_i L} = 692.5^\circ C - \frac{294\frac{W}{m}\times 0.02\frac{m^2\cdot K}{W}}{\pi(0.005m)}$$

$$T_i = 318.2^\circ C.$$ <

COMMENTS: Use of the critical insulation thickness in lieu of a thin coating has the effect of reducing the maximum insulation temperature from 778.7°C to 318.2°C. Use of the critical insulation thickness also reduces the cable surface temperature to 692.5°C from 778.7°C with no insulation or from 1153°C with a thin coating.

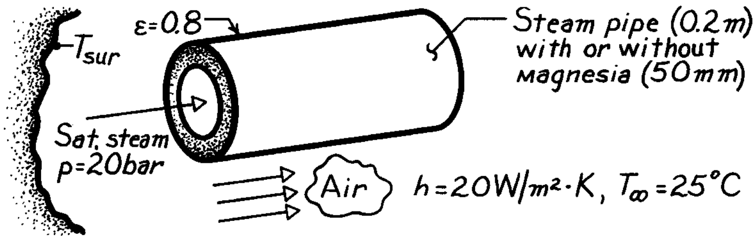
PROBLEM 3.48

KNOWN: Saturated steam conditions in a pipe with prescribed surroundings.

FIND: (a) Heat loss per unit length from bare pipe and from insulated pipe, (b) Pay back period for insulation.

SCHEMATIC:

Steam Costs:
$4 for 10^9 J
Insulation Cost:
$100 per meter
Operation time:
7500 h/yr



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible pipe wall resistance, (5) Negligible steam side convection resistance (pipe inner surface temperature is equal to steam temperature), (6) Negligible contact resistance, (7) $T_{sur} = T_\infty$.

PROPERTIES: *Table A-6*, Saturated water (p = 20 bar): $T_{sat} = T_s = 486K$; *Table A-3*, Magnesia, 85% (T ≈ 392K): k = 0.058 W/m·K.

ANALYSIS: (a) Without the insulation, the heat loss may be expressed in terms of radiation and convection rates,

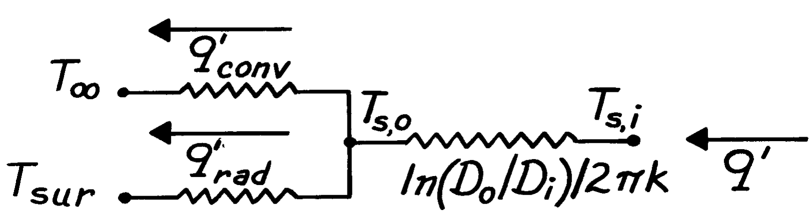
$$q' = \varepsilon\pi D\sigma\left(T_s^4 - T_{sur}^4\right) + h(\pi D)(T_s - T_\infty)$$

$$q' = 0.8\pi(0.2m)5.67\times10^{-8}\ \frac{W}{m^2\cdot K^4}\left(486^4 - 298^4\right)K^4$$

$$+20\frac{W}{m^2\cdot K}(\pi\times0.2m)\ (486\text{-}298)K$$

$$q' = (1365+2362)\,W/m = 3727\ W/m.$$ <

With the insulation, the thermal circuit is of the form



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From an energy balance at the outer surface of the insulation,

$$q'_{cond} = q'_{conv} + q'_{rad}$$

$$\frac{T_{s,i} - T_{s,o}}{\ln\left(D_o / D_i\right)/2\pi\ k} = h\pi\ D_o\left(T_{s,o} - T_\infty\right) + \varepsilon\sigma\pi\ D_o\left(T_{s,o}^4 - T_{sur}^4\right)$$

$$\frac{\left(486 - T_{s,o}\right)K}{\dfrac{\ln\left(0.3\text{m}/0.2\text{m}\right)}{2\pi\left(0.058\ \text{W/m}\cdot\text{K}\right)}} = 20\frac{\text{W}}{\text{m}^2\cdot\text{K}}\pi\left(0.3\text{m}\right)\left(T_{s,o} - 298\text{K}\right)$$

$$+0.8\times5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\pi\left(0.3\text{m}\right)\left(T_{s,o}^4 - 298^4\right)\text{K}^4.$$

By trial and error, we obtain

$$T_{s,o} \approx 305\text{K}$$

in which case

$$q' = \frac{\left(486\text{-}305\right)\text{K}}{\dfrac{\ln\left(0.3\text{m}/0.2\text{m}\right)}{2\pi\left(0.055\ \text{W/m}\cdot\text{K}\right)}} = 163\ \text{W/m}.$$ <

(b) The yearly energy savings per unit length of pipe due to use of the insulation is

$$\frac{\text{Savings}}{\text{Yr}\cdot\text{m}} = \frac{\text{Energy Savings}}{\text{Yr.}}\times\frac{\text{Cost}}{\text{Energy}}$$

$$\frac{\text{Savings}}{\text{Yr}\cdot\text{m}} = \left(3727 - 163\right)\frac{\text{J}}{\text{s}\cdot\text{m}}\times3600\frac{\text{s}}{\text{h}}\times7500\frac{\text{h}}{\text{Yr}}\times\frac{\$4}{10^9\text{J}}$$

$$\frac{\text{Savings}}{\text{Yr}\cdot\text{m}} = \$385/\text{Yr}\cdot\text{m}.$$

The pay back period is then

$$\text{Pay Back Period} = \frac{\text{Insulation Costs}}{\text{Savings/Yr.}\cdot\text{m}} = \frac{\$100/\text{m}}{\$385/\text{Yr}\cdot\text{m}}$$

Pay Back Period = 0.26 Yr = 3.1 mo. <

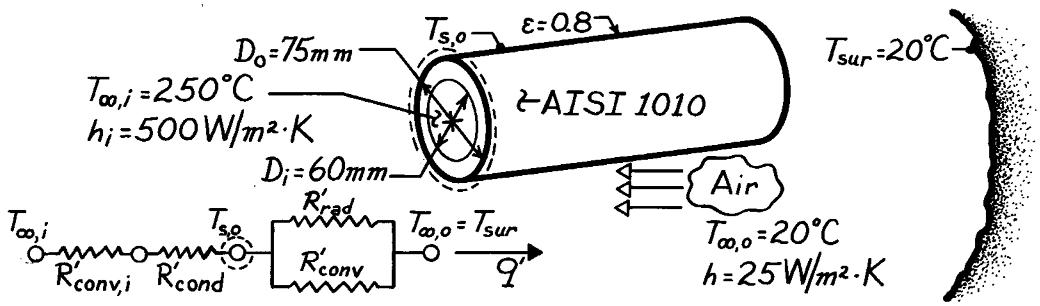
COMMENTS: Such a low pay back period is more than sufficient to justify investing in the insulation.

PROBLEM 3.49

KNOWN: Temperature and convection coefficient associated with steam flow through a pipe of prescribed inner and outer diameters. Outer surface emissivity and convection coefficient. Temperature of ambient air and surroundings.

FIND: Heat loss per unit length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Surroundings form a large enclosure about pipe.

PROPERTIES: *Table A-1*, Steel, AISI 1010 (T ≈ 450 K): k = 56.5 W/m·K.

ANALYSIS: Referring to the thermal circuit, it follows from an energy balance on the outer surface that

$$\frac{T_{\infty,i}-T_{s,o}}{R_{conv,i}+R_{cond}}=\frac{T_{s,o}-T_{\infty,o}}{R_{conv,o}}+\frac{T_{s,o}-T_{sur}}{R_{rad}}$$

or from Eqs. 3.9, 3.28 and 1.7,

$$\frac{T_{\infty,i}-T_{s,o}}{(1/\pi\ D_i h_i)+\ln(D_o/D_i)/2\pi k}=\frac{T_{s,o}-T_{\infty,o}}{(1/\pi\ D_o h_o)}+\varepsilon\pi\ D_o\sigma\left(T_{s,o}^4-T_{sur}^4\right)$$

$$\frac{523\text{K}\ -\ T_{s,o}}{\left(\pi\times 0.6\text{m}\times 500\ \text{W/m}^2\cdot\text{K}\right)^{-1}+\dfrac{\ln\ (75/60)}{2\pi\times 56.5\ \text{W/m}\cdot\text{K}}}=\frac{T_{s,o}-293\text{K}}{\left(\pi\times 0.075\text{m}\times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1}}$$

$$+0.8\pi\times(0.075\text{m})\times 5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left[T_{s,o}^4-293^4\right]\text{K}^4$$

$$\frac{523-T_{s,o}}{0.0106+0.0006}=\frac{T_{s,o}-293}{0.170}+1.07\times 10^{-8}\left[T_{s,o}^4-293^4\right].$$

From a trial-and-error solution, $T_{s,o}\approx 502\text{K}$. Hence the heat loss is

$$q'=\pi\ D_o h_o\left(T_{s,o}-T_{\infty,o}\right)+\varepsilon\pi\ D_o\sigma\left(T_{s,o}^4-T_{sur}^4\right)$$

$$q'=\pi\,(0.075\text{m})\,25\ \text{W/m}^2\cdot\text{K}\,(502\text{-}293)+0.8\ \pi\,(0.075\text{m})\,5.67\times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[502^4-243^4\right]\text{K}^4$$

$$q'=1231\ \text{W/m}+600\ \text{W/m}=1831\ \text{W/m}.$$ <

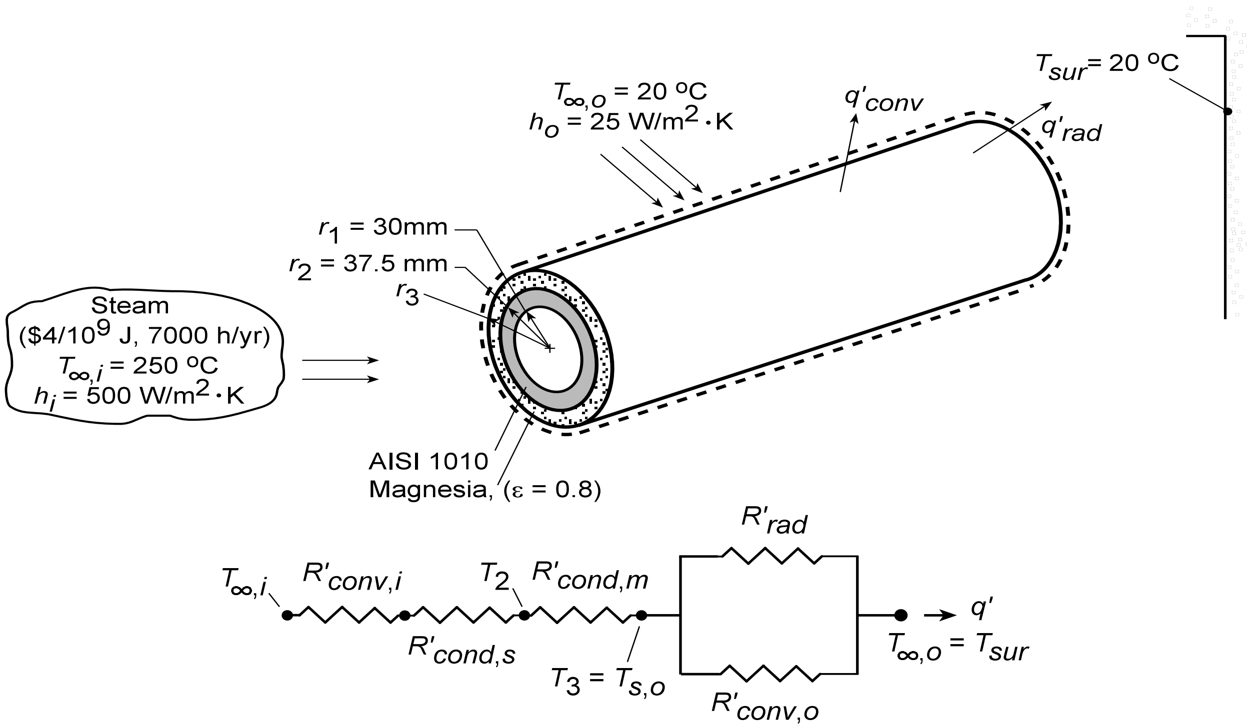
COMMENTS: The thermal resistance between the outer surface and the surroundings is much larger than that between the outer surface and the steam.

PROBLEM 3.50

KNOWN: Temperature and convection coefficient associated with steam flow through a pipe of prescribed inner and outer radii. Emissivity of outer surface magnesia insulation, and convection coefficient. Temperature of ambient air and surroundings.

FIND: Heat loss per unit length q' and outer surface temperature $T_{s,o}$ as a function of insulation thickness. Recommended insulation thickness. Corresponding annual savings and temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Surroundings form a large enclosure about pipe.

PROPERTIES: *Table A-1*, Steel, AISI 1010 (T ≈ 450 K): k_s = 56.5 W/m·K. *Table A-3*, Magnesia, 85% (T ≈ 365 K): k_m = 0.055 W/m·K.

ANALYSIS: Referring to the thermal circuit, it follows from an energy balance on the outer surface that

$$\frac{T_{\infty,i}-T_{s,o}}{R'_{conv,i}+R'_{cond,s}+R'_{cond,m}}=\frac{T_{s,o}-T_{\infty,o}}{R'_{conv,o}}+\frac{T_{s,o}-T_{sur}}{R'_{rad}}$$

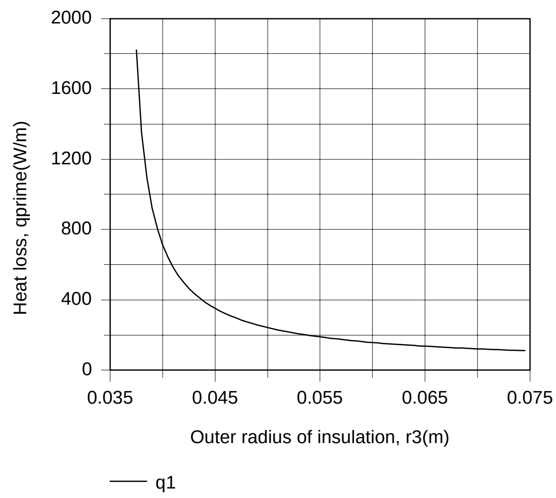
or from Eqs. 3.9, 3.28 and 1.7,

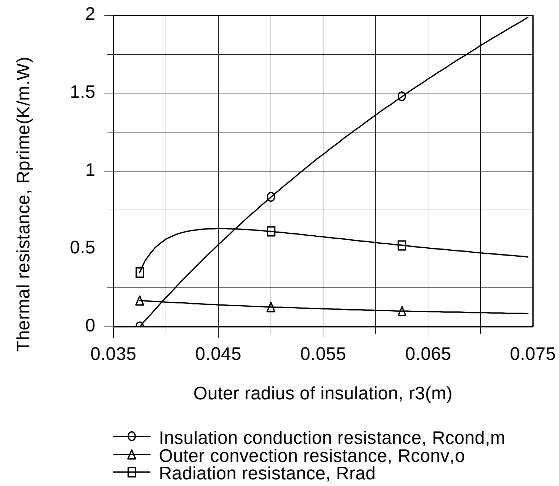
$$\frac{T_{\infty,i}-T_{s,o}}{\left(1/2\pi r_1 h_i\right)+\ln\left(r_2/r_1\right)/2\pi k_s+\ln\left(r_3/r_2\right)/2\pi k_m}=\frac{T_{s,o}-T_{\infty,o}}{\left(1/2\pi r_3 h_o\right)}+\frac{T_{s,o}-T_{sur}}{\left[\left(2\pi r_3\right)\varepsilon\sigma\left(T_{s,o}+T_{sur}\right)\left(T_{s,o}^2+T_{sur}^2\right)\right]^{-1}}$$

This expression may be solved for $T_{s,o}$ as a function of r_3, and the heat loss may then be determined by evaluating either the left-or right-hand side of the energy balance equation. The results are plotted as follows.

Continued...

PROBLEM 3.50 (Cont.)



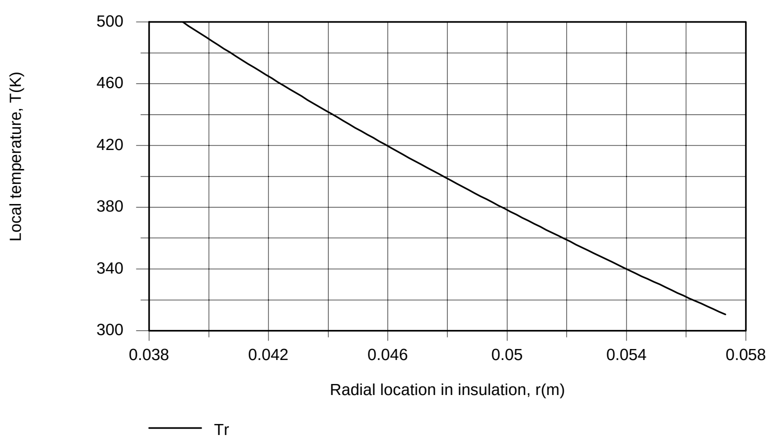


The rapid decay in q' with increasing r_3 is attributable to the dominant contribution which the insulation begins to make to the total thermal resistance. The inside convection and tube wall conduction resistances are fixed at 0.0106 m·K/W and 6.29×10^{-4} m·K/W, respectively, while the resistance of the insulation increases to approximately 2 m·K/W at r_3 = 0.075 m.

The heat loss may be reduced by almost 91% from a value of approximately 1830 W/m at $r_3 = r_2$ = 0.0375 m (no insulation) to 172 W/m at r_3 = 0.0575 m and by only an additional 3% if the insulation thickness is increased to r_3 = 0.0775 m. Hence, an insulation thickness of $(r_3 - r_2)$ = 0.020 m is recommended, for which q' = 172 W/m. The corresponding annual savings (AS) in energy costs is therefore

$$AS = \left[(1830 - 172)\,W/m\right]\frac{\$4}{10^9\,J}\times 7000\frac{h}{y}\times 3600\frac{s}{h} = \$167/m \qquad <$$

The corresponding temperature distribution is



The temperature in the insulation decreases from $T(r) = T_2$ = 521 K at $r = r_2$ = 0.0375 m to $T(r) = T_3$ = 309 K at $r = r_3$ = 0.0575 m.

Continued...

PROBLEM 3.50 (Cont.)

COMMENTS: 1. The annual energy and costs savings associated with insulating the steam line are substantial, as is the reduction in the outer surface temperature (from $T_{s,o} \approx 502$ K for $r_3 = r_2$, to 309 K for $r_3 = 0.0575$ m).

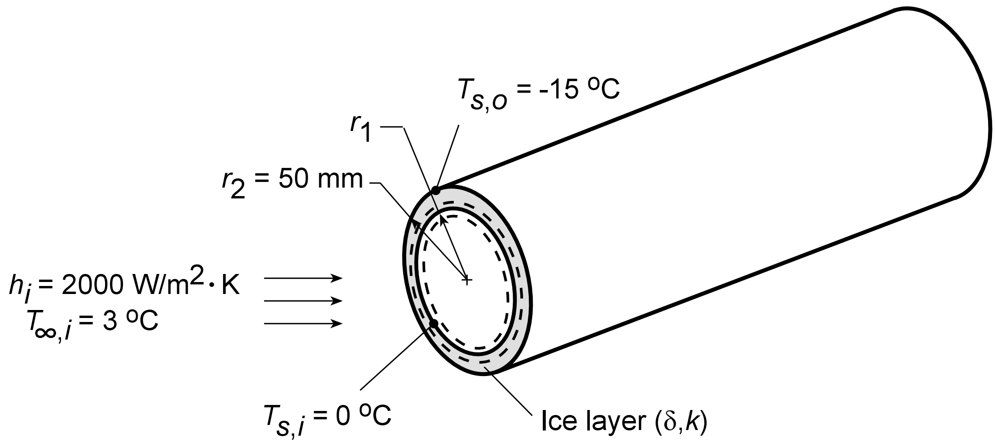
2. The increase in R'_{rad} to a maximum value of 0.63 m·K/W at $r_3 = 0.0455$ m and the subsequent decay is due to the competing effects of h_{rad} and $A'_3 = (1/2\pi r_3)$. Because the initial decay in $T_3 = T_{s,o}$ with increasing r_3, and hence, the reduction in h_{rad}, is more pronounced than the increase in A'_3, R'_{rad} increases with r_3. However, as the decay in $T_{s,o}$, and hence h_{rad}, becomes less pronounced, the increase in A'_3 becomes more pronounced and R'_{rad} decreases with increasing r_3.

PROBLEM 3.51

KNOWN: Pipe wall temperature and convection conditions associated with water flow through the pipe and ice layer formation on the inner surface.

FIND: Ice layer thickness δ.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Negligible pipe wall thermal resistance, (3) negligible ice/wall contact resistance, (4) Constant k.

PROPERTIES: *Table A.3*, Ice (T = 265 K): k ≈ 1.94 W/m·K.

ANALYSIS: Performing an energy balance for a control surface about the ice/water interface, it follows that, for a unit length of pipe,

$$q'_{conv} = q'_{cond}$$

$$h_i\,(2\pi r_1)\left(T_{\infty,i} - T_{s,i}\right) = \frac{T_{s,i} - T_{s,o}}{\ln\left(r_2/r_1\right)/2\pi k}$$

Dividing both sides of the equation by r_2,

$$\frac{\ln\left(r_2/r_1\right)}{\left(r_2/r_1\right)} = \frac{k}{h_i r_2} \times \frac{T_{s,i} - T_{s,o}}{T_{\infty,i} - T_{s,i}} = \frac{1.94\,\text{W/m}\cdot\text{K}}{\left(2000\,\text{W/m}^2\cdot\text{K}\right)(0.05\,\text{m})} \times \frac{15^\circ\text{C}}{3^\circ\text{C}} = 0.097$$

The equation is satisfied by $r_2/r_1 = 1.114$, in which case $r_1 = 0.050\text{ m}/1.114 = 0.045\text{ m}$, and the ice layer thickness is

$$\delta = r_2 - r_1 = 0.005\,\text{m} = 5\,\text{mm}$$ <

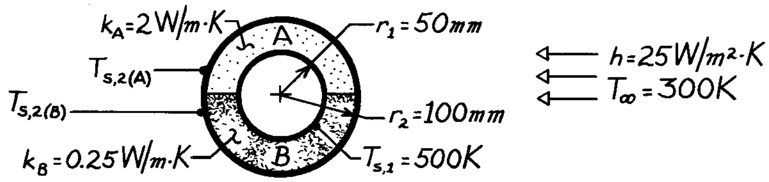
COMMENTS: With no flow, $h_i \rightarrow 0$, in which case $r_1 \rightarrow 0$ and complete blockage could occur. The pipe should be insulated.

PROBLEM 3.52

KNOWN: Inner surface temperature of insulation blanket comprised of two semi-cylindrical shells of different materials. Ambient air conditions.

FIND: (a) Equivalent thermal circuit, (b) Total heat loss and material outer surface temperatures.

SCHEMATIC:



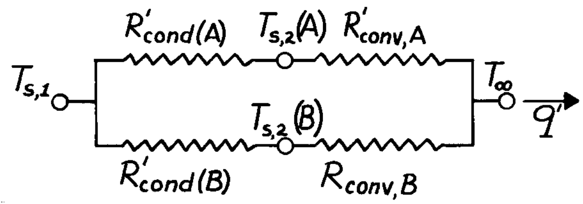
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial conduction, (3) Infinite contact resistance between materials, (4) Constant properties.

ANALYSIS: (a) The thermal circuit is,

$$R'_{conv,A} = R'_{conv,B} = 1/\pi\, r_2 h$$

$$R'_{cond(A)} = \frac{\ln(r_2/r_1)}{\pi\, k_A}$$

$$R'_{cond(\mathbf{B})} = \frac{\ln(r_2/r_i)}{\pi\, k_B}$$



<

The conduction resistances follow from Section 3.3.1 and Eq. 3.28. Each resistance is larger by a factor of 2 than the result of Eq. 3.28 due to the reduced area.

(b) Evaluating the thermal resistances and the heat rate $(q' = q'_A + q'_B)$,

$$R'_{conv} = \left(\pi \times 0.1\text{m} \times 25\ \text{W/m}^2\cdot\text{K}\right)^{-1} = 0.1273\ \text{m}\cdot\text{K/W}$$

$$R'_{cond(A)} = \frac{\ln(0.1\text{m}/0.05\text{m})}{\pi \times 2\ \text{W/m}\cdot\text{K}} = 0.1103\ \text{m}\cdot\text{K/W} \quad R'_{cond(B)} = 8\ R'_{cond(A)} = 0.8825\ \text{m}\cdot\text{K/W}$$

$$q' = \frac{T_{s,1} - T_\infty}{R'_{cond(A)} + R'_{conv}} + \frac{T_{s,1} - T_\infty}{R'_{cond(B)} + R'_{conv}}$$

$$q' = \frac{(500-300)\text{K}}{(0.1103+0.1273)\text{m}\cdot\text{K/W}} + \frac{(500-300)\text{K}}{(0.8825+0.1273)\text{m}\cdot\text{K/W}} = (842+198)\text{W/m} = 1040\ \text{W/m}.$$ <

Hence, the temperatures are

$$T_{s,2(A)} = T_{s,1} - q'_A R'_{cond(A)} = 500\text{K} - 842\frac{\text{W}}{\text{m}} \times 0.1103\frac{\text{m}\cdot\text{K}}{\text{W}} = 407\text{K}$$ <

$$T_{s,2(B)} = T_{s,1} - q'_B R'_{cond(B)} = 500\text{K} - 198\frac{\text{W}}{\text{m}} \times 0.8825\frac{\text{m}\cdot\text{K}}{\text{W}} = 325\text{K}.$$ <

COMMENTS: The total heat loss can also be computed from $q' = (T_{s,1} - T_\infty)/R_{equiv}$,

where $R_{equiv} = \left[\left(R'_{cond(A)} + R'_{conv,A}\right)^{-1} + \left(R'_{cond(B)} + R'_{conv,B}\right)^{-1}\right]^{-1} = 0.1923\ \text{m}\cdot\text{K/W}.$

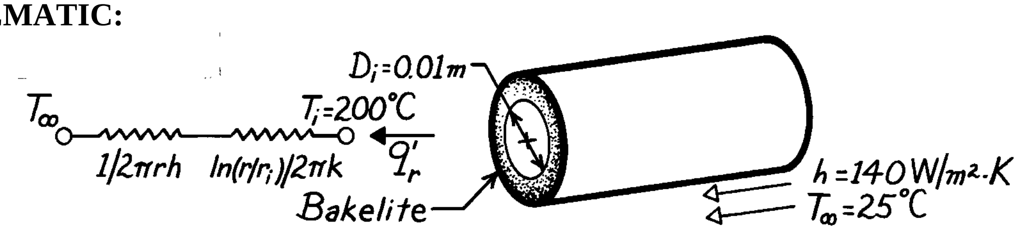
Hence $q' = (500-300)\text{K}/0.1923\ \text{m}\cdot\text{K/W} = 1040\ \text{W/m}.$

PROBLEM 3.53

KNOWN: Surface temperature of a circular rod coated with bakelite and adjoining fluid conditions.

FIND: (a) Critical insulation radius, (b) Heat transfer per unit length for bare rod and for insulation at critical radius, (c) Insulation thickness needed for 25% heat rate reduction.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r, (3) Constant properties, (4) Negligible radiation and contact resistance.

PROPERTIES: *Table A-3*, Bakelite (300K): k = 1.4 W/m·K.

ANALYSIS: (a) From Example 3.4, the critical radius is

$$r_{cr} = \frac{k}{h} = \frac{1.4\ \text{W/m}\cdot\text{K}}{140\ \text{W/m}^2\cdot\text{K}} = 0.01\text{m}. \qquad <$$

(b) For the bare rod,

$$q' = h\left(\pi\ D_i\right)\ \left(T_i - T_\infty\right)$$

$$q' = 140\frac{\text{W}}{\text{m}^2\cdot\text{K}}\left(\pi\times 0.01\text{m}\right)\ \left(200-25\right)^\circ\text{C} = 770\ \text{W/m} \qquad <$$

For the critical insulation thickness,

$$q' = \frac{T_i - T_\infty}{\dfrac{1}{2\pi\ r_{cr}h} + \dfrac{\ln\left(r_{cr}/r_i\right)}{2\pi\ k}} = \frac{\left(200-25\right)^\circ\text{C}}{\dfrac{1}{2\pi\times\left(0.01\text{m}\right)\times 140\ \text{W/m}^2\cdot\text{K}} + \dfrac{\ln\left(0.01\text{m}/0.005\text{m}\right)}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}}$$

$$q' = \frac{175^\circ\text{C}}{\left(0.1137+0.0788\right)\ \text{m}\cdot\text{K/W}} = 909\ \text{W/m} \qquad <$$

(c) The insulation thickness needed to reduce the heat rate to 577 W/m is obtained from

$$q' = \frac{T_i - T_\infty}{\dfrac{1}{2\pi\ rh} + \dfrac{\ln\left(r/r_i\right)}{2\pi\ k}} = \frac{\left(200-25\right)^\circ\text{C}}{\dfrac{1}{2\pi\left(r\right)140\ \text{W/m}^2\cdot\text{K}} + \dfrac{\ln\left(r/0.005\text{m}\right)}{2\pi\times 1.4\ \text{W/m}\cdot\text{K}}} = 577\frac{\text{W}}{\text{m}}$$

From a trial-and-error solution, find

$$r \approx 0.06\ \text{m}.$$

The desired insulation thickness is then

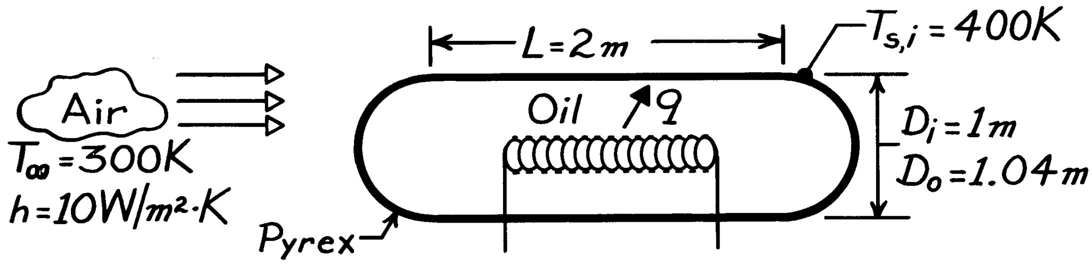
$$\delta = \left(r - r_i\right) \approx \left(0.06 - 0.005\right)\text{m} = 55\ \text{mm}. \qquad <$$

PROBLEM 3.54

KNOWN: Geometry of an oil storage tank. Temperature of stored oil and environmental conditions.

FIND: Heater power required to maintain a prescribed inner surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in radial direction, (3) Constant properties, (4) Negligible radiation.

PROPERTIES: *Table A-3*, Pyrex (300K): k = 1.4 W/m·K.

ANALYSIS: The rate at which heat must be supplied is equal to the loss through the cylindrical and hemispherical sections. Hence,

$$q = q_{cyl} + 2q_{hemi} = q_{cyl} + q_{spher}$$

or, from Eqs. 3.28 and 3.36,

$$q = \frac{T_{s,i} - T_\infty}{\dfrac{\ln(D_o/D_i)}{2\pi\, Lk} + \dfrac{1}{\pi\, D_o L h}} + \frac{T_{s,i} - T_\infty}{\dfrac{1}{2\pi\, k}\left[\dfrac{1}{D_i} - \dfrac{1}{D_o}\right] + \dfrac{1}{\pi D_o^2 h}}$$

$$q = \frac{(400-300)\text{K}}{\dfrac{\ln 1.04}{2\pi(2\text{m})1.4\ \text{W/m}\cdot\text{K}} + \dfrac{1}{\pi(1.04\text{m})2\text{m}\left(10\ \text{W/m}^2\cdot\text{K}\right)}}$$

$$+ \frac{(400-300)\text{K}}{\dfrac{1}{2\pi(1.4\ \text{W/m}\cdot\text{K})}(1-0.962)\text{m}^{-1} + \dfrac{1}{\pi(1.04\text{m})^2 10\ \text{W/m}^2\cdot\text{K}}}$$

$$q = \frac{100\text{K}}{2.23\times10^{-3}\ \text{K/W} + 15.30\times10^{-3}\ \text{K/W}} + \frac{100\text{K}}{4.32\times10^{-3}\ \text{K/W} + 29.43\times10^{-3}}$$

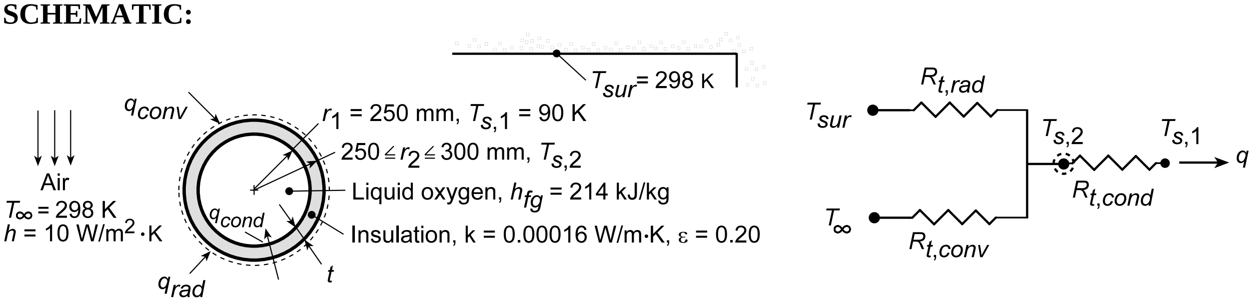
q = 5705W + 2963W = 8668W. <

PROBLEM 3.55

KNOWN: Diameter of a spherical container used to store liquid oxygen and properties of insulating material. Environmental conditions.

FIND: (a) Reduction in evaporative oxygen loss associated with a prescribed insulation thickness, (b) Effect of insulation thickness on evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, one-dimensional conduction, (2) Negligible conduction resistance of container wall and contact resistance between wall and insulation, (3) Container wall at boiling point of liquid oxygen.

ANALYSIS: (a) Applying an energy balance to a control surface about the insulation, $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that $q_{conv} + q_{rad} = q_{cond} = q$. Hence,

$$\frac{T_\infty - T_{s,2}}{R_{t,conv}} + \frac{T_{sur} - T_{s,2}}{R_{t,rad}} = \frac{T_{s,2} - T_{s,1}}{R_{t,cond}} = q \tag{1}$$

where $R_{t,conv} = \left(4\pi r_2^2 h\right)^{-1}$, $R_{t,rad} = \left(4\pi r_2^2 h_r\right)^{-1}$, $R_{t,cond} = (1/4\pi k)\left[(1/r_1) - (1/r_2)\right]$, and, from Eq. 1.9, the radiation coefficient is $h_r = \varepsilon\sigma\left(T_{s,2} + T_{sur}\right)\left(T_{s,2}^2 + T_{sur}^2\right)$. With t = 10 mm ($r_2$ = 260 mm), ε = 0.2 and $T_\infty = T_{sur}$ = 298 K, an iterative solution of the energy balance equation yields $T_{s,2} \approx 297.7$ K, where $R_{t,conv}$ = 0.118 K/W, $R_{t,rad}$ = 0.982 K/W and $R_{t,cond}$ = 76.5 K/W. With the insulation, it follows that the heat gain is

$$q_w \approx 2.72 \text{ W}$$

Without the insulation, the heat gain is

$$q_{wo} = \frac{T_\infty - T_{s,1}}{R_{t,conv}} + \frac{T_{sur} - T_{s,1}}{R_{t,rad}}$$

where, with $r_2 = r_1$, $T_{s,1}$ = 90 K, $R_{t,conv}$ = 0.127 K/W and $R_{t,rad}$ = 3.14 K/W. Hence,

$$q_{wo} = 1702 \text{ W}$$

With the oxygen mass evaporation rate given by $\dot{m} = q/h_{fg}$, the percent reduction in evaporated oxygen is

$$\%\ \text{Reduction} = \frac{\dot{m}_{wo} - \dot{m}_w}{\dot{m}_{wo}} \times 100\% = \frac{q_{wo} - q_w}{q_{wo}} \times 100\%$$

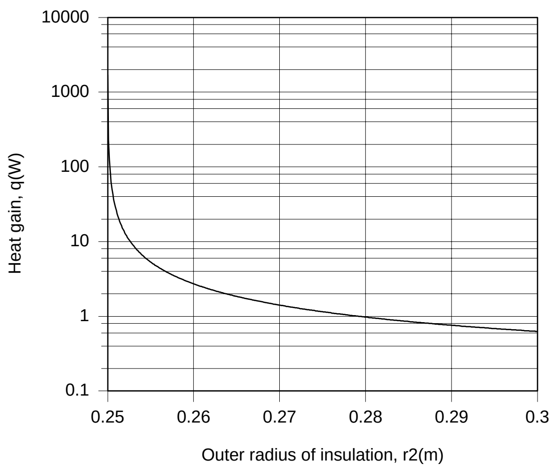
Hence,

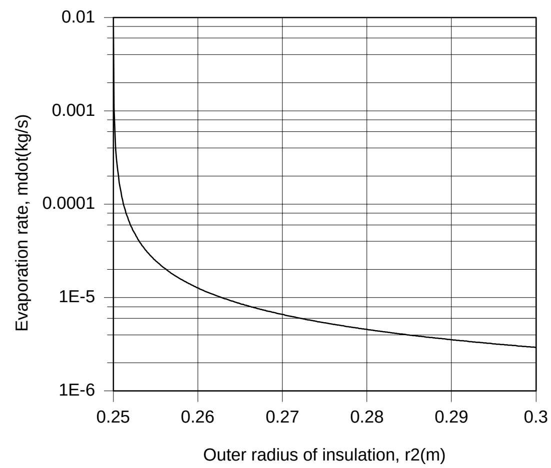
$$\%\ \text{Reduction} = \frac{(1702 - 2.7)\text{ W}}{1702\text{ W}} \times 100\% = 99.8\%$$ <

Continued...

PROBLEM 3.55 (Cont.)

(b) Using Equation (1) to compute $T_{s,2}$ and q as a function of r_2, the corresponding evaporation rate, $\dot{m} = q/h_{fg}$, may be determined. Variations of q and $\dot{m}$ with r_2 are plotted as follows.





Because of its extremely low thermal conductivity, significant benefits are associated with using even a thin layer of insulation. Nearly three-order magnitude reductions in q and $\dot{m}$ are achieved with $r_2 = 0.26$ m. With increasing r_2, q and $\dot{m}$ decrease from values of 1702 W and 8×10^{-3} kg/s at $r_2 = 0.25$ m to 0.627 W and 2.9×10^{-6} kg/s at $r_2 = 0.30$ m.

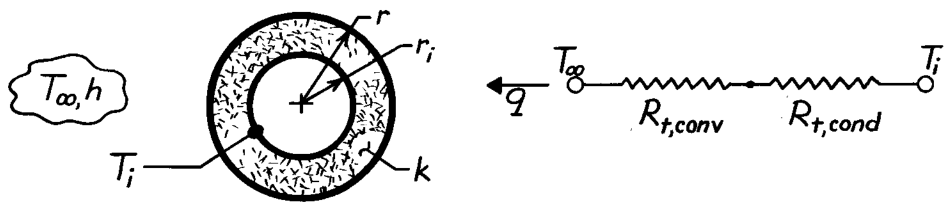
COMMENTS: Laminated metallic-foil/glass-mat insulations are extremely effective and corresponding conduction resistances are typically much larger than those normally associated with surface convection and radiation.

PROBLEM 3.56

KNOWN: Sphere of radius r_i, covered with insulation whose outer surface is exposed to a convection process.

FIND: Critical insulation radius, r_{cr}.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (spherical) conduction, (3) Constant properties, (4) Negligible radiation at surface.

ANALYSIS: The heat rate follows from the thermal circuit shown in the schematic,

$$q = (T_i - T_\infty)/R_{tot}$$

where $R_{tot} = R_{t,conv} + R_{t,cond}$ and

$$R_{t,conv} = \frac{1}{hA_s} = \frac{1}{4\pi\ hr^2} \qquad (3.9)$$

$$R_{t,cond} = \frac{1}{4\pi\ k}\left[\frac{1}{r_i} - \frac{1}{r}\right] \qquad (3.36)$$

If q is a maximum or minimum, we need to find the condition for which

$$\frac{d\ R_{tot}}{dr} = 0.$$

It follows that

$$\frac{d}{dr}\left[\frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r}\right] + \frac{1}{4\pi\ hr^2}\right] = \left[+\frac{1}{4\pi\ k}\frac{1}{r^2} - \frac{1}{2\pi\ h}\frac{1}{r^3}\right] = 0$$

giving

$$r_{cr} = 2\frac{k}{h}$$

The second derivative, evaluated at $r = r_{cr}$, is

$$\frac{d}{dr}\left[\frac{dR_{tot}}{dr}\right] = -\frac{1}{2\pi\ k}\ \frac{1}{r^3} + \frac{3}{2\pi\ h}\ \frac{1}{r^4}\Bigg]_{r=r_{cr}}$$

$$= \frac{1}{(2k/h)^3}\left\{-\frac{1}{2\pi\ k} + \frac{3}{2\pi\ h}\frac{1}{2k/h}\right\} = \frac{1}{(2k/h)^3}\frac{1}{2\pi\ k}\left\{-1 + \frac{3}{2}\right\} > 0$$

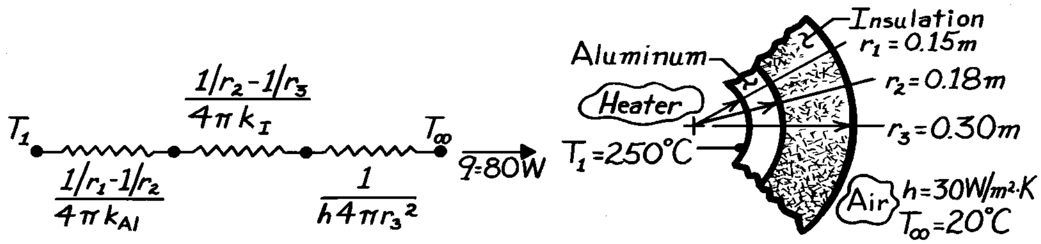
Hence, it follows no optimum R_{tot} exists. We refer to this condition as the critical insulation radius. See Example 3.4 which considers this situation for a cylindrical system.

PROBLEM 3.57

KNOWN: Thickness of hollow aluminum sphere and insulation layer. Heat rate and inner surface temperature. Ambient air temperature and convection coefficient.

FIND: Thermal conductivity of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation exchange at outer surface.

PROPERTIES: *Table A-1*, Aluminum (523K): $k \approx 230$ W/m·K.

ANALYSIS: From the thermal circuit,

$$q = \frac{T_1 - T_\infty}{R_{tot}} = \frac{T_1 - T_\infty}{\frac{1/r_1 - 1/r_2}{4\pi\, k_{A1}} + \frac{1/r_2 - 1/r_3}{4\pi\, k_I} + \frac{1}{h4\pi\, r_3^2}}$$

$$q = \frac{(250-20)^\circ \mathrm{C}}{\left[\frac{1/0.15 - 1/0.18}{4\pi(230)} + \frac{1/0.18 - 1/0.30}{4\pi\, k_I} + \frac{1}{30(4\pi)(0.3)^2}\right]\frac{\mathrm{K}}{\mathrm{W}}} = 80\ \mathrm{W}$$

or

$$3.84\times10^{-4} + \frac{0.177}{k_I} + 0.029 = \frac{230}{80} = 2.875.$$

Solving for the unknown thermal conductivity, find

$$k_I = 0.062\ \mathrm{W/m\cdot K}. \qquad <$$

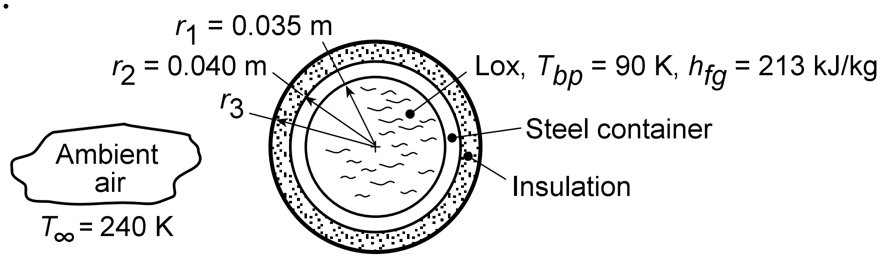
COMMENTS: The dominant contribution to the total thermal resistance is made by the insulation. Hence uncertainties in knowledge of h or k_{A1} have a negligible effect on the accuracy of the k_I measurement.

PROBLEM 3.58

KNOWN: Dimensions of spherical, stainless steel liquid oxygen (LOX) storage container. Boiling point and latent heat of fusion of LOX. Environmental temperature.

FIND: Thermal isolation system which maintains boil-off below 1 kg/day.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible thermal resistances associated with internal and external convection, conduction in the container wall, and contact between wall and insulation, (3) Negligible radiation at exterior surface, (4) Constant insulation thermal conductivity.

PROPERTIES: *Table A.1*, 304 Stainless steel (T = 100 K): k_s = 9.2 W/m·K; *Table A.3*, Reflective, aluminum foil-glass paper insulation (T = 150 K): k_i = 0.000017 W/m·K.

ANALYSIS: The heat gain associated with a loss of 1 kg/day is

$$q = \dot{m}h_{fg} = \frac{1\,\text{kg/day}}{86,400\,\text{s/day}}\left(2.13\times10^{5}\ \text{J/kg}\right) = 2.47\,\text{W}$$

With an overall temperature difference of $\left(T_\infty - T_{bp}\right)$ = 150 K, the corresponding total thermal resistance is

$$R_{tot} = \frac{\Delta T}{q} = \frac{150\,\text{K}}{2.47\,\text{W}} = 60.7\,\text{K/W}$$

Since the conduction resistance of the steel wall is

$$R_{t,cond,s} = \frac{1}{4\pi k_s}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{4\pi\left(9.2\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.35\,\text{m}} - \frac{1}{0.40\,\text{m}}\right) = 2.4\times10^{-3}\ \text{K/W}$$

it is clear that exclusive reliance must be placed on the insulation and that a special insulation of very low thermal conductivity should be selected. The best choice is a highly reflective foil/glass matted insulation which was developed for cryogenic applications. It follows that

$$R_{t,cond,i} = 60.7\,\text{K/W} = \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) = \frac{1}{4\pi\left(0.000017\,\text{W/m}\cdot\text{K}\right)}\left(\frac{1}{0.40\,\text{m}} - \frac{1}{r_3}\right)$$

which yields r_3 = 0.4021 m. The minimum insulation thickness is therefore $\delta = (r_3 - r_2)$ = 2.1 mm.

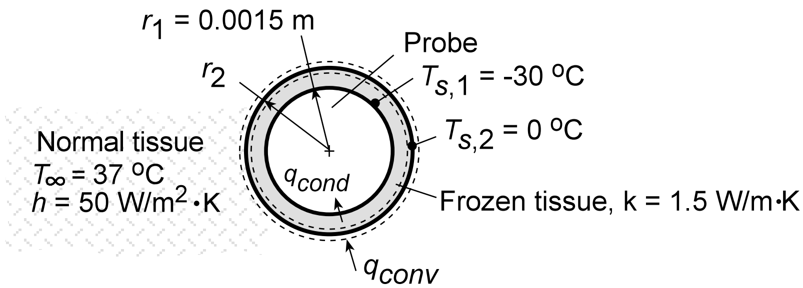
COMMENTS: The heat loss could be reduced well below the maximum allowable by adding more insulation. Also, in view of weight restrictions associated with launching space vehicles, consideration should be given to fabricating the LOX container from a lighter material.

PROBLEM 3.59

KNOWN: Diameter and surface temperature of a spherical cryoprobe. Temperature of surrounding tissue and effective convection coefficient at interface between frozen and normal tissue.

FIND: Thickness of frozen tissue layer.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Negligible contact resistance between probe and frozen tissue, (3) Constant properties.

ANALYSIS: Performing an energy balance for a control surface about the phase front, it follows that

$$q_{conv} - q_{cond} = 0$$

Hence,

$$h\left(4\pi r_2^2\right)\left(T_\infty - T_{s,2}\right) = \frac{T_{s,2} - T_{s,1}}{\left[(1/r_1) - (1/r_2)\right]/4\pi k}$$

$$r_2^2\left[(1/r_1) - (1/r_2)\right] = \frac{k}{h}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)}$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = \frac{k}{hr_1}\frac{\left(T_{s,2} - T_{s,1}\right)}{\left(T_\infty - T_{s,2}\right)} = \frac{1.5\,\text{W/m}\cdot\text{K}}{\left(50\,\text{W/m}^2\cdot\text{K}\right)(0.0015\,\text{m})}\left(\frac{30}{37}\right)$$

$$\left(\frac{r_2}{r_1}\right)\left[\left(\frac{r_2}{r_1}\right) - 1\right] = 16.2$$

$$\left(r_2/r_1\right) = 4.56$$

It follows that r_2 = 6.84 mm and the thickness of the frozen tissue is

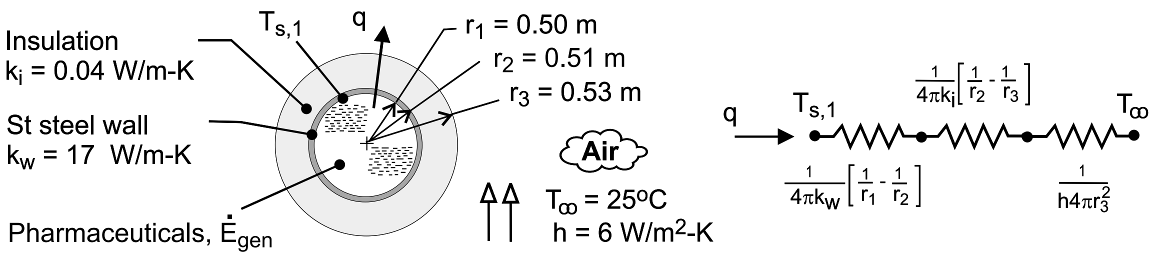
$$\delta = r_2 - r_1 = 5.34\,\text{mm}$$ <

PROBLEM 3.60

KNOWN: Inner diameter, wall thickness and thermal conductivity of spherical vessel containing heat generating medium. Inner surface temperature without insulation. Thickness and thermal conductivity of insulation. Ambient air temperature and convection coefficient.

FIND: (a) Thermal energy generated within vessel, (b) Inner surface temperature of vessel with insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional, radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

ANALYSIS: (a) From an energy balance performed at an instant for a control surface about the pharmaceuticals, $\dot{E}_g = q$, in which case, without the insulation

$$\dot{E}_g = q = \frac{T_{s,1} - T_\infty}{\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi r_2^2 h}} = \frac{(50-25)°C}{\frac{1}{4\pi\,(17\,W/m\cdot K)}\left(\frac{1}{0.50m} - \frac{1}{0.51m}\right) + \frac{1}{4\pi\,(0.51m)^2\,6\,W/m^2\cdot K}}$$

$$\dot{E}_g = q = \frac{25°C}{\left(1.84\times10^{-4} + 5.10\times10^{-2}\right)K/W} = 489\,W$$ <

(b) With the insulation,

$$T_{s,1} = T_\infty + q\left[\frac{1}{4\pi k_w}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \frac{1}{4\pi k_i}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \frac{1}{4\pi r_3^2 h}\right]$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + \frac{1}{4\pi\,(0.04)}\left(\frac{1}{0.51} - \frac{1}{0.53}\right) + \frac{1}{4\pi\,(0.53)^2\,6}\right]\frac{K}{W}$$

$$T_{s,1} = 25°C + 489\,W\left[1.84\times10^{-4} + 0.147 + 0.047\right]\frac{K}{W} = 120°C$$ <

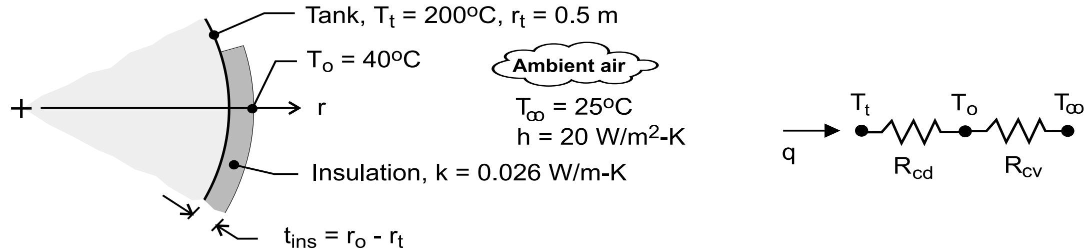
COMMENTS: The thermal resistance associated with the vessel wall is negligible, and without the insulation the dominant resistance is due to convection. The thermal resistance of the insulation is approximately three times that due to convection.

PROBLEM 3.61

KNOWN: Spherical tank of 1-m diameter containing an exothermic reaction and is at 200°C when the ambient air is at 25°C. Convection coefficient on outer surface is 20 $W/m^2 \cdot K$.

FIND: Determine the thickness of urethane foam required to reduce the exterior temperature to 40°C. Determine the percentage reduction in the heat rate achieved using the insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, (3) Convection coefficient is the same for bare and insulated exterior surface, and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings.

PROPERTIES: *Table A-3,* urethane, rigid foam (300 K): k = 0.026 W/m·K.

ANALYSIS: (a) The heat transfer situation for the heat rate from the tank can be represented by the thermal circuit shown above. The heat rate from the tank is

$$q = \frac{T_t - T_\infty}{R_{cd} + R_{cv}}$$

where the thermal resistances associated with conduction within the insulation (Eq. 3.35) and convection for the exterior surface, respectively, are

$$R_{cd} = \frac{(1/r_t - 1/r_o)}{4\pi k} = \frac{(1/0.5 - 1/r_o)}{4\pi \times 0.026\ W/m \cdot K} = \frac{(1/0.5 - 1/r_o)}{0.3267} K/W$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{4\pi h r_o^2} = \frac{1}{4\pi \times 20\ W/m^2 \cdot K \times r_o^2} = 3.979 \times 10^{-3} r_o^{-2} K/W$$

To determine the required insulation thickness so that T_o = 40°C, perform an energy balance on the o-node.

$$\frac{T_t - T_o}{R_{cd}} + \frac{T_\infty - T_o}{R_{cv}} = 0$$

$$\frac{(200 - 40)K}{(1/0.5 - 1/r_o)/0.3267\ K/W} + \frac{(25 - 40)K}{3.979 \times 10^{-3} r_o^2\ K/W} = 0$$

$$r_o = 0.5135\ m \qquad t = r_o - r_i = (0.5135 - 0.5000)\ m = 13.5\ mm$$ <

From the rate equation, for the bare and insulated surfaces, respectively,

$$q_o = \frac{T_t - T_\infty}{1/4\pi h r_t^2} = \frac{(200 - 25)K}{0.01592\ K/W} = 10.99\ kW$$

$$q_{ins} = \frac{T_t - T_\infty}{R_{cd} + R_{cv}} = \frac{(200 - 25)}{(0.161 + 0.01592)K/W} = 0.994\ kW$$

Hence, the percentage reduction in heat loss achieved with the insulation is,

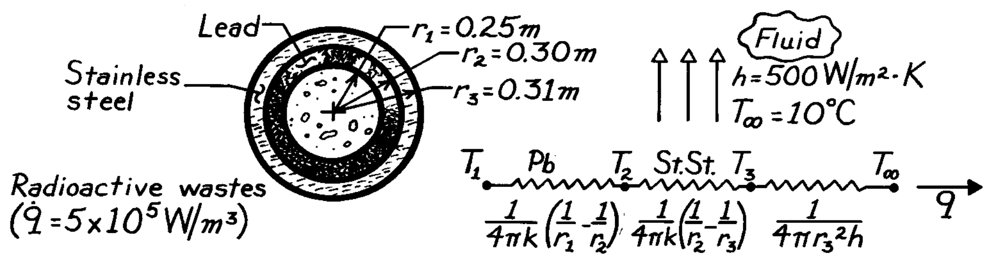
$$\frac{q_{ins} - q_o}{q_o} \times 100 = -\frac{0.994 - 10.99}{10.99} \times 100 = 91\%$$ <

PROBLEM 3.62

KNOWN: Dimensions and materials used for composite spherical shell. Heat generation associated with stored material.

FIND: Inner surface temperature, T_1, of lead (proposal is flawed if this temperature exceeds the melting point).

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300K, (4) Negligible contact resistance.

PROPERTIES: *Table A-1*, Lead: k = 35.3 W/m·K, MP = 601K; St.St.: 15.1 W/m·K.

ANALYSIS: From the thermal circuit, it follows that

$$q = \frac{T_1 - T_\infty}{R_{tot}} = \dot{q}\left[\frac{4}{3}\pi\, r_1^3\right]$$

Evaluate the thermal resistances,

$$R_{Pb} = \left[1/\left(4\pi \times 35.3\ \text{W/m}\cdot\text{K}\right)\right]\ \left[\frac{1}{0.25\text{m}} - \frac{1}{0.30\text{m}}\right] = 0.00150\ \text{K/W}$$

$$R_{St.St.} = \left[1/\left(4\pi \times 15.1\ \text{W/m}\cdot\text{K}\right)\right]\ \left[\frac{1}{0.30\text{m}} - \frac{1}{0.31\text{m}}\right] = 0.000567\ \text{K/W}$$

$$R_{conv} = \left[1/\left(4\pi \times 0.31^2\,\text{m}^2 \times 500\ \text{W/m}^2\cdot\text{K}\right)\right] = 0.00166\ \text{K/W}$$

$$R_{tot} = 0.00372\ \text{K/W.}$$

The heat rate is $q = 5\times10^5\ \text{W/m}^3\left(4\pi/3\right)\left(0.25\text{m}\right)^3 = 32{,}725\ \text{W}$. The inner surface temperature is

$$T_1 = T_\infty + R_{tot}\ q = 283\text{K} + 0.00372\text{K/W}\left(32{,}725\ \text{W}\right)$$

$$T_1 = 405\ \text{K} < \text{MP} = 601\text{K}.$$ <

Hence, from the thermal standpoint, the proposal is adequate.

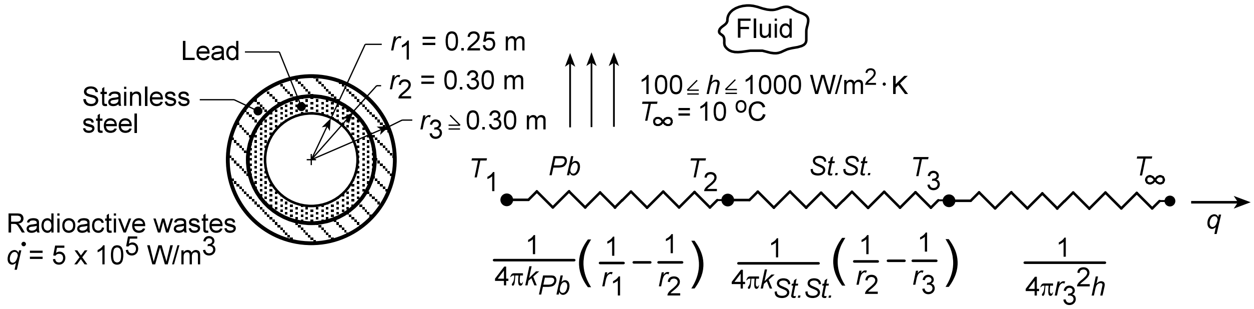
COMMENTS: In fabrication, attention should be given to maintaining a good thermal contact. A protective outer coating should be applied to prevent long term corrosion of the stainless steel.

PROBLEM 3.63

KNOWN: Dimensions and materials of composite (lead and stainless steel) spherical shell used to store radioactive wastes with constant heat generation. Range of convection coefficients h available for cooling.

FIND: (a) Variation of maximum lead temperature with h. Minimum allowable value of h to maintain maximum lead temperature at or below 500 K. (b) Effect of outer radius of stainless steel shell on maximum lead temperature for h = 300, 500 and 1000 $W/m^2 \cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Steady-state conditions, (3) Constant properties at 300 K, (4) Negligible contact resistance.

PROPERTIES: *Table A-1*, Lead: k = 35.3 W/m·K, St. St.: 15.1 W/m·K.

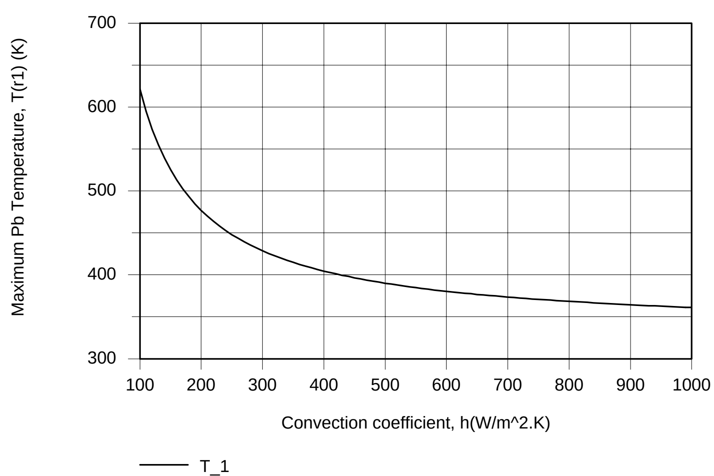
ANALYSIS: (a) From the schematic, the maximum lead temperature T_1 corresponds to $r = r_1$, and from the thermal circuit, it may be expressed as

$$T_1 = T_\infty + R_{tot} q$$

where $q = \dot{q}(4/3)\pi r_1^3 = 5\times10^5\,W/m^3\,(4\pi/3)(0.25\,m)^3 = 32,725\,W$. The total thermal resistance is

$$R_{tot} = R_{cond,Pb} + R_{cond,St.St} + R_{conv}$$

where expressions for the component resistances are provided in the schematic. Using the *Resistance Network* model and *Thermal Resistance* tool pad of IHT, the following result is obtained for the variation of T_1 with h.



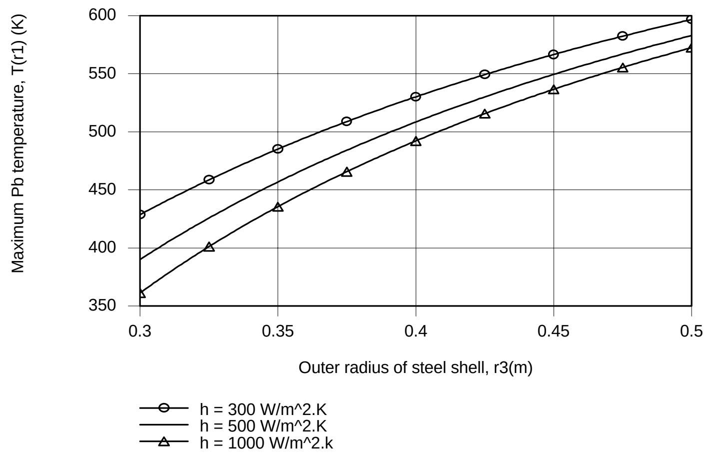
Continued...

PROBLEM 3.63 (Cont.)

To maintain T_1 below 500 K, the convection coefficient must be maintained at

$$h \geq 181\ \text{W/m}^2\cdot\text{K} \qquad <$$

(b) The effect of varying the outer shell radius over the range $0.3 \leq r_3 \leq 0.5$ m is shown below.



For h = 300, 500 and 1000 $\text{W/m}^2\cdot\text{K}$, the maximum allowable values of the outer radius are $r_3 = 0.365$, 0.391 and 0.408 m, respectively.

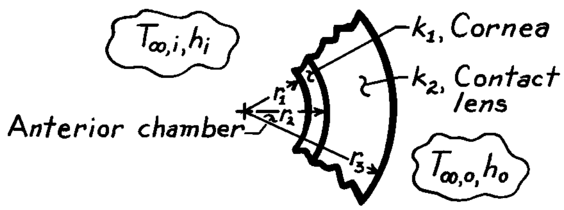
COMMENTS: For a maximum allowable value of $T_1 = 500$ K, the maximum allowable value of the total thermal resistance is $R_{tot} = (T_1 - T_\infty)/q$, or $R_{tot} = (500 - 283)\text{K}/32{,}725\ \text{W} = 0.00663$ K/W. Hence, any increase in $R_{cond,St.St}$ due to increasing r_3 must be accompanied by an equivalent reduction in R_{conv}.

PROBLEM 3.64

KNOWN: Representation of the eye with a contact lens as a composite spherical system subjected to convection processes at the boundaries.

FIND: (a) Thermal circuits with and without contact lens in place, (b) Heat loss from anterior chamber for both cases, and (c) Implications of the heat loss calculations.

SCHEMATIC:

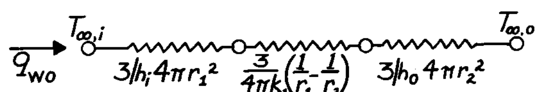


r_1=10.2mm k_1=0.35 W/m·K
r_2=12.7mm k_2=0.80 W/m·K
r_3=16.5mm
$T_{\infty,i}$=37°C h_i=12 W/m^2·K
$T_{\infty,o}$=21°C h_o=6 W/m^2·K

ASSUMPTIONS: (1) Steady-state conditions, (2) Eye is represented as 1/3 sphere, (3) Convection coefficient, h_o, unchanged with or without lens present, (4) Negligible contact resistance.

ANALYSIS: (a) Using Eqs. 3.9 and 3.36 to express the resistance terms, the thermal circuits are:

Without lens:



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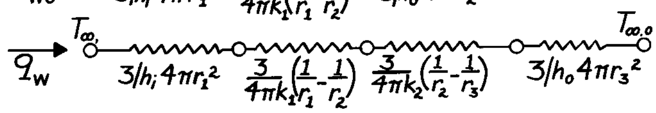
With lens:



<

(b) The heat losses for both cases can be determined as q = ($T_{\infty,i}$ - $T_{\infty,o}$)/R_t, where R_t is the thermal resistance from the above circuits.

Without lens: $$R_{t,wo}=\frac{3}{12\text{W/m}^2\cdot\text{K}4\pi\left(10.2\times10^{-3}\text{m}\right)^2}+\frac{3}{4\pi\times0.35\text{ W/m}\cdot\text{K}}\left[\frac{1}{10.2}-\frac{1}{12.7}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{ W/m}^2\cdot\text{K}4\pi\left(12.7\times10^{-3}\text{m}\right)^2}=191.2\text{ K/W}+13.2\text{ K/W}+246.7\text{ K/W}=451.1\text{ K/W}$$

With lens: $$R_{t,w}=191.2\text{ K/W}+13.2\text{ K/W}+\frac{3}{4\pi\times0.80\text{ W/m}\cdot\text{K}}\left[\frac{1}{12.7}-\frac{1}{16.5}\right]\frac{1}{10^{-3}}\text{m}$$

$$+\frac{3}{6\text{W/m}^2\cdot\text{K}4\pi\left(16.5\times10^{-3}\text{m}\right)^2}=191.2\text{ K/W}+13.2\text{ K/W}+5.41\text{ K/W}+146.2\text{ K/W}=356.0\text{ K/W}$$

Hence the heat loss rates from the anterior chamber are

Without lens: $q_{wo}=(37-21)^\circ\text{C}/451.1\text{ K/W}=35.5\text{mW}$ <

With lens: $q_w=(37-21)^\circ\text{C}/356.0\text{ K/W}=44.9\text{mW}$ <

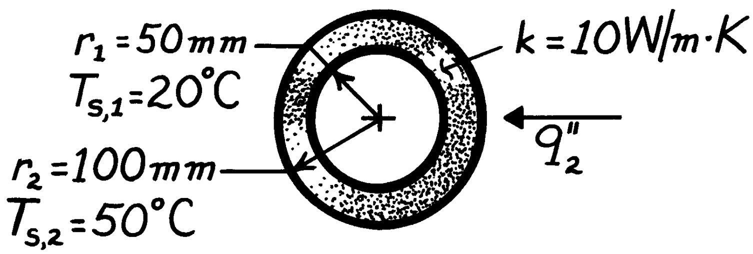
(c) The heat loss from the anterior chamber increases by approximately 20% when the contact lens is in place, implying that the outer radius, r_3, is less than the critical radius.

PROBLEM 3.65

KNOWN: Thermal conductivity and inner and outer radii of a hollow sphere subjected to a uniform heat flux at its outer surface and maintained at a uniform temperature on the inner surface.

FIND: (a) Expression for radial temperature distribution, (b) Heat flux required to maintain prescribed surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) No generation, (4) Constant properties.

ANALYSIS: (a) For the assumptions, the temperature distribution may be obtained by integrating Fourier's law, Eq. 3.33. That is,

$$\frac{q_r}{4\pi}\int_{r_1}^{r}\frac{dr}{r^2} = -k\int_{T_{s,1}}^{T} dT \quad \text{or} \quad -\frac{q_r}{4\pi}\left.\frac{1}{r}\right|_{r_1}^{r} = -k\left(T - T_{s,1}\right).$$

Hence,

$$T(r) = T_{s,1} + \frac{q_r}{4\pi\, k}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$

or, with $q_2'' \equiv q_r / 4\pi\, r_2^2$,

$$T(r) = T_{s,1} + \frac{q_2'' r_2^2}{k}\left[\frac{1}{r} - \frac{1}{r_1}\right]$$ <

(b) Applying the above result at r_2,

$$q_2'' = \frac{k\left(T_{s,2} - T_{s,1}\right)}{r_2^2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]} = \frac{10\ \text{W/m}\cdot\text{K}\ (50-20)^\circ\text{C}}{(0.1\text{m})^2\left[\frac{1}{0.1} - \frac{1}{0.05}\right]\frac{1}{\text{m}}} = -3000\ \text{W/m}^2.$$ <

COMMENTS: (1) The desired temperature distribution could also be obtained by solving the appropriate form of the heat equation,

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right] = 0$$

and applying the boundary conditions $T(r_1) = T_{s,1}$ and $-k\left.\frac{dT}{dr}\right]_{r_2} = q_2''$.

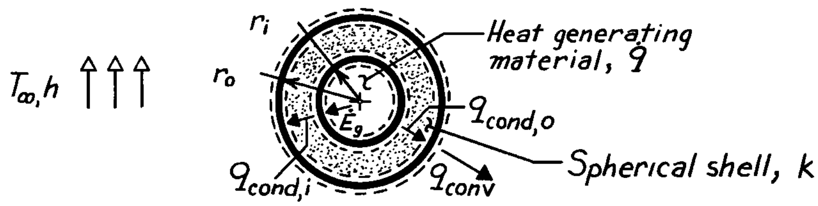
(2) The negative sign on q_2'' implies heat transfer in the negative r direction.

PROBLEM 3.66

KNOWN: Volumetric heat generation occurring within the cavity of a spherical shell of prescribed dimensions. Convection conditions at outer surface.

FIND: Expression for steady-state temperature distribution in shell.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction, (2) Steady-state conditions, (3) Constant properties, (4) Uniform generation within the shell cavity, (5) Negligible radiation.

ANALYSIS: For the prescribed conditions, the appropriate form of the heat equation is

$$\frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=0$$

Integrate twice to obtain,

$$r^2\frac{dT}{dr}=C_1 \qquad \text{and} \qquad T=-\frac{C_1}{r}+C_2. \qquad (1,2)$$

The boundary conditions may be obtained from energy balances at the inner and outer surfaces. At the inner surface (r_i),

$$\dot{E}_g=\dot{q}\left(4/3\pi\, r_i^3\right)=q_{cond,i}=-k\left(4\pi\, r_i^2\right)dT/dr)_{r_i} \qquad dT/dr)_{r_i}=-\dot{q}r_i/3k. \qquad (3)$$

At the outer surface (r_o),

$$q_{cond,o}=-k4\pi\, r_o^2\ dT/dr)_{r_o}=q_{conv}=h4\pi\, r_o^2\left[T\left(r_o\right)-T_\infty\right]$$

$$dT/dr)_{r_o}=-\left(h/k\right)\ \left[T\left(r_o\right)-T_\infty\right]. \qquad (4)$$

From Eqs. (1) and (3), $C_1=-\dot{q}r_i^3/3k$. From Eqs. (1), (2) and (4)

$$-\frac{\dot{q}r_i^3}{3kr_o^2}=-\left[\frac{h}{k}\right]\left[\frac{\dot{q}r_i^3}{3r_o k}+C_2-T_\infty\right]$$

$$C_2=\frac{\dot{q}r_i^3}{3hr_o^2}-\frac{\dot{q}r_i^3}{3r_o k}+T_\infty.$$

Hence, the temperature distribution is

$$T=\frac{\dot{q}r_i^3}{3k}\left[\frac{1}{r}-\frac{1}{r_o}\right]+\frac{\dot{q}r_i^3}{3hr_o^2}+T_\infty. \qquad <$$

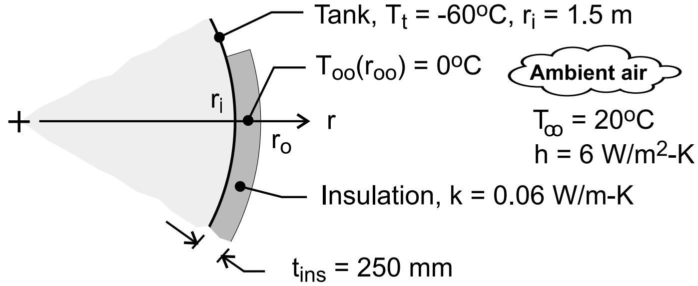
COMMENTS: Note that $\dot{E}_g=q_{cond,i}=q_{cond,o}=q_{conv}$.

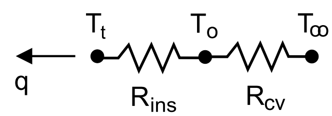
PROBLEM 3.67

KNOWN: Spherical tank of 3-m diameter containing LP gas at -60°C with 250 mm thickness of insulation having thermal conductivity of 0.06 W/m·K. Ambient air temperature and convection coefficient on the outer surface are 20°C and 6 $W/m^2 \cdot K$, respectively.

FIND: (a) Determine the radial position in the insulation at which the temperature is 0°C and (b) If the insulation is pervious to moisture, what conclusions can be reached about ice formation? What effect will ice formation have on the heat gain? How can this situation be avoided?

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional, radial (spherical) conduction through the insulation, and (3) Negligible radiation exchange between the insulation outer surface and the ambient surroundings.

ANALYSIS: (a) The heat transfer situation can be represented by the thermal circuit shown above. The heat gain to the tank is

$$q = \frac{T_\infty - T_t}{R_{ins} + R_{cv}} = \frac{\left[20 - (-60)\right] K}{\left(0.1263 + 4.33 \times 10^{-3}\right) K/W} = 612.4 \text{ W}$$

where the thermal resistances for the insulation (see Table 3.3) and the convection process on the outer surface are, respectively,

$$R_{ins} = \frac{1/r_i - 1/r_o}{4\pi k} = \frac{(1/1.50 - 1/1.75)\text{m}^{-1}}{4\pi \times 0.06 \text{ W/m} \cdot \text{K}} = 0.1263 \text{ K/W}$$

$$R_{cv} = \frac{1}{hA_s} = \frac{1}{h 4\pi r_o^2} = \frac{1}{6 \text{ W/m}^2 \cdot \text{K} \times 4\pi (1.75 \text{ m})^2} = 4.33 \times 10^{-3} \text{K/W}$$

To determine the location within the insulation where T_{oo} (r_{oo}) = 0°C, use the conduction rate equation, Eq. 3.35,

$$q = \frac{4\pi k (T_{oo} - T_t)}{(1/r_i - 1/r_{oo})} \qquad r_{oo} = \left[\frac{1}{r_i} - \frac{4\pi k (T_{oo} - T_t)}{q}\right]^{-1}$$

and substituting numerical values, find

$$r_{oo} = \left[\frac{1}{1.5 \text{ m}} - \frac{4\pi \times 0.06 \text{ W/m} \cdot \text{K} \left(0 - (-60)\right) \text{K}}{612.4 \text{ W}}\right]^{-1} = 1.687 \text{ m}$$ <

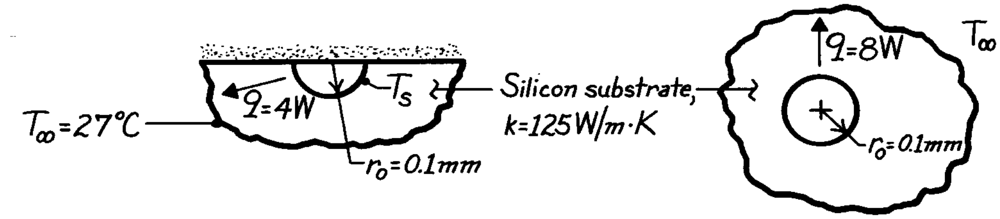
(b) With r_{oo} = 1.687 m, we'd expect the region of the insulation $r_i \le r \le r_{oo}$ to be filled with ice formations if the insulation is pervious to water vapor. The effect of the ice formation is to substantially increase the heat gain since k_{ice} is nearly twice that of k_{ins}, and the ice region is of thickness (1.687 – 1.50)m = 187 mm. To avoid ice formation, a vapor barrier should be installed at a radius larger than r_{oo}.

PROBLEM 3.68

KNOWN: Radius and heat dissipation of a hemispherical source embedded in a substrate of prescribed thermal conductivity. Source and substrate boundary conditions.

FIND: Substrate temperature distribution and surface temperature of heat source.

SCHEMATIC:



ASSUMPTIONS: (1) Top surface is adiabatic. Hence, hemispherical source in semi-infinite medium is equivalent to spherical source in infinite medium (with q = 8 W) and heat transfer is one-dimensional in the radial direction, (2) Steady-state conditions, (3) Constant properties, (4) No generation.

ANALYSIS: Heat equation reduces to

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right)=0 \qquad r^2 dT/dr=C_1$$

$$T(r)=-C_1/r+C_2.$$

Boundary conditions:

$$T(\infty)=T_\infty \qquad T(r_o)=T_s$$

Hence, $C_2 = T_\infty$ and

$$T_s=-C_1/r_o+T_\infty \quad \text{and} \quad C_1=r_o\left(T_\infty-T_s\right).$$

The temperature distribution has the form

$$T(r)=T_\infty+\left(T_s-T_\infty\right)r_o/r \qquad <$$

and the heat rate is

$$q=-kA dT/dr=-k2\pi\, r^2\left[-\left(T_s-T_\infty\right)r_o/r^2\right]=k2\pi\, r_o\left(T_s-T_\infty\right)$$

It follows that

$$T_s-T_\infty=\frac{q}{k2\pi\, r_o}=\frac{4\text{ W}}{125\text{ W/m}\cdot\text{K}\ 2\pi\left(10^{-4}\text{ m}\right)}=50.9^\circ\text{C}$$

$$T_s=77.9^\circ\text{C}. \qquad <$$

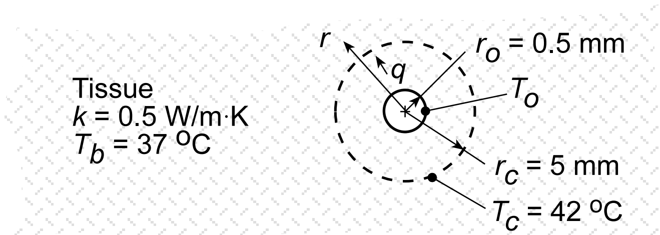
COMMENTS: For the semi-infinite (or infinite) medium approximation to be valid, the substrate dimensions must be much larger than those of the transistor.

PROBLEM 3.69

KNOWN: Critical and normal tissue temperatures. Radius of spherical heat source and radius of tissue to be maintained above the critical temperature. Tissue thermal conductivity.

FIND: General expression for radial temperature distribution in tissue. Heat rate required to maintain prescribed thermal conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Constant k.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=0$$

Integrating twice,

$$\frac{dT}{dr}=\frac{C_1}{r^2}$$

$$T(r)=-\frac{C_1}{r}+C_2$$

Since $T \rightarrow T_b$ as $r \rightarrow \infty$, $C_2 = T_b$. At $r = r_o$, $q = -k\left(4\pi r_o^2\right)dT/dr\big|_{r_o} = -4\pi k r_o^2 C_1/r_o^2 = -4\pi k C_1$.

Hence, $C_1 = -q/4\pi k$ and the temperature distribution is

$$T(r)=\frac{q}{4\pi kr}+T_b \quad <$$

It follows that

$$q=4\pi kr\left[T(r)-T_b\right]$$

Applying this result at $r = r_c$,

$$q=4\pi\left(0.5\,W/m\cdot K\right)\left(0.005\,m\right)\left(42-37\right)^\circ C=0.157\,W \quad <$$

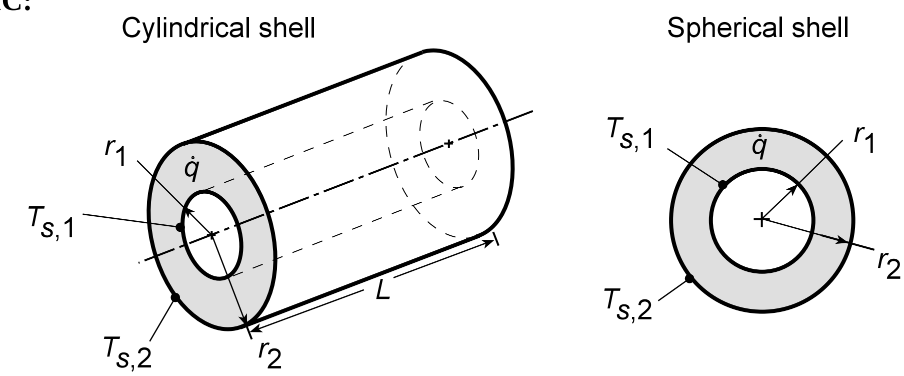
COMMENTS: At $r_o = 0.0005$ m, $T(r_o) = \left[q/\left(4\pi k r_o\right)\right]+T_b = 92°C$. Proximity of this temperature to the boiling point of water suggests the need to operate at a lower power dissipation level.

PROBLEM 3.70

KNOWN: Cylindrical and spherical shells with uniform heat generation and surface temperatures.

FIND: Radial distributions of temperature, heat flux and heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction, (2) Uniform heat generation, (3) Constant k.

ANALYSIS: (a) For the *cylindrical shell*, the appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

The general solution is

$$T(r)=-\frac{\dot{q}}{4k}r^2+C_1\ln r+C_2$$

Applying the boundary conditions, it follows that

$$T(r_1)=T_{s,1}=-\frac{\dot{q}}{4k}r_1^2+C_1\ln r_1+C_2$$

$$T(r_2)=T_{s,2}=-\frac{\dot{q}}{4k}r_2^2+C_1\ln r_2+C_2$$

which may be solved for

$$C_1=\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\Big/\ln(r_2/r_1)$$

$$C_2=T_{s,2}+(\dot{q}/4k)r_2^2-C_1\ln r_2$$

Hence,

$$T(r)=T_{s,2}+(\dot{q}/4k)\left(r_2^2-r^2\right)+\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]\frac{\ln(r/r_2)}{\ln(r_2/r_1)}$$ <

With $q''=-k\,dT/dr$, the heat flux distribution is

$$q''(r)=\frac{\dot{q}}{2}r-\frac{k\left[(\dot{q}/4k)\left(r_2^2-r_1^2\right)+\left(T_{s,2}-T_{s,1}\right)\right]}{r\ln(r_2/r_1)}$$ <

Continued...

PROBLEM 3.70 (Cont.)

Similarly, with q = q'' A(r) = q'' (2πrL), the heat rate distribution is

$$q(r) = \pi L\dot{q}r^2 - \frac{2\pi Lk\left[(\dot{q}/4k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]}{\ln(r_2/r_1)} \quad <$$

(b) For the *spherical shell*, the heat equation and general solution are

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{q}}{k} = 0$$

$$T(r) = -(\dot{q}/6k)r^2 - C_1/r + C_2$$

Applying the boundary conditions, it follows that

$$T(r_1) = T_{s,1} = -(\dot{q}/6k)r_1^2 - C_1/r_1 + C_2$$

$$T(r_2) = T_{s,2} = -(\dot{q}/6k)r_2^2 - C_1/r_2 + C_2$$

Hence,

$$C_1 = \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right] \Big/ \left[(1/r_1) - (1/r_2)\right]$$

$$C_2 = T_{s,2} + (\dot{q}/6k)r_2^2 + C_1/r_2$$

and

$$T(r) = T_{s,2} + (\dot{q}/6k)\left(r_2^2 - r^2\right) - \left[(\dot{q}/6k)\left(r_2^2 - r_1^2\right) + \left(T_{s,2} - T_{s,1}\right)\right]\frac{(1/r) - (1/r_2)}{(1/r_1) - (1/r_2)} \quad <$$

With q''(r) = - k dT/dr, the heat flux distribution is

$$q''(r) = \frac{\dot{q}}{3}r - \frac{\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)}\frac{1}{r^2} \quad <$$

and, with q = $q''\left(4\pi r^2\right)$, the heat rate distribution is

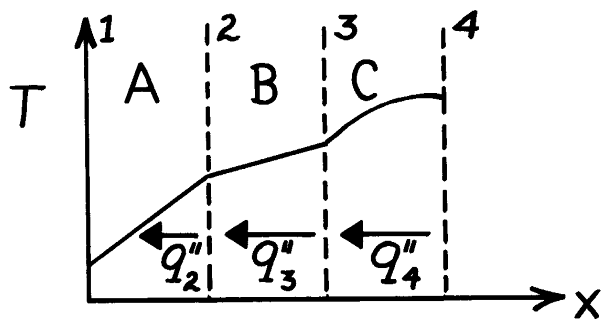
$$q(r) = \frac{4\pi\dot{q}}{3}r^3 - \frac{4\pi\left[(\dot{q}/6)\left(r_2^2 - r_1^2\right) + k\left(T_{s,2} - T_{s,1}\right)\right]}{(1/r_1) - (1/r_2)} \quad <$$

PROBLEM 3.71

KNOWN: Temperature distribution in a composite wall.

FIND: (a) Relative magnitudes of interfacial heat fluxes, (b) Relative magnitudes of thermal conductivities, and (c) Heat flux as a function of distance x.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: (a) For the prescribed conditions (one-dimensional, steady-state, constant k), the parabolic temperature distribution in C implies the existence of heat generation. Hence, since dT/dx *increases* with *decreasing* x, the heat flux in C *increases* with *decreasing* x. Hence,

$$q_3'' > q_4''$$

However, the linear temperature distributions in A and B indicate no generation, in which case

$$q_2'' = q_3''$$

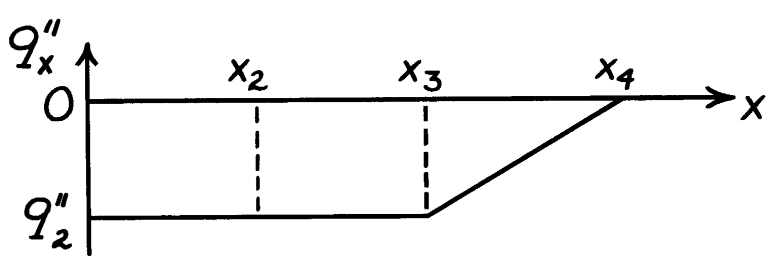
(b) Since conservation of energy requires that $q''_{3,B} = q''_{3,C}$ and $dT/dx)_B < dT/dx)_C$, it follows from Fourier's law that

$$k_B > k_C.$$

Similarly, since $q''_{2,A} = q''_{2,B}$ and $dT/dx)_A > dT/dx)_B$, it follows that

$$k_A < k_B.$$

(c) It follows that the flux distribution appears as shown below.



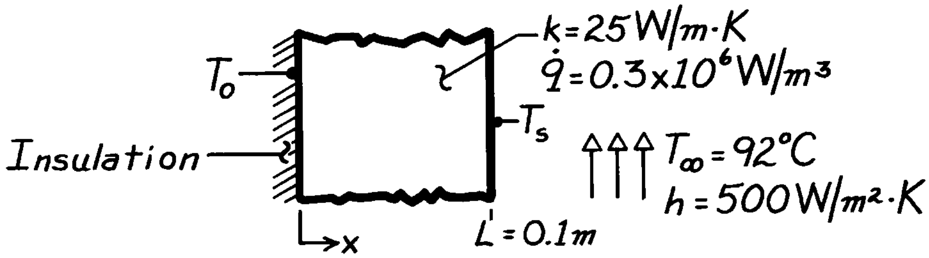
COMMENTS: Note that, with $dT/dx)_{4,C} = 0$, the interface at 4 is adiabatic.

PROBLEM 3.72

KNOWN: Plane wall with internal heat generation which is insulated at the inner surface and subjected to a convection process at the outer surface.

FIND: Maximum temperature in the wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform volumetric heat generation, (3) Inner surface is adiabatic.

ANALYSIS: From Eq. 3.42, the temperature at the inner surface is given by Eq. 3.43 and is the maximum temperature within the wall,

$$T_o = \dot{q}L^2/2k + T_s.$$

The outer surface temperature follows from Eq. 3.46,

$$T_s = T_\infty + \dot{q}L/h$$

$$T_s = 92^\circ C + 0.3\times10^6 \frac{W}{m^3}\times 0.1m/500 W/m^2\cdot K = 92^\circ C + 60^\circ C = 152^\circ C.$$

It follows that

$$T_o = 0.3\times10^6 W/m^3 \times (0.1m)^2/2\times25 W/m\cdot K + 152^\circ C$$

$$T_o = 60^\circ C + 152^\circ C = 212^\circ C.$$ <

COMMENTS: The heat flux leaving the wall can be determined from knowledge of h, T_s and T_∞ using Newton's law of cooling.

$$q''_{conv} = h(T_s - T_\infty) = 500 W/m^2\cdot K(152-92)^\circ C = 30 kW/m^2.$$

This same result can be determined from an energy balance on the entire wall, which has the form

$$\dot{E}_g - \dot{E}_{out} = 0$$

where

$$\dot{E}_g = \dot{q}AL \qquad \text{and} \qquad \dot{E}_{out} = q''_{conv}\cdot A.$$

Hence,

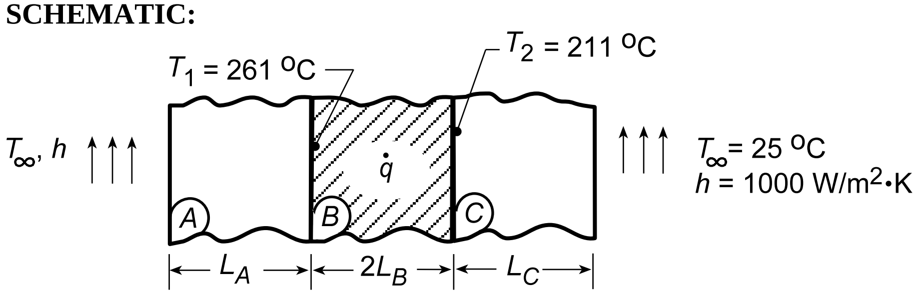
$$q''_{conv} = \dot{q}L = 0.3\times10^6 W/m^3 \times 0.1m = 30 kW/m^2.$$

PROBLEM 3.73

KNOWN: Composite wall with outer surfaces exposed to convection process.

FIND: (a) Volumetric heat generation and thermal conductivity for material B required for special conditions, (b) Plot of temperature distribution, (c) T_1 and T_2, as well as temperature distributions corresponding to loss of coolant condition where h = 0 on surface A.

SCHEMATIC:



L_A = 30 mm
L_B = 30 mm
L_C = 20 mm
k_A = 25 W/m·K
k_C = 50 W/m·K

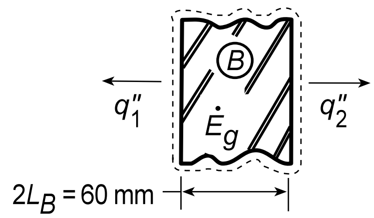
ASSUMPTIONS: (1) Steady-state, one-dimensional heat transfer, (2) Negligible contact resistance at interfaces, (3) Uniform generation in B; zero in A and C.

ANALYSIS: (a) From an energy balance on wall B,

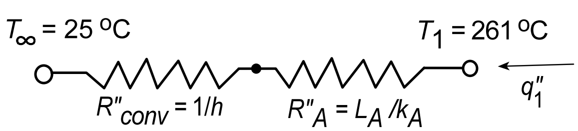
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

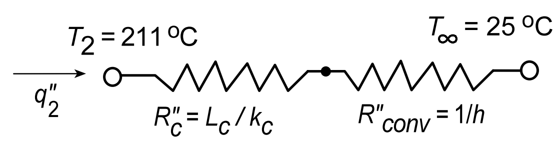
$$-q_1'' - q_2'' + 2\dot{q}L_B = 0$$

$$\dot{q}_B = \left(q_1'' + q_2''\right)/2L_B \; .$$



To determine the heat fluxes, q_1'' and q_2'', construct thermal circuits for A and C:





$$q_1'' = \left(T_1 - T_\infty\right)/\left(1/h + L_A/k_A\right)$$

$$q_1'' = \left(261-25\right)^\circ\text{C}\Big/\left(\frac{1}{1000\,\text{W/m}^2\cdot\text{K}} + \frac{0.030\,\text{m}}{25\,\text{W/m}\cdot\text{K}}\right)$$

$$q_1'' = 236^\circ\text{C}/\left(0.001+0.0012\right)\text{m}^2\cdot\text{K/W}$$

$$q_1'' = 107,273\,\text{W/m}^2$$

$$q_2'' = \left(T_2 - T_\infty\right)/\left(L_C/k_C + 1/h\right)$$

$$q_2'' = \left(211-25\right)^\circ\text{C}\Big/\left(\frac{0.020\,\text{m}}{50\,\text{W/m}\cdot\text{K}} + \frac{1}{1000\,\text{W/m}^2\cdot\text{K}}\right)$$

$$q_2'' = 186^\circ\text{C}/\left(0.0004+0.001\right)\text{m}^2\cdot\text{K/W}$$

$$q_2'' = 132,857\,\text{W/m}^2$$

Using the values for q_1'' and q_2'' in Eq. (1), find

$$\dot{q}_B = \left(106,818 + 132,143\,\text{W/m}^2\right)\Big/2\times 0.030\,\text{m} = 4.00\times10^6\,\text{W/m}^3 \; .$$ <

To determine k_B, use the general form of the temperature and heat flux distributions in wall B,

$$T(x) = -\frac{\dot{q}_B}{2k_B}x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left[-\frac{\dot{q}}{k_B}x + C_1\right] \qquad (1,2)$$

there are 3 unknowns, C_1, C_2 and k_B, which can be evaluated using three conditions,

Continued...

PROBLEM 3.73 (Cont.)

$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \qquad \text{where } T_1 = 261°C \qquad (3)$$

$$T(+L_B) = T_2 = -\frac{\dot{q}_B}{2k_B}(+L_B)^2 + C_1 L_B + C_2 \qquad \text{where } T_2 = 211°C \qquad (4)$$

$$q''_x(-L_B) = -q''_1 = -k_B\left[-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right] \qquad \text{where } q''_1 = 107{,}273\ \text{W/m}^2 \qquad (5)$$

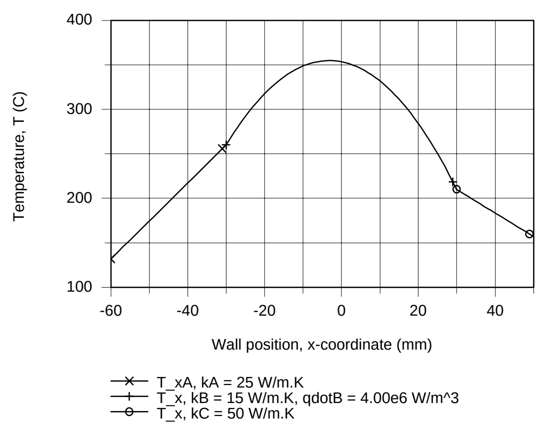
Using IHT to solve Eqs. (3), (4) and (5) simultaneously with $\dot{q}_B = 4.00 \times 10^6$ W/m^3, find

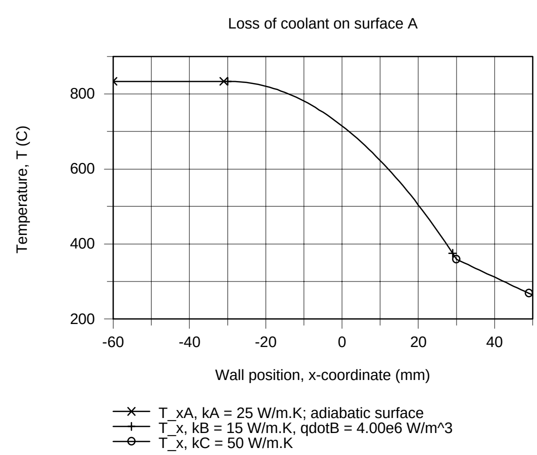
$$k_B = 15.3\ \text{W/m} \cdot \text{K}$$ <

(b) Following the method of analysis in the *IHT Example 3.6, User-Defined Functions*, the temperature distribution is shown in the plot below. The important features are (1) Distribution is quadratic in B, but non-symmetrical; linear in A and C; (2) Because thermal conductivities of the materials are different, discontinuities exist at each interface; (3) By comparison of gradients at x = $-L_B$ and $+L_B$, find $q''_2 > q''_1$.

(c) Using the same method of analysis as for Part (c), the temperature distribution is shown in the plot below when h = 0 on the surface of A. Since the left boundary is adiabatic, material A will be isothermal at T_1. Find

$T_1 = 835°C \qquad T_2 = 360°C$ <



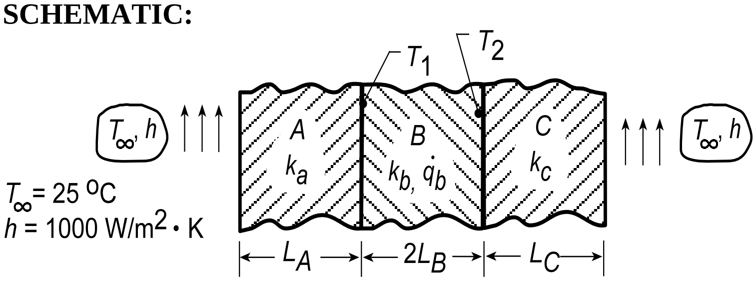


PROBLEM 3.74

KNOWN: Composite wall exposed to convection process; inside wall experiences a uniform heat generation.

FIND: (a) Neglecting interfacial thermal resistances, determine T_1 and T_2, as well as the heat fluxes through walls A and C, and (b) Determine the same parameters, but consider the interfacial contact resistances. Plot temperature distributions.

SCHEMATIC:



$k_A = 25\,W/m\cdot K \qquad L_A = 30\,mm$

$k_B = 15\,W/m\cdot K \qquad L_B = 30\,mm$

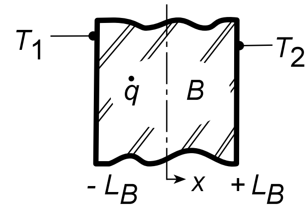
$k_C = 50\,W/m\cdot K \qquad L_C = 20\,mm$

$\dot{q}_B = 4\times 10^6\,W/m^3$

ASSUMPTIONS: (1) One-dimensional, steady-state heat flow, (2) Negligible contact resistance between walls, part (a), (3) Uniform heat generation in B, zero in A and C, (4) Uniform properties, (5) Negligible radiation at outer surfaces.

ANALYSIS: (a) The temperature distribution in wall B follows from Eq. 3.41,

$$T(x) = \frac{\dot{q}_B L_B^2}{2k_B}\left(1 - \frac{x^2}{L_B^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L_B} + \frac{T_1 - T_2}{2}. \qquad (1)$$

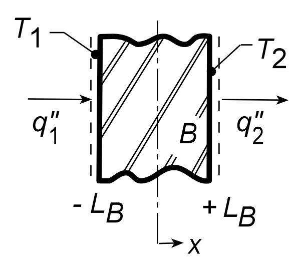


The heat fluxes to the neighboring walls are found using Fourier's law,

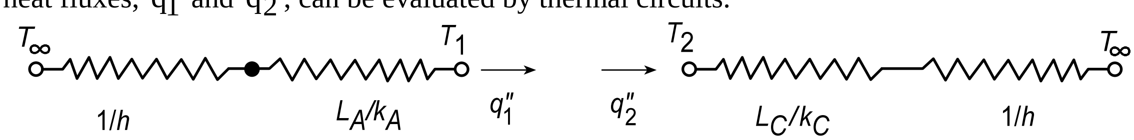
$$q''_x = -k\frac{dT}{dx}.$$

$$\text{At } x = -L_B: \quad q''_x(-L_B) - k_B\left[+\frac{\dot{q}_B}{k_B}(L_B) + \frac{T_2 - T_1}{2L_B}\right] = q''_1 \quad (2)$$

$$\text{At } x = +L_B: \quad q''_x(L_B) - k_B\left[-\frac{\dot{q}_B}{k_B}(L_B) + \frac{T_2 - T_1}{2L_B}\right] = q''_2 \quad (3)$$



The heat fluxes, q''_1 and q''_2, can be evaluated by thermal circuits.



Substituting numerical values, find

$$q''_1 = (T_\infty - T_1)^\circ C\Big/(1/h + L_A/k_A) = (25 - T_1)^\circ C\Big/\left(1/1000\,W/m^2\cdot K + 0.03\,m/25\,W/m\cdot K\right)$$

$$q''_1 = (25 - T_1)^\circ C\Big/(0.001 + 0.0012)\,K/W = 454.6(25 - T_1) \qquad (4)$$

$$q''_2 = (T_2 - T_\infty)^\circ C\Big/(1/h + L_C/k_C) = (T_2 - 25)^\circ C\Big/\left(1/1000\,W/m^2\cdot K + 0.02\,m/50\,W/m\cdot K\right)$$

$$q''_2 = (T_2 - 25)^\circ C\Big/(0.001 + 0.0004)\,K/W = 714.3(T_2 - 25). \qquad (5)$$

Continued...

Substituting the expressions for the heat fluxes, Eqs. (4) and (5), into Eqs. (2) and (3), a system of two equations with two unknowns is obtained.

Eq. (2): $$-4\times10^6\ \text{W/m}^3\times0.03\,\text{m}+15\ \text{W/m}\cdot\text{K}\,\frac{T_2-T_1}{2\times0.03\,\text{m}}=q''_1$$

$$-1.2\times10^5\ \text{W/m}^2-2.5\times10^2\,(T_2-T_1)\,\text{W/m}^2=454.6(25-T_1)$$

$$704.6\,T_1-250\,T_2=131{,}365 \qquad (6)$$

Eq. (3): $$+4\times10^6\ \text{W/m}^3\times0.03\,\text{m}-15\ \text{W/m}\cdot\text{K}\,\frac{T_2-T_1}{2\times0.03\,\text{m}}=q''_2$$

$$+1.2\times10^5\ \text{W/m}^2-2.5\times10^2\,(T_2-T_1)\,\text{W/m}^2=714.3(T_2-25)$$

$$250\,T_1-964\,T_2=-137{,}857 \qquad (7)$$

Solving Eqs. (6) and (7) simultaneously, find

T_1 = 260.9°C $\qquad$ T_2 = 210.0°C $\qquad$ <

From Eqs. (4) and (5), the heat fluxes at the interfaces and through walls A and C are, respectively,

$$q''_1=454.6(25-260.9)=-107{,}240\ \text{W/m}^2$$ <

$$q''_2=714.3(210-25)=+132{,}146\ \text{W/m}^2\,.$$ <

Note directions of the heat fluxes.

(b) Considering interfacial contact resistances, we will use a different approach. The general solution for the temperature and heat flux distributions in each of the materials is

$$T_A(x)=C_1x+C_2 \qquad q''_x=-k_AC_1 \qquad -(L_A+L_B)\le x\le -L_B \qquad (1,2)$$

$$T_B(x)=-\frac{\dot{q}_B}{2k_B}x^2+C_3x+C_4 \qquad q''_x=-\frac{\dot{q}_B}{k_B}x+C_3 \qquad -L_B\le x\le L_B \qquad (3,4)$$

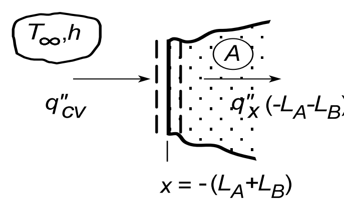
$$T_C(x)=C_5x+C_6 \qquad q''_x=-k_CC_5 \qquad +L_B\le x\le (L_B+L_C) \qquad (5,6)$$

To determine C_1 ... C_6 and the distributions, we need to identify boundary conditions using surface energy balances.

At $x = -(L_A + L_B)$:

$$-q''_x(-L_A-L_B)+q''_{cv}=0 \qquad (7)$$

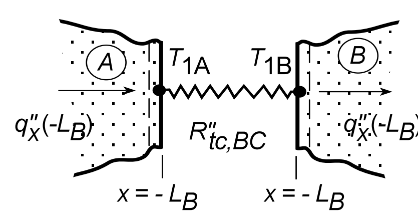
$$-(-k_AC_1)+h\left[T_\infty-T_A(-L_A-L_B)\right] \qquad (8)$$



At $x = -L_B$: The heat flux must be continuous, but the temperature will be discontinuous across the contact resistance.

$$q''_{x,A}(-L_B)=q''_{x,B}(-L_B) \qquad (9)$$

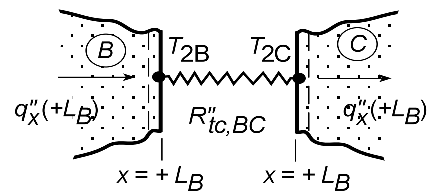
$$q''_{x,A}(-L_B)=\left[T_{1A}(-L_B)-T_{1B}(-L_B)\right]/R''_{tc,AB} \qquad (10)$$



Continued...

PROBLEM 3.74 (Cont.)

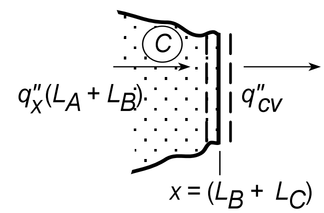
At x = + L_B: The same conditions apply as for x = $-L_B$,



$$q''_{x,B}(+L_B) = q''_{x,C}(+L_B) \quad (11)$$

$$q''_{x,B}(+L_B) = \left[T_{2B}(+L_B) - T_{2C}(+L_B)\right]\Big/R''_{tc,BC} \quad (12)$$

At x = +(L_B + L_C):

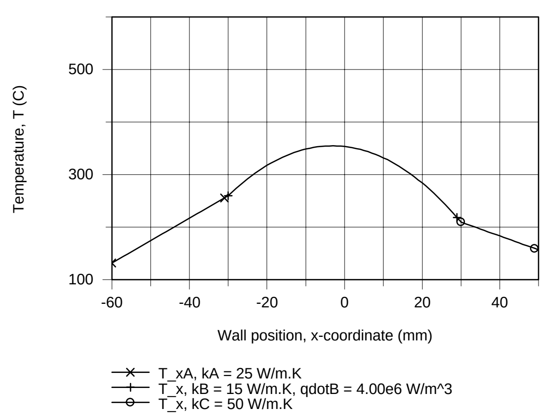


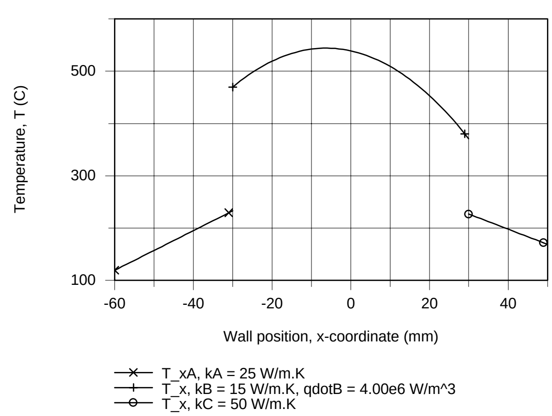
$$-q_{x,C}(L_B + L_C) - q''_{cv} = 0 \quad (13)$$

$$-(-k_C C_5) - h\left[T_C(L_B + L_C) - T_\infty\right] = 0 \quad (14)$$

Following the method of analysis in IHT Example 3.6, User-Defined Functions, we solve the system of equations above for the constants C_1 ... C_6 for conditions with negligible and prescribed values for the interfacial constant resistances. The results are tabulated and plotted below; q''_1 and q''_2 represent heat fluxes leaving surfaces A and C, respectively.

Conditions	T_{1A} (°C)	T_{1B} (°C)	T_{2B} (°C)	T_{2C} (°C)	q''_1 (kW/m^2)	q''_2 (kW/m^2)
$R''_{tc} = 0$	260	260	210	210	106.8	132.0
$R''_{tc} \neq 0$	233	470	371	227	94.6	144.2

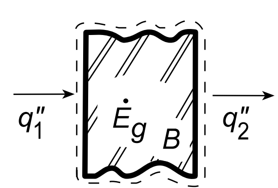




COMMENTS: (1) The results for part (a) can be checked using an energy balance on wall B,

$$\dot{E}_{in} - \dot{E}_{out} = -\dot{E}_g$$

$$q''_1 - q''_2 = -\dot{q}_B \times 2L_B$$



where

$$q''_1 - q''_2 = -107,240 - 132,146 = 239,386\ \text{W}\big/\text{m}^2$$

$$-\dot{q}_B L_B = -4\times10^6\ \text{W}\big/\text{m}^3 \times 2(0.03\,\text{m}) = -240,000\ \text{W}\big/\text{m}^2 .$$

Hence, we have confirmed proper solution of Eqs. (6) and (7).

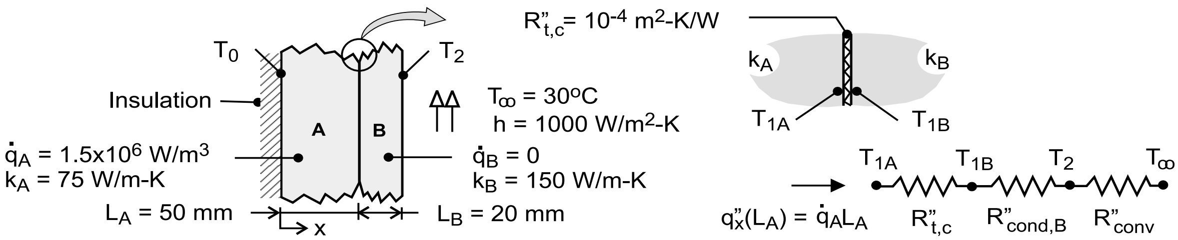
(2) Note that the effect of the interfacial contact resistance is to increase the temperature at all locations. The total heat flux leaving the composite wall (q_1 + q_2) will of course be the same for both cases.

PROBLEM 3.75

KNOWN: Composite wall of materials A and B. Wall of material A has uniform generation, while wall B has no generation. The inner wall of material A is insulated, while the outer surface of material B experiences convection cooling. Thermal contact resistance between the materials is $R''_{t,c} = 10^{-4} m^2 \cdot K/W$. See Ex. 3.6 that considers the case without contact resistance.

FIND: Compute and plot the temperature distribution in the composite wall.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with constant properties, and (3) Inner surface of material A is adiabatic.

ANALYSIS: From the analysis of Ex. 3.6, we know the temperature distribution in material A is parabolic with zero slope at the inner boundary, and that the distribution in material B is linear. At the interface between the two materials, $x = L_A$, the temperature distribution will show a discontinuity.

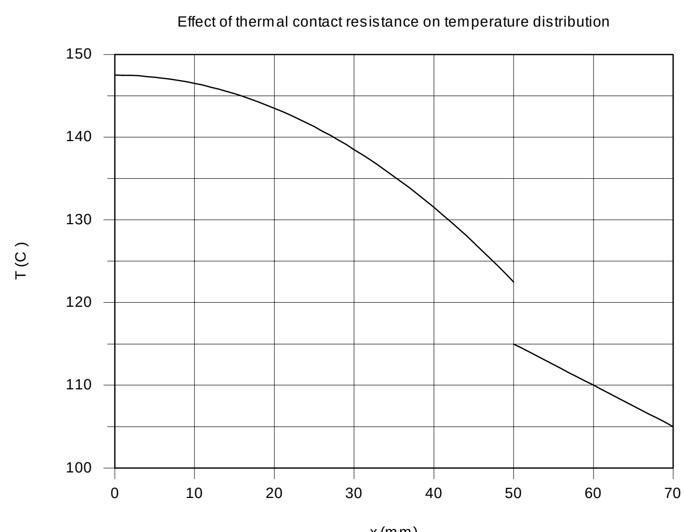
$$T_A(x) = \frac{\dot{q} L_A^2}{2 k_A}\left(1 - \frac{x^2}{L_A^2}\right) + T_{1A} \qquad 0 \le x \le L_A$$

$$T_B(x) = T_{1B} - (T_{1B} - T_2)\frac{x - L_A}{L_B} \qquad L_A \le x \le L_A + L_B$$

Considering the thermal circuit above (see also Ex. 3.6) including the thermal contact resistance,

$$q'' = \dot{q} L_A = \frac{T_{1A} - T_\infty}{R''_{tot}} = \frac{T_{1B} - T_\infty}{R''_{cond,B} + R''_{conv}} = \frac{T_2 - T_\infty}{R''_{conv}}$$

find $T_A(0) = 147.5°C$, $T_{1A} = 122.5°C$, $T_{1B} = 115°C$, and $T_2 = 105°C$. Using the foregoing equations in IHT, the temperature distributions for each of the materials can be calculated and are plotted on the graph below.



COMMENTS: (1) The effect of the thermal contact resistance between the materials is to increase the maximum temperature of the system.

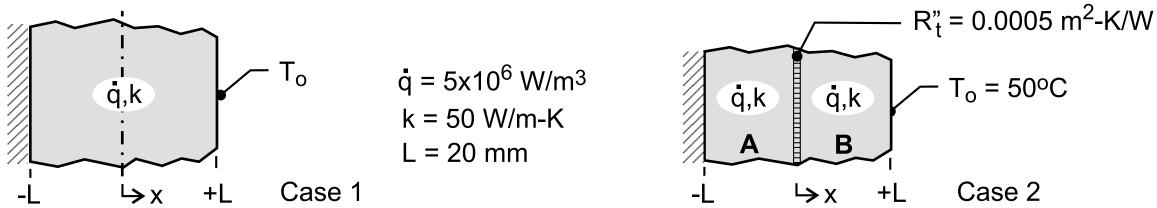
(2) Can you explain why the temperature distribution in the material B is not affected by the presence of the thermal contact resistance at the materials' interface?

PROBLEM 3.76

KNOWN: Plane wall of thickness 2L, thermal conductivity k with uniform energy generation $\dot{q}$. For case 1, boundary at x = -L is perfectly insulated, while boundary at x = +L is maintained at T_o = 50°C. For case 2, the boundary conditions are the same, but a thin dielectric strip with thermal resistance $R''_t = 0.0005\ m^2 \cdot K/W$ is inserted at the mid-plane.

FIND: (a) Sketch the temperature distribution for case 1 on T-x coordinates and describe key features; identify and calculate the maximum temperature in the wall, (b) Sketch the temperature distribution for case 2 on the same T-x coordinates and describe the key features; (c) What is the temperature difference between the two walls at x = 0 for case 2? And (d) What is the location of the maximum temperature of the composite wall in case 2; calculate this temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in the plane and composite walls, and (3) Constant properties.

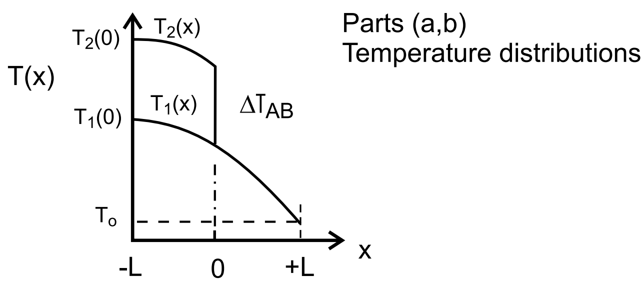
ANALYSIS: (a) For case 1, the temperature distribution, $T_1(x)$ vs. x, is parabolic as shown in the schematic below and the gradient is zero at the insulated boundary, x = -L. From Eq. 3.43,

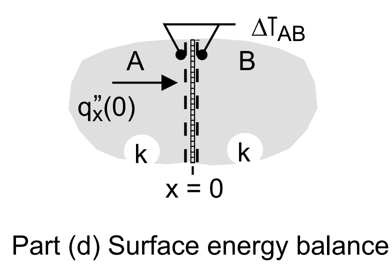
$$T_1(-L) - T_1(+L) = \frac{\dot{q}(2L)^2}{2k} = \frac{5\times10^6\ W/m^3 (2\times0.020\ m)^2}{2\times50\ W/m\cdot K} = 80°C$$

and since $T_1(+L) = T_o$ = 50°C, the maximum temperature occurs at x = -L,

$$T_1(-L) = T_1(+L) + 80°C = 130°C$$

(b) For case 2, the temperature distribution, $T_2(x)$ vs. x, is piece-wise parabolic, with zero gradient at x = -L and a drop across the dielectric strip, ΔT_{AB}. The temperature gradients at either side of the dielectric strip are equal.





(c) For case 2, the temperature drop across the thin dielectric strip follows from the surface energy balance shown above.

$$q''_x(0) = \Delta T_{AB} / R''_t \qquad\qquad q''_x(0) = \dot{q}L$$

$$\Delta T_{AB} = R''_t\ \dot{q}L = 0.0005\ m^2 \cdot K/W \times 5\times10^6 W/m^3 \times 0.020\ m = 50°C.$$

(d) For case 2, the maximum temperature in the composite wall occurs at x = -L, with the value,

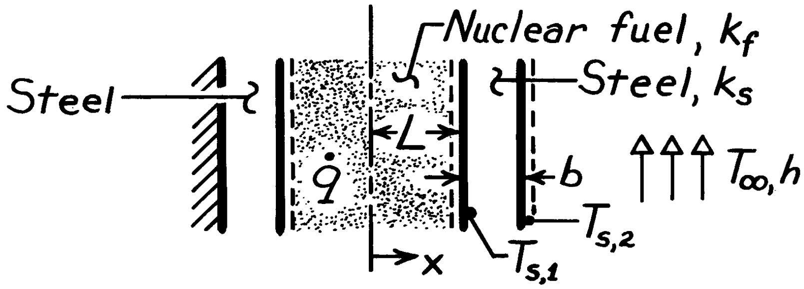
$$T_2(-L) = T_1(-L) + \Delta T_{AB} = 130°C + 50°C = 180°C$$ <

PROBLEM 3.77

KNOWN: Geometry and boundary conditions of a nuclear fuel element.

FIND: (a) Expression for the temperature distribution in the fuel, (b) Form of temperature distribution for the entire system.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state conditions, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance between fuel and cladding.

ANALYSIS: (a) The general solution to the heat equation, Eq. 3.39,

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k_f} = 0 \qquad (-L \le x \le +L)$$

is $\quad T = -\dfrac{\dot{q}}{2k_f}x^2 + C_1x + C_2.$

The insulated wall at x = - (L+b) dictates that the heat flux at x = - L is zero (for an energy balance applied to a control volume about the wall, $\dot{E}_{in} = \dot{E}_{out} = 0$). Hence

$$\left.\frac{dT}{dx}\right]_{x=-L} = -\frac{\dot{q}}{k_f}(-L) + C_1 = 0 \qquad \text{or} \qquad C_1 = -\frac{\dot{q}L}{k_f}$$

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + C_2.$$

The value of $T_{s,1}$ may be determined from the energy conservation requirement that $\dot{E}_g = q_{cond} = q_{conv}$, or on a unit area basis.

$$\dot{q}(2L) = \frac{k_s}{b}\left(T_{s,1} - T_{s,2}\right) = h\left(T_{s,2} - T_\infty\right).$$

Hence,

$$T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + T_{s,2} \qquad \text{where} \qquad T_{s,2} = \frac{\dot{q}(2L)}{h} + T_\infty$$

$$T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2L)}{h} + T_\infty.$$

Continued …..

Hence from Eq. (1),

$$T(L) = T_{s,1} = \frac{\dot{q}(2\,Lb)}{k_s} + \frac{\dot{q}(2\,L)}{h} + T_\infty = -\frac{3}{2}\;\frac{\dot{q}\left(L^2\right)}{k_f} + C_2$$

which yields

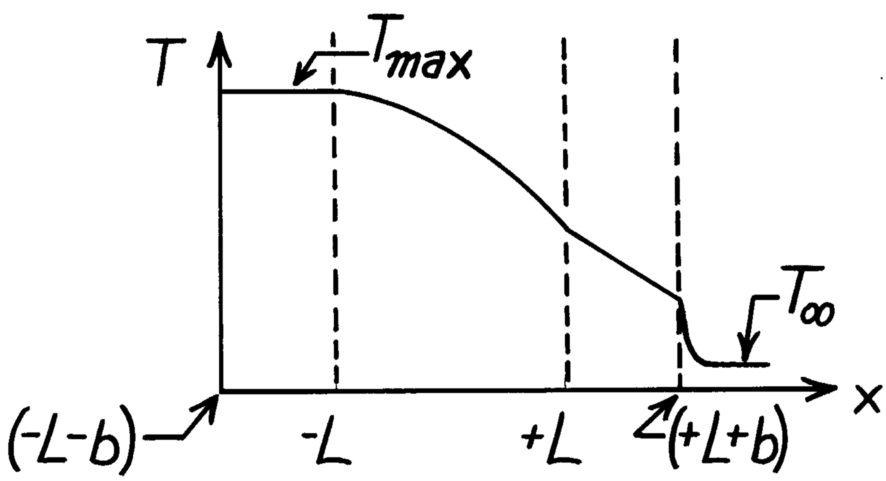
$$C_2 = T_\infty + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\;\frac{L}{k_f}\right]$$

Hence, the temperature distribution for $(-L \le x \le +L)$ is

$$T = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left[\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\;\frac{L}{k_f}\right] + T_\infty$$ <

(b) For the temperature distribution shown below,

$(-L-b) \le x \le -L$: $dT/dx = 0$, $T = T_{max}$
$-L \le x \le +L$: $|dT/dx| \uparrow$ with $\uparrow$ x
$+L \le x \le L+b$: (dT/dx) is const.

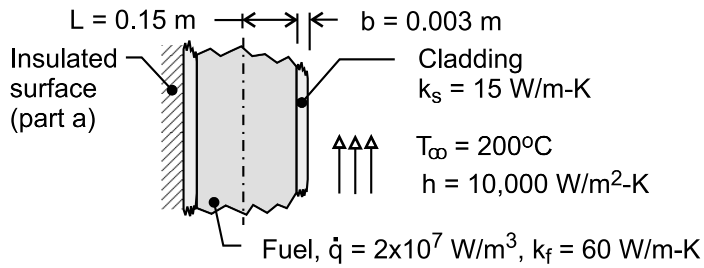


PROBLEM 3.78

KNOWN: Thermal conductivity, heat generation and thickness of fuel element. Thickness and thermal conductivity of cladding. Surface convection conditions.

FIND: (a) Temperature distribution in fuel element with one surface insulated and the other cooled by convection. Largest and smallest temperatures and corresponding locations. (b) Same as part (a) but with equivalent convection conditions at both surfaces, (c) Plot of temperature distributions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional heat transfer, (2) Steady-state, (3) Uniform generation, (4) Constant properties, (5) Negligible contact resistance.

ANALYSIS: (a) From Eq. C.1,

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \qquad (1)$$

With an insulated surface at x = -L, Eq. C.10 yields

$$T_{s,1} - T_{s,2} = \frac{2\dot{q}L^2}{k_f} \qquad (2)$$

and with convection at x = L + b, Eq. C.13 yields

$$U\left(T_{s,2} - T_\infty\right) = \dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \qquad (3)$$

Substracting Eq. (2) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty\right) - \frac{4\dot{q}L^2}{k_f}$$

$$T_{s,2} = T_\infty + \frac{2\dot{q}L}{U} \qquad (4)$$

Continued …..

and substituting into Eq. (2)

$$T_{s,1} = T_\infty + 2\dot{q}L\left(\frac{L}{k_f} + \frac{1}{U}\right) \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (1),

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2}{U} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty$$

or, with $U^{-1} = h^{-1} + b/k_s$,

$$T(x) = -\frac{\dot{q}}{2k_f}x^2 - \frac{\dot{q}L}{k_f}x + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty \tag{6}$$ <

The maximum temperature occurs at x = - L and is

$$T(-L) = 2\dot{q}L\left(\frac{b}{k_s} + \frac{1}{h} + \frac{L}{k_f}\right) + T_\infty$$

$$T(-L) = 2\times 2\times 10^7\ W/m^3 \times 0.015\,m\left(\frac{0.003m}{15\,W/m\cdot K} + \frac{1}{10,000\,W/m^2\cdot K} + \frac{0.015\,m}{60\,W/m\cdot K}\right) + 200°C = 530°C$$ <

The lowest temperature is at x = + L and is

$$T(+L) = -\frac{3}{2}\frac{\dot{q}L^2}{k_f} + \dot{q}L\left(\frac{2b}{k_s} + \frac{2}{h} + \frac{3}{2}\frac{L}{k_f}\right) + T_\infty = 380°C$$ <

(b) If a convection condition is maintained at x = - L, Eq. C.12 reduces to

$$U\left(T_\infty - T_{s,1}\right) = -\dot{q}L - \frac{k_f}{2L}\left(T_{s,2} - T_{s,1}\right)$$

$$T_{s,1} - T_{s,2} = \frac{2LU}{k_f}\left(T_{s,1} - T_\infty\right) - \frac{2\dot{q}L^2}{k_f} \tag{7}$$

Subtracting Eq. (7) from Eq. (3),

$$0 = \frac{2LU}{k_f}\left(T_{s,2} - T_\infty - T_{s,1} + T_\infty\right) \qquad \text{or} \qquad T_{s,1} = T_{s,2}$$

Hence, from Eq. (7)

Continued …..

PROBLEM 3.78 (Cont.)

$$T_{s,1} = T_{s,2} = \frac{\dot{q}L}{U} + T_\infty = \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \qquad (8)$$

Substituting into Eq. (1), the temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k_f}\left(1 - \frac{x^2}{L^2}\right) + \dot{q}L\left(\frac{1}{h} + \frac{b}{k_s}\right) + T_\infty \qquad (9)$$ <

The maximum temperature is at x = 0 and is

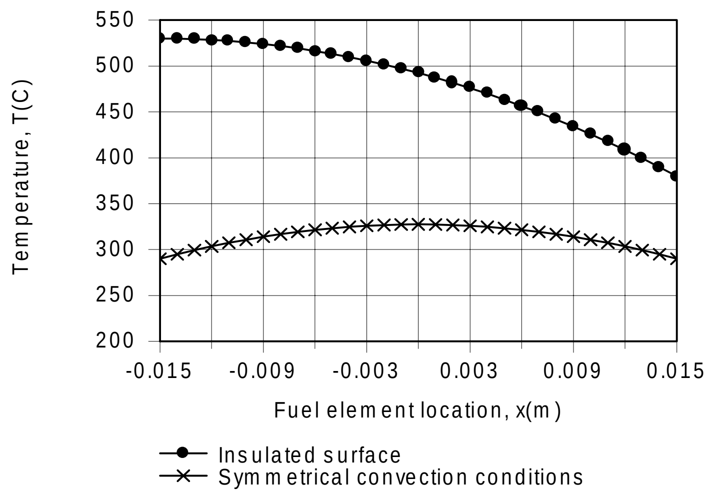
$$T(0) = \frac{2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})^2}{2\times 60\ \mathrm{W/m\cdot K}} + 2\times10^7\ \mathrm{W/m^3}\times 0.015\,\mathrm{m}\left(\frac{1}{10,000\ \mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\ \mathrm{W/m\cdot K}}\right) + 200°\mathrm{C}$$

$$T(0) = 37.5°\mathrm{C} + 90°\mathrm{C} + 200°\mathrm{C} = 327.5°\mathrm{C}$$ <

The minimum temperature at x = ± L is

$$T_{s,1} = T_{s,2} = 2\times10^7\ \mathrm{W/m^3}\,(0.015\,\mathrm{m})\left(\frac{1}{10,000\ \mathrm{W/m^2\cdot K}} + \frac{0.003\,\mathrm{m}}{15\,\mathrm{W/m\cdot K}}\right) + 200°\mathrm{C} = 290°\mathrm{C}$$ <

(c) The temperature distributions are as shown.



The amount of heat generation is the same for both cases, but the ability to transfer heat from both surfaces for case (b) results in lower temperatures throughout the fuel element.

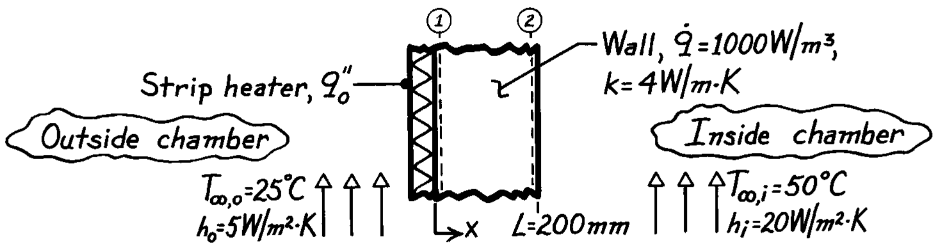
COMMENTS: Note that for case (a), the temperature in the insulated cladding is constant and equivalent to $T_{s,1}$ = 530°C.

PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

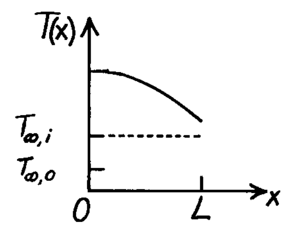
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}=0$ but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \tag{1}$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \tag{2}$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \tag{3}$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \tag{4}$$

Continued …..

and from Eq. (3) with x = L and T(L) = T_2,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \qquad (5,6)$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C} + 1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2 \cdot \text{K} = 50^\circ\text{C} + 10^\circ\text{C} = 60^\circ\text{C}$$ <

$$T_1 = 60^\circ\text{C} + 1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2 \times 4\ \text{W/m} \cdot \text{K} = 65^\circ\text{C}.$$ <

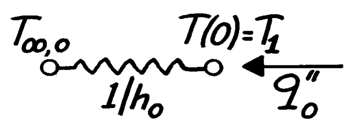
Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

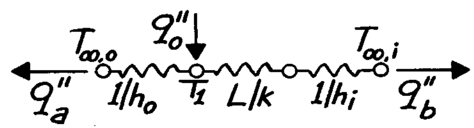
(c) The value of q''_o when T(0) = T_1 = 65°C follows from the circuit



$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2 \cdot \text{K}\,(65\text{-}25)^\circ\text{C} = 200\ \text{W/m}^2.$$ <

(d) With $\dot{q} = 0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

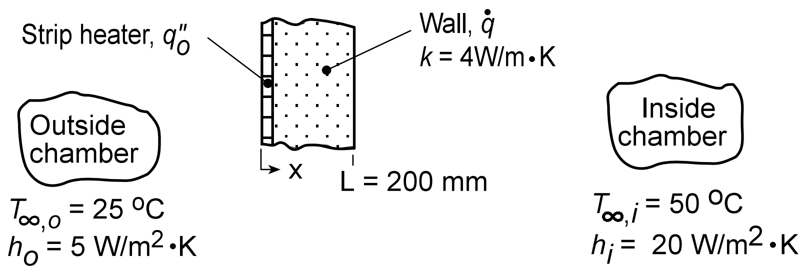
$$T_1 = 55^\circ\text{C}.$$ <

PROBLEM 3.80

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation and strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions ($T_{\infty,i}$, h_i, $T_{\infty,o}$, h_o). Strip heater acts to guard against heat losses from the wall to the outside.

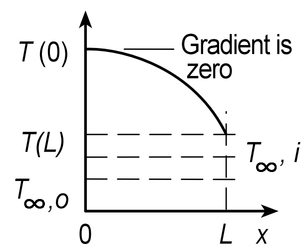
FIND: Compute and plot q''_o and T(0) as a function of $\dot{q}$ for $200 \le \dot{q} \le 2000$ W/m^3 and $T_{\infty,i}$ = 30, 50 and 70°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: If no heat generated within the wall will be lost to the outside of the chamber, the gradient at the position x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution must be parabolic as shown in the sketch.



To determine the required heater flux q''_o as a function of the operation conditions $\dot{q}$ and $T_{\infty,i}$, the analysis begins by considering the temperature distribution in the wall and then surface energy balances at the two wall surfaces. The analysis is organized for easy treatment with equation-solving software.

Temperature distribution in the wall, T(x): The general solution for the temperature distribution in the wall is, Eq. 3.40,

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$$

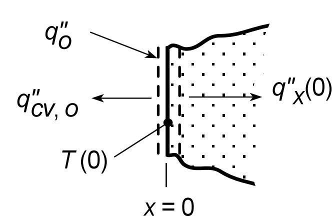
and the guard condition at the outer wall, x = 0, requires that the conduction heat flux be zero. Using Fourier's law,

$$q''_x(0) = -k\frac{dT}{dx}\bigg)_{x=0} = -kC_1 = 0 \qquad (C_1 = 0) \qquad (1)$$

At the outer wall, x = 0,

$$T(0) = C_2 \qquad (2)$$

Surface energy balance, x = 0:



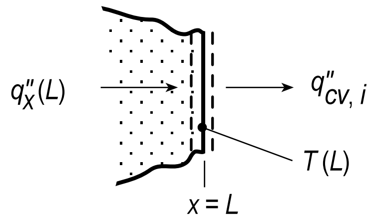
$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_o - q''_{cv,o} - q''_x(0) = 0 \qquad (3)$$

$$q''_{cv,o} = h\left(T(0) - T_{\infty,o}\right), q''_x(0) = 0 \qquad (4a,b)$$

Continued...

Surface energy balance, x = L:



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

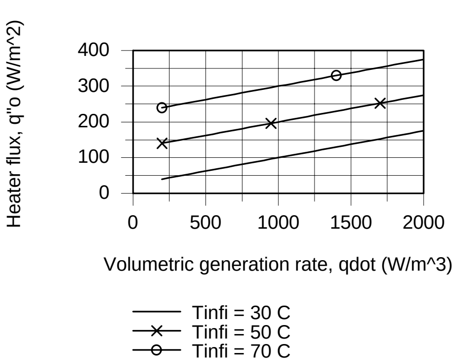
$$q''_x(L) - q''_{cv,i} = 0 \qquad (5)$$

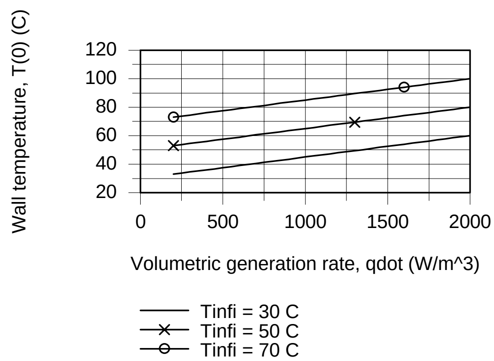
$$q''_x(L) = -k\left.\frac{dT}{dx}\right)_{x=L} = +\dot{q}L \qquad (6)$$

$$q''_{cv,i} = h\left[T(L) - T_{\infty,i}\right]$$

$$q''_{cv,i} = h\left[-\frac{\dot{q}}{2k}L^2 + T(0) - T_{\infty,i}\right] \qquad (7)$$

Solving Eqs. (1) through (7) simultaneously with appropriate numerical values and performing the parametric analysis, the results are plotted below.





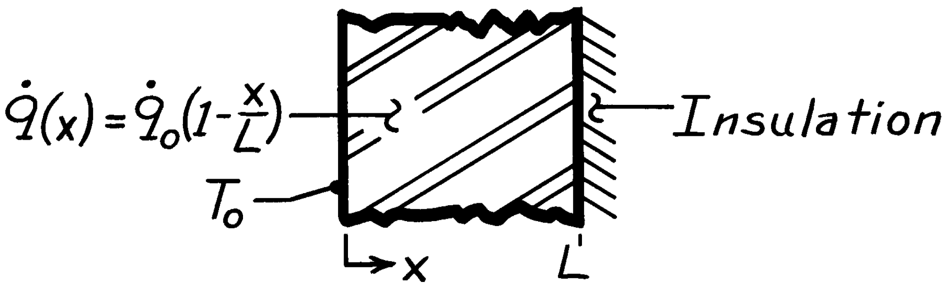
From the first plot, the heater flux q''_o is a linear function of the volumetric generation rate $\dot{q}$. As expected, the higher $\dot{q}$ and $T_{\infty,i}$, the higher the heat flux required to maintain the guard condition ($q''_x(0) = 0$). Notice that for any $\dot{q}$ condition, equal changes in $T_{\infty,i}$ result in equal changes in the required q''_o. The outer wall temperature T(0) is also linearly dependent upon $\dot{q}$. From our knowledge of the temperature distribution, it follows that for any $\dot{q}$ condition, the outer wall temperature T(0) will track changes in $T_{\infty,i}$.

PROBLEM 3.81

KNOWN: Plane wall with prescribed nonuniform volumetric generation having one boundary insulated and the other isothermal.

FIND: Temperature distribution, T(x), in terms of x, L, k, $\dot{q}_o$ and T_o.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction, (3) Constant properties.

ANALYSIS: The appropriate form the heat diffusion equation is

$$\frac{d}{dx}\left[\frac{dT}{dx}\right]+\frac{\dot{q}}{k}=0.$$

Noting that $\dot{q}=\dot{q}(x)=\dot{q}_o\ (1-x/L)$, substitute for $\dot{q}(x)$ into the above equation, separate variables and then integrate,

$$d\left[\frac{dT}{dx}\right]=-\frac{\dot{q}_o}{k}\left[1-\frac{x}{L}\right]dx \qquad \frac{dT}{dx}=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]+C_1.$$

Separate variables and integrate again to obtain the general form of the temperature distribution in the wall,

$$dT=-\frac{\dot{q}_o}{k}\left[x-\frac{x^2}{2L}\right]dx+C_1dx \qquad T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+C_1x+C_2.$$

Identify the boundary conditions at x = 0 and x = L to evaluate C_1 and C_2. At x = 0,

$$T(0)=T_o=-\frac{\dot{q}_o}{k}(0-0)+C_1\cdot 0+C_2 \qquad \text{hence, } C_2=T_o$$

At x = L,

$$\left.\frac{dT}{dx}\right]_{x=L}=0=-\frac{\dot{q}_o}{k}\left[L-\frac{L^2}{2L}\right]+C_1 \qquad \text{hence, } C_1=\frac{\dot{q}_oL}{2k}$$

The temperature distribution is

$$T(x)=-\frac{\dot{q}_o}{k}\left[\frac{x^2}{2}-\frac{x^3}{6L}\right]+\frac{\dot{q}_oL}{2k}x+T_o.$$ <

COMMENTS: It is good practice to test the final result for satisfying BCs. The heat flux at x = 0 can be found using Fourier's law or from an overall energy balance

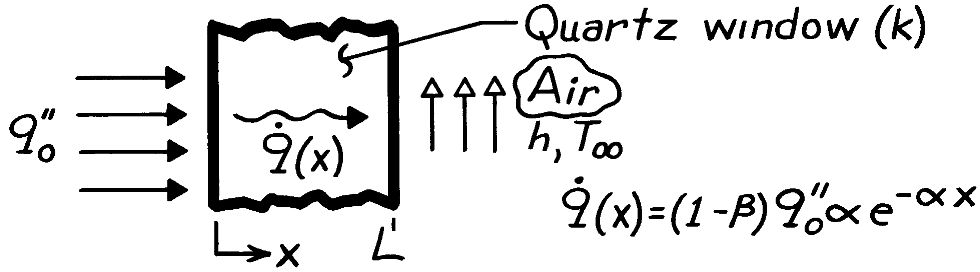
$$\dot{E}_{out}=\dot{E}_g=\int_0^L \dot{q}dV \qquad \text{to obtain} \qquad q''_{out}=\dot{q}_oL/2.$$

PROBLEM 3.82

KNOWN: Distribution of volumetric heating and surface conditions associated with a quartz window.

FIND: Temperature distribution in the quartz.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible radiation emission and convection at inner surface (x = 0) and negligible emission from outer surface, (4) Constant properties.

ANALYSIS: The appropriate form of the heat equation for the quartz is obtained by substituting the prescribed form of $\dot{q}$ into Eq. 3.39.

$$\frac{d^2T}{dx^2}+\frac{\alpha(1-\beta)q''_o}{k}e^{-\alpha x}=0$$

Integrating,

$$\frac{dT}{dx}=+\frac{(1-\beta)q''_o}{k}e^{-\alpha x}+C_1 \qquad T=-\frac{(1-\beta)}{k\alpha}q''_o e^{-\alpha x}+C_1x+C_2$$

Boundary Conditions:

$$-k\, dT/dx)_{x=o}=\beta q''_o$$
$$-k\, dT/dx)_{x=L}=h\left[T(L)-T_\infty\right]$$

Hence, at x = 0:

$$-k\left[\frac{(1-\beta)}{k}q''_o+C_1\right]=\beta q''_o$$
$$C_1=-q''_o/k$$

At x = L:

$$-k\left[\frac{(1-\beta)}{k}q''_o e^{-\alpha L}+C_1\right]=h\left[-\frac{(1-\beta)}{k\alpha}q''_o e^{-\alpha L}+C_1L+C_2-T_\infty\right]$$

Substituting for C_1 and solving for C_2,

$$C_2=\frac{q''_o}{h}\left[1-(1-\beta)e^{-\alpha L}\right]+\frac{q''_o}{k}+\frac{q''_o(1-\beta)}{k\alpha}e^{-\alpha L}+T_\infty.$$

Hence,

$$T(x)=\frac{(1-\beta)q''_o}{k\alpha}\left[e^{-\alpha L}-e^{-\alpha x}\right]+\frac{q''_o}{k}(L-x)+\frac{q''_o}{h}\left[1-(1-\beta)e^{-\alpha L}\right]+T_\infty. \quad <$$

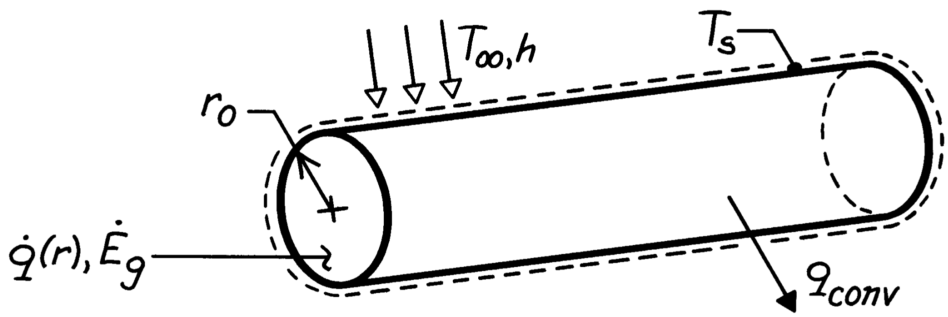
COMMENTS: The temperature distribution depends strongly on the radiative coefficients, α and β. For $\alpha \to \infty$ or β = 1, the heating occurs entirely at x = 0 (no volumetric heating).

PROBLEM 3.83

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left(1-\frac{r^2}{r_o^2}\right)$$

$$r\frac{dT}{dr} = -\frac{\dot{q}_o r^2}{2k} + \frac{\dot{q} r^4}{4kr_o^2} + C_1 \qquad T = -\frac{\dot{q}_o r^2}{4k} + \frac{\dot{q}_o r^4}{16kr_o^2} + C_1 \ln r + C_2.$$

From the boundary conditions,

$$\frac{dT}{dr}\Big|_{r=0} = 0 \rightarrow C_1 = 0 \qquad -k\frac{dT}{dr}\Big|_{r=r_o} = h\left[T(r_o) - T_\infty\right]$$

$$+\frac{\dot{q}_o r_o}{2} - \frac{\dot{q}_o r_o}{4} = h\left[-\frac{\dot{q}_o r_o^2}{4k} + \frac{\dot{q}_o r_o^2}{16k} + C_2 - T_\infty\right]$$

$$C_2 = \frac{\dot{q}_o r_o}{4h} + \frac{3\dot{q}_o r_o^2}{16k} + T_\infty.$$

Hence

$$T(r) = T_\infty + \frac{\dot{q}_o r_o}{4h} + \frac{\dot{q}_o r_o^2}{k}\left[\frac{3}{16} - \frac{1}{4}\left(\frac{r}{r_o}\right)^2 + \frac{1}{16}\left(\frac{r}{r_o}\right)^4\right]. \qquad <$$

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + (\dot{q}_o r_o)/4h$$

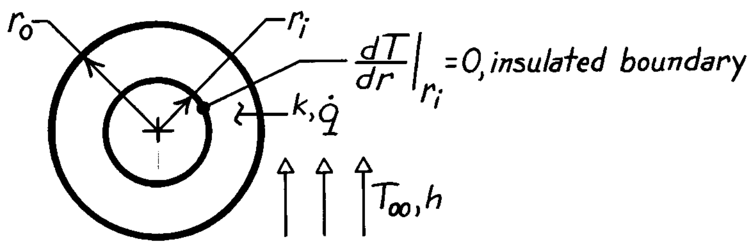
The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.84

KNOWN: Cylindrical shell with uniform volumetric generation is insulated at inner surface and exposed to convection on the outer surface.

FIND: (a) Temperature distribution in the shell in terms of r_i, r_o, $\dot{q}$, h, T_∞ and k, (b) Expression for the heat rate per unit length at the outer radius, $q'(r_o)$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial (cylindrical) conduction in shell, (3) Uniform generation, (4) Constant properties.

ANALYSIS: (a) The general form of the temperature distribution and boundary conditions are

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

at $r = r_i$: $\qquad \left.\frac{dT}{dr}\right)_{r_i} = 0 = -\frac{\dot{q}}{2k}r_i + C_1\frac{1}{r_i} + 0 \qquad C_1 = \frac{\dot{q}}{2k}r_i^2$

at $r = r_o$: $\qquad -k\left.\frac{dT}{dr}\right)_{r_o} = h\left[T(r_o) - T_\infty\right] \qquad$ surface energy balance

$$k\left[-\frac{\dot{q}}{2k}r_o + \left(\frac{\dot{q}}{2k}r_i^2 \cdot \frac{1}{r_o}\right)\right] = h\left[-\frac{\dot{q}}{4k}r_o^2 + \left(\frac{\dot{q}}{2k}r_i^2\right)\ln r_o + C_2 - T_\infty\right]$$

$$C_2 = -\frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + \frac{\dot{q}r_o^2}{2k}\left[\frac{1}{2} - \left(\frac{r_i}{r_o}\right)^2 \ln r_o\right] + T_\infty$$

Hence,

$$T(r) = \frac{\dot{q}}{4k}\left(r_o^2 - r^2\right) + \frac{\dot{q}r_i^2}{2k}\ln\left(\frac{r}{r_o}\right) - \frac{\dot{q}r_o}{2h}\left[1 + \left(\frac{r_i}{r_o}\right)^2\right] + T_\infty. \qquad <$$

(b) From an overall energy balance on the shell,

$$q_r'(r_o) = \dot{E}_g' = \dot{q}\pi\left(r_o^2 - r_i^2\right). \qquad <$$

Alternatively, the heat rate may be found using Fourier's law and the temperature distribution,

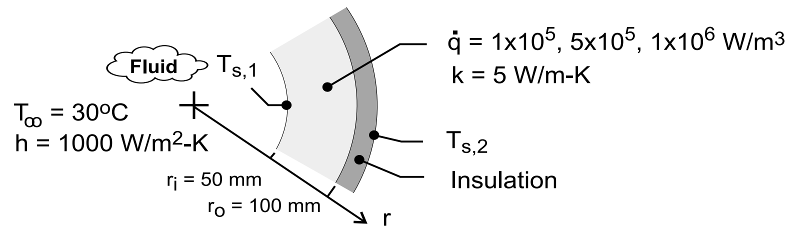
$$q_r'(r) = -k(2\pi r_o)\left.\frac{dT}{dr}\right)_{r_o} = -2\pi k r_o\left[-\frac{\dot{q}}{2k}r_o + \frac{\dot{q}r_i^2}{2k}\frac{1}{r_o} + 0 + 0\right] = \dot{q}\pi\left(r_o^2 - r_i^2\right)$$

PROBLEM 3.85

KNOWN: The solid tube of Example 3.7 with inner and outer radii, 50 and 100 mm, and a thermal conductivity of 5 W/m·K. The inner surface is cooled by a fluid at 30°C with a convection coefficient of 1000 $W/m^2 \cdot K$.

FIND: Calculate and plot the temperature distributions for volumetric generation rates of 1×10^5, 5×10^5, and 1×10^6 W/m^3. Use Eq. (7) with Eq. (10) of the Example 3.7 in the *IHT Workspace*.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties and (4) Uniform volumetric generation.

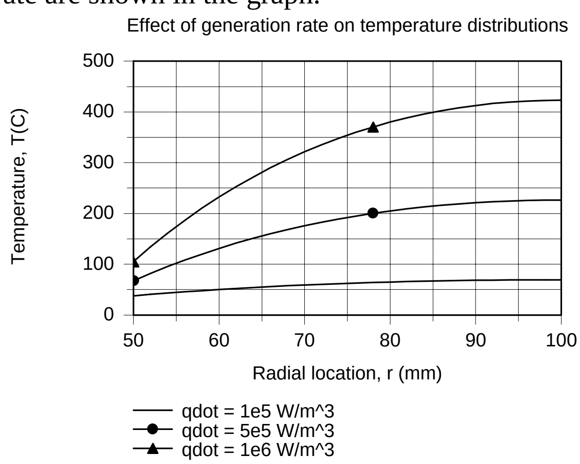
ANALYSIS: From Example 3.7, the temperature distribution in the tube is given by Eq. (7),

$$T(r) = T_{s,2} + \frac{\dot{q}}{4k}\left(r_2^2 - r^2\right) - \frac{\dot{q}}{2k} r_2^2 \ell n\left(\frac{r_2}{r}\right) \qquad r_1 \le r \le r_2 \qquad (1)$$

The temperature at the inner boundary, $T_{s,1}$, follows from the surface energy balance, Eq. (10),

$$\pi\dot{q}\left(r_2^2 - r_1^2\right) = h 2\pi r_1\left(T_{s,1} - T_\infty\right) \qquad (2)$$

For the conditions prescribed in the schematic with $\dot{q} = 1\times10^5 W/m^3$, Eqs. (1) and (2), with r = r_1 and T(r) = $T_{s,1}$, are solved simultaneously to find $T_{s,2}$ = 69.3°C. Eq. (1), with $T_{s,2}$ now a known parameter, can be used to determine the temperature distribution, T(r). The results for different values of the generation rate are shown in the graph.



COMMENTS: (1) The temperature distributions are parabolic with a zero gradient at the insulated outer boundary, r = r_2. The effect of increasing $\dot{q}$ is to increase the maximum temperature in the tube, which always occurs at the outer boundary.

(2) The equations used to generate the graphical result in the *IHT Workspace* are shown below.

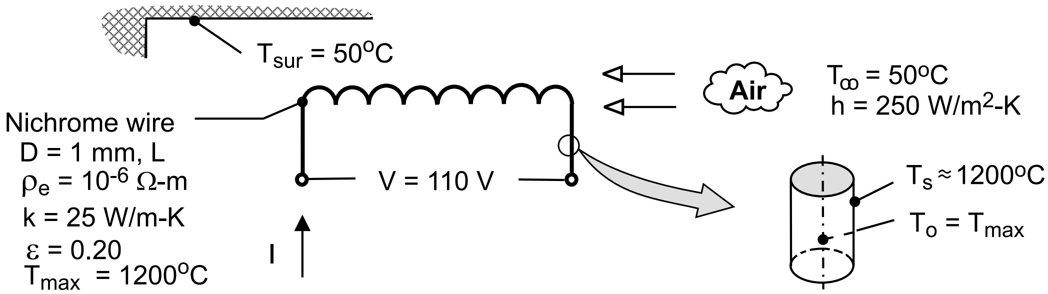
```
// The temperature distribution, from Eq. 7, Example 3.7
T_r = Ts2 + qdot/(4*k) * (r2^2 – r^2) – qgot / (2*k) * r2^2*ln (r2/r)
// The temperature at the inner surface, from Eq. 7
Ts1 = Ts2 + qdot / (4*k) * (r2^2 – r1^2) – qdot / (2*k) * r2^2 * ln (r2/r1)
// The energy balance on the surface, from Eq. 10
pi * qdot * (r2^2 – r1^2) = h * 2 * pi * r1 * (Ts1 – Tinf)
```

PROBLEM 3.86

KNOWN: Diameter, resistivity, thermal conductivity, emissivity, voltage, and maximum temperature of heater wire. Convection coefficient and air exit temperature. Temperature of surroundings.

FIND: Maximum operating current, heater length and power rating.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Uniform wire temperature, (3) Constant properties, (4) Radiation exchange with large surroundings.

ANALYSIS: Assuming a uniform wire temperature, $T_{max} = T(r = 0) \equiv T_o \approx T_s$, the maximum volumetric heat generation may be obtained from Eq. (3.55), but with the total heat transfer coefficient, $h_t = h + h_r$, used in lieu of the convection coefficient h. With

$$h_r = \varepsilon\sigma\left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right) = 0.20\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(1473+323\right)\text{K}\left(1473^2+323\right)^2\text{K}^2 = 46.3\,\text{W/m}^2\cdot\text{K}$$

$$h_t = \left(250 + 46.3\right)\text{W/m}^2\cdot\text{K} = 296.3\,\text{W/m}^2\cdot\text{K}$$

$$\dot{q}_{max} = \frac{2h_t}{r_o}\left(T_s - T_\infty\right) = \frac{2\left(296.3\,\text{W/m}^2\cdot\text{K}\right)}{0.0005\text{m}}\left(1150°\text{C}\right) = 1.36\times 10^9\ \text{W/m}^3$$

Hence, with $\quad \dot{q} = \frac{I^2 R_e}{\forall} = \frac{I^2\left(\rho_e L/A_c\right)}{LA_c} = \frac{I^2\rho_e}{A_c^2} = \frac{I^2\rho_e}{\left(\pi D^2/4\right)^2}$

$$I_{max} = \left(\frac{\dot{q}_{max}}{\rho_e}\right)^{1/2}\frac{\pi D^2}{4} = \left(\frac{1.36\times 10^9\,\text{W/m}^3}{10^{-6}\,\Omega\cdot\text{m}}\right)^{1/2}\frac{\pi\left(0.001\text{m}\right)^2}{4} = 29.0\,\text{A}$$ <

Also, with $\Delta E = I\,R_e = I\,(\rho_e L/A_c)$,

$$L = \frac{\Delta E\cdot A_c}{I_{max}\,\rho_e} = \frac{110\,\text{V}\left[\pi\left(0.001\text{m}\right)^2/4\right]}{29.0\,\text{A}\left(10^{-6}\,\Omega\cdot\text{m}\right)} = 2.98\text{m}$$ <

and the power rating is

$$P_{elec} = \Delta E\cdot I_{max} = 110\,\text{V}\left(29\,\text{A}\right) = 3190\,\text{W} = 3.19\,\text{kW}$$ <

COMMENTS: To assess the validity of assuming a uniform wire temperature, Eq. (3.53) may be used to compute the centerline temperature corresponding to $\dot{q}_{max}$ and a surface temperature of 1200°C. It follows that $T_o = \frac{\dot{q}\,r_o^2}{4k} + T_s = \frac{1.36\times 10^9\ \text{W/m}^3\left(0.0005\text{m}\right)^2}{4\left(25\,\text{W/m}\cdot\text{K}\right)} + 1200°\text{C} = 1203°\text{C}$. With only a

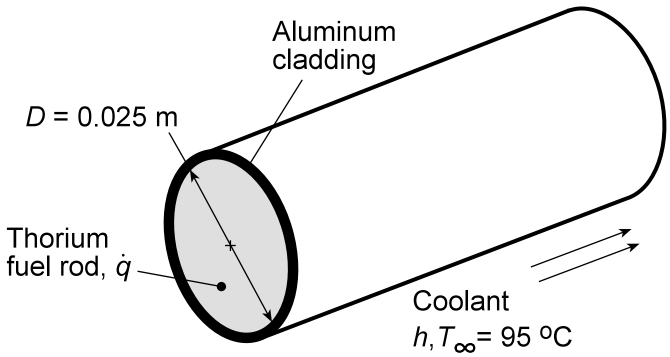
3°C temperature difference between the centerline and surface of the wire, the assumption is *excellent*.

PROBLEM 3.87

KNOWN: Energy generation in an aluminum-clad, thorium fuel rod under specified operating conditions.

FIND: (a) Whether prescribed operating conditions are acceptable, (b) Effect of $\dot{q}$ and h on acceptable operating conditions.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in r-direction, (2) Steady-state conditions, (3) Constant properties, (4) Negligible temperature gradients in aluminum and contact resistance between aluminum and thorium.

PROPERTIES: *Table A-1*, Aluminum, pure: M.P. = 933 K; *Table A-1*, Thorium: M.P. = 2023 K, k ≈ 60 W/m·K.

ANALYSIS: (a) System failure would occur if the melting point of either the thorium or the aluminum were exceeded. From Eq. 3.53, the maximum thorium temperature, which exists at r = 0, is

$$T(0) = \frac{\dot{q} r_o^2}{4k} + T_s = T_{Th,max}$$

where, from the energy balance equation, Eq. 3.55, the surface temperature, which is also the aluminum temperature, is

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h} = T_{Al}$$

Hence,

$$T_{Al} = T_s = 95^\circ C + \frac{7\times10^8\, W/m^3 \times 0.0125\, m}{14,000\, W/m^2\cdot K} = 720^\circ C = 993\, K$$

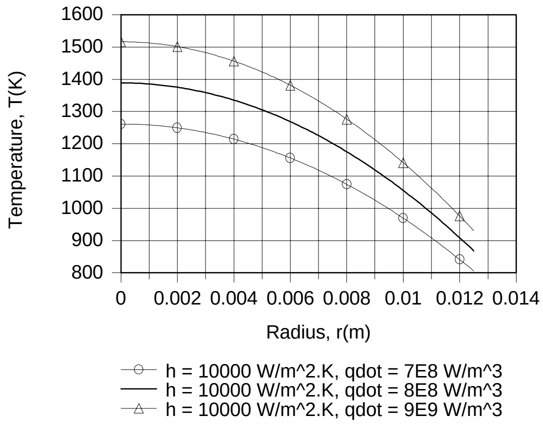
$$T_{Th,max} = \frac{7\times10^8\, W/m^3 (0.0125 m)^2}{4\times60\, W/m\cdot K} + 993\, K = 1449\, K \quad <$$

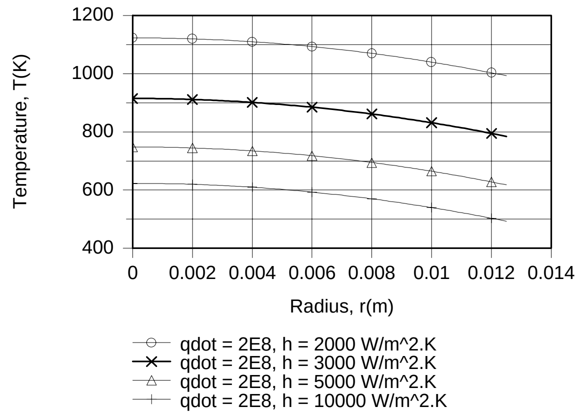
Although $T_{Th,max}$ < $M.P._{Th}$ and the thorium would not melt, T_{al} > $M.P._{Al}$ and the cladding would melt under the proposed operating conditions. The problem could be eliminated by *decreasing* $\dot{q}$, *increasing* h or using a cladding material with a higher melting point.

(b) Using the one-dimensional, steady-state conduction model (solid cylinder) of the IHT software, the following radial temperature distributions were obtained for parametric variations in $\dot{q}$ and h.

Continued...

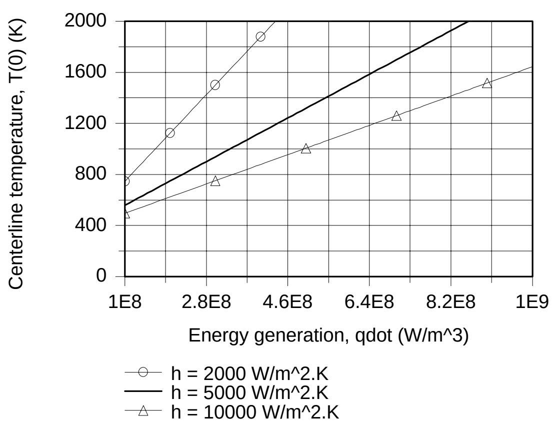
PROBLEM 3.87 (Cont.)

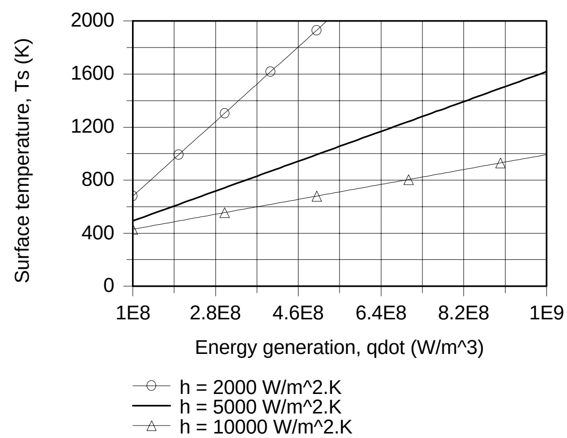




For h = 10,000 W/m^2·K, which represents a reasonable upper limit with water cooling, the temperature of the aluminum would be well below its melting point for $\dot{q} = 7 \times 10^8$ W/m^3, but would be close to the melting point for $\dot{q} = 8 \times 10^8$ W/m^3 and would exceed it for $\dot{q} = 9 \times 10^8$ W/m^3. Hence, under the best of conditions, $\dot{q} \approx 7 \times 10^8$ W/m^3 corresponds to the maximum allowable energy generation. However, if coolant flow conditions are constrained to provide values of h < 10,000 W/m^2·K, volumetric heating would have to be reduced. Even for $\dot{q}$ as low as 2×10^8 W/m^3, operation could not be sustained for h = 2000 W/m^2·K.

The effects of $\dot{q}$ and h on the centerline and surface temperatures are shown below.





For h = 2000 and 5000 W/m^2·K, the melting point of thorium would be approached for $\dot{q} \approx 4.4 \times 10^8$ and 8.5×10^8 W/m^3, respectively. For h = 2000, 5000 and 10,000 W/m^2·K, the melting point of aluminum would be approached for $\dot{q} \approx 1.6 \times 10^8$, 4.3×10^8 and 8.7×10^8 W/m^3. Hence, the envelope of acceptable operating conditions must call for a reduction in $\dot{q}$ with decreasing h, from a maximum of $\dot{q} \approx 7 \times 10^8$ W/m^3 for h = 10,000 W/m^2·K.

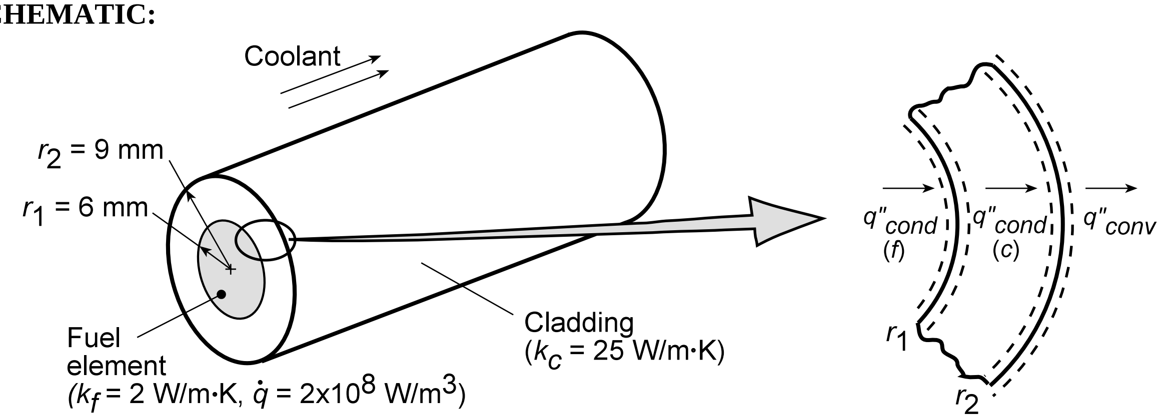
COMMENTS: Note the problem which would arise in the event of a *loss of coolant*, for which case h would *decrease* drastically.

PROBLEM 3.88

KNOWN: Radii and thermal conductivities of reactor fuel element and cladding. Fuel heat generation rate. Temperature and convection coefficient of coolant.

FIND: (a) Expressions for temperature distributions in fuel and cladding, (b) Maximum fuel element temperature for prescribed conditions, (c) Effect of h on temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Negligible contact resistance, (4) Constant properties.

ANALYSIS: (a) From Eqs. 3.49 and 3.23, the heat equations for the fuel (f) and cladding (c) are

$$\frac{1}{r}\frac{d}{dr}\left(\frac{dT_f}{dr}\right)=-\frac{\dot{q}}{k_f} \qquad (0 \le r \le r_1) \qquad\qquad \frac{1}{r}\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0 \qquad (r_1 \le r \le r_2)$$

Hence, integrating both equations twice,

$$\frac{dT_f}{dr}=-\frac{\dot{q}r}{2k_f}+\frac{C_1}{k_f r} \qquad T_f=-\frac{\dot{q}r^2}{4k_f}+\frac{C_1}{k_f}\ln r+C_2 \qquad (1,2)$$

$$\frac{dT_c}{dr}=\frac{C_3}{k_c r} \qquad T_c=\frac{C_3}{k_c}\ln r+C_4 \qquad (3,4)$$

The corresponding boundary conditions are:

$$dT_f/dr)_{r=0}=0 \qquad T_f(r_1)=T_c(r_1) \qquad (5,6)$$

$$-k_f\left.\frac{dT_f}{dr}\right)_{r=r_1}=-k_c\left.\frac{dT_c}{dr}\right)_{r=r_1} \qquad -k_c\left.\frac{dT_c}{dr}\right)_{r=r_2}=h\left[T_c(r_2)-T_\infty\right] \qquad (7,8)$$

Note that Eqs. (7) and (8) are obtained from surface energy balances at r_1 and r_2, respectively. Applying Eq. (5) to Eq. (1), it follows that $C_1 = 0$. Hence,

$$T_f=-\frac{\dot{q}r^2}{4k_f}+C_2 \qquad (9)$$

From Eq. (6), it follows that

$$-\frac{\dot{q}r_1^2}{4k_f}+C_2=\frac{C_3\ln r_1}{k_c}+C_4 \qquad (10)$$

Continued...

Also, from Eq. (7),

$$\frac{\dot{q}r_1}{2} = -\frac{C_3}{r_1} \qquad \text{or} \qquad C_3 = -\frac{\dot{q}r_1^2}{2} \tag{11}$$

Finally, from Eq. (8), $-\frac{C_3}{r_2} = h\left[\frac{C_3}{k_c}\ln r_2 + C_4 - T_\infty\right]$ or, substituting for C_3 and solving for C_4

$$C_4 = \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty \tag{12}$$

Substituting Eqs. (11) and (12) into (10), it follows that

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} - \frac{\dot{q}r_1^2 \ln r_1}{2k_c} + \frac{\dot{q}r_1^2}{2r_2 h} + \frac{\dot{q}r_1^2}{2k_c}\ln r_2 + T_\infty$$

$$C_2 = \frac{\dot{q}r_1^2}{4k_f} + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} T_\infty \tag{13}$$

Substituting Eq. (13) into (9),

$$T_f = \frac{\dot{q}}{4k_f}\left(r_1^2 - r^2\right) + \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r_1} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \tag{14}$$ <

Substituting Eqs. (11) and (12) into (4),

$$T_c = \frac{\dot{q}r_1^2}{2k_c}\ln\frac{r_2}{r} + \frac{\dot{q}r_1^2}{2r_2 h} + T_\infty \, . \tag{15}$$ <

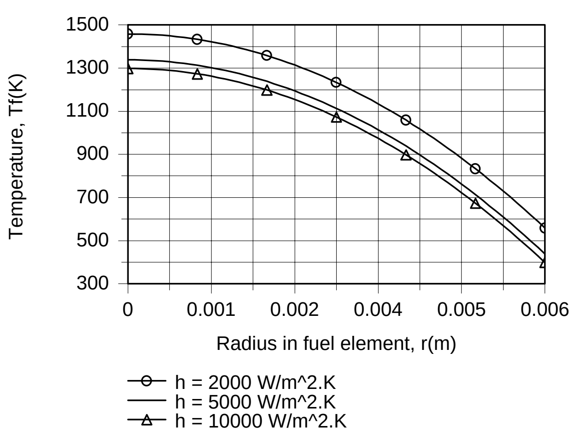
(b) Applying Eq. (14) at r = 0, the maximum fuel temperature for h = 2000 $W/m^2 \cdot K$ is

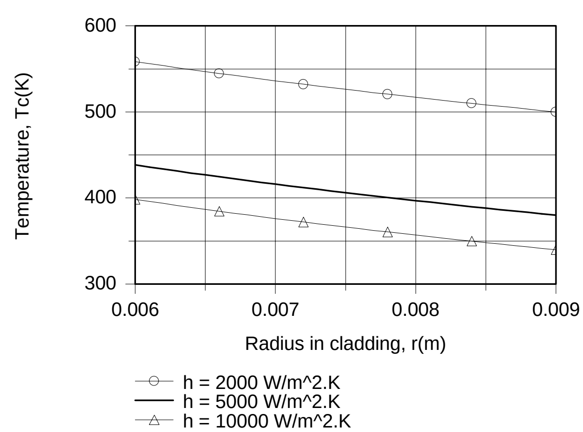
$$T_f(0) = \frac{2\times10^8\ W/m^3 \times (0.006\,m)^2}{4\times 2\ W/m\cdot K} + \frac{2\times10^8\ W/m^3 \times (0.006\,m)^2}{2\times 25\ W/m\cdot K}\ln\frac{0.009\,m}{0.006\,m}$$

$$+\frac{2\times10^8\ W/m^3\,(0.006\,m)^2}{2\times(0.09\,m)\,2000\ W/m^2\cdot K} + 300\,K$$

$$T_f(0) = (900 + 58.4 + 200 + 300)\,K = 1458\,K\ .$$ <

(c) Temperature distributions for the prescribed values of h are as follows:





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PROBLEM 3.88 (Cont.)

Clearly, the ability to control the maximum fuel temperature by increasing h is limited, and even for $h \rightarrow \infty$, $T_f(0)$ exceeds 1000 K. The overall temperature drop, $T_f(0) - T_\infty$, is influenced principally by the low thermal conductivity of the fuel material.

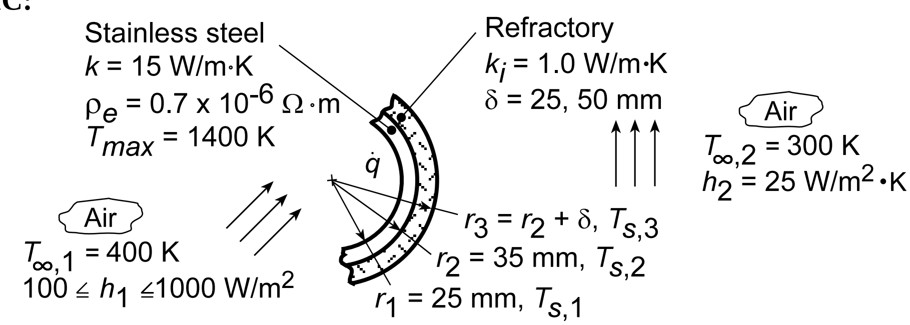
COMMENTS: For the prescribed conditions, Eq. (14) yields, $T_f(0) - T_f(r_1) = \dot{q}r_1^2/4k_f = (2\times10^8\ W/m^3)(0.006\ m)^3/8\ W/m\cdot K = 900\ K$, in which case, with no cladding and $h \rightarrow \infty$, $T_f(0) = 1200\ K$. To reduce $T_f(0)$ below 1000 K for the prescribed material, it is necessary to reduce $\dot{q}$.

PROBLEM 3.89

KNOWN: Dimensions and properties of tubular heater and external insulation. Internal and external convection conditions. Maximum allowable tube temperature.

FIND: (a) Maximum allowable heater current for adiabatic outer surface, (3) Effect of internal convection coefficient on heater temperature distribution, (c) Extent of heat loss at outer surface.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform heat generation, (4) Negligible radiation at outer surface, (5) Negligible contact resistance.

ANALYSIS: (a) From Eqs. 7 and 10, respectively, of Example 3.7, we know that

$$T_{s,2} - T_{s,1} = \frac{\dot{q}}{2k} r_2^2 \ln\frac{r_2}{r_1} - \frac{\dot{q}}{4k}\left(r_2^2 - r_1^2\right) \qquad (1)$$

and

$$T_{s,1} = T_{\infty,1} + \frac{\dot{q}\left(r_2^2 - r_1^2\right)}{2h_1 r_1} \qquad (2)$$

Hence, eliminating $T_{s,1}$, we obtain

$$T_{s,2} - T_{\infty,1} = \frac{\dot{q} r_2^2}{2k}\left[\ln\frac{r_2}{r_1} - \frac{1}{2}\left(1 - r_1^2/r_2^2\right) + \frac{k}{h_1 r_1}\left(1 - r_1^2/r_2^2\right)\right]$$

Substituting the prescribed conditions (h_1 = 100 W/m^2·K),

$$T_{s,2} - T_{\infty,1} = 1.237\times10^{-4}\left(m^3\cdot K/W\right)\dot{q}\left(W/m^3\right)$$

Hence, with T_{max} corresponding to $T_{s,2}$, the maximum allowable value of $\dot{q}$ is

$$\dot{q}_{max} = \frac{1400 - 400}{1.237\times10^{-4}} = 8.084\times10^6 \ W/m^3$$

with

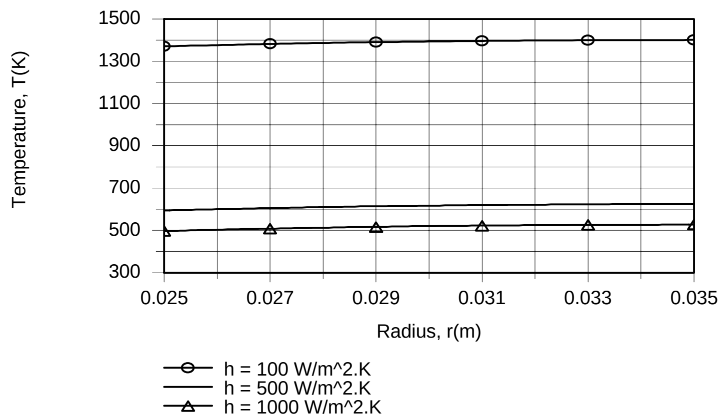
$$\dot{q} = \frac{I^2 Re}{\forall} = \frac{I^2 \rho_e L/A_c}{LA_c} = \frac{\rho_e I^2}{\left[\pi\left(r_2^2 - r_1\right)\right]^2}$$

$$I_{max} = \pi\left(r_2^2 - r_1^2\right)\left(\frac{\dot{q}}{\rho_e}\right)^{1/2} = \pi\left(0.035^2 - 0.025^2\right)m^2\left(\frac{8.084\times10^6 \ W/m^3}{0.7\times10^{-6}\Omega\cdot m}\right)^{1/2} = 6406\,A \quad <$$

Continued …..

PROBLEM 3.89 (Cont.)

(b) Using the one-dimensional, steady-state conduction model of IHT (hollow cylinder; convection at inner surface and adiabatic outer surface), the following temperature distributions were obtained.



The results are consistent with key implications of Eqs. (1) and (2), namely that the value of h_1 has no effect on the temperature drop across the tube ($T_{s,2}$ - $T_{s,1}$ = 30 K, irrespective of h_1), while $T_{s,1}$ decreases with increasing h_1. For h_1 = 100, 500 and 1000 $W/m^2 \cdot K$, respectively, the ratio of the temperature drop between the inner surface and the air to the temperature drop across the tube, $(T_{s,1} - T_{\infty,1})/(T_{s,2} - T_{s,1})$, decreases from 970/30 = 32.3 to 194/30 = 6.5 and 97/30 = 3.2. Because the outer surface is insulated, the heat rate to the airflow is fixed by the value of $\dot{q}$ and, irrespective of h_1,

$$q'(r_1) = \pi\left(r_2^2 - r_1^2\right)\dot{q} = -15,240\,W$$ <

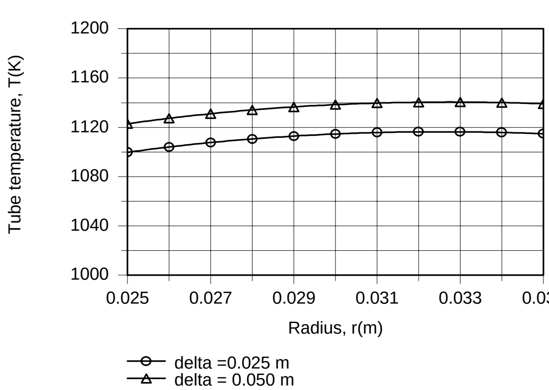
(c) Heat loss from the outer surface of the tube to the surroundings depends on the total thermal resistance

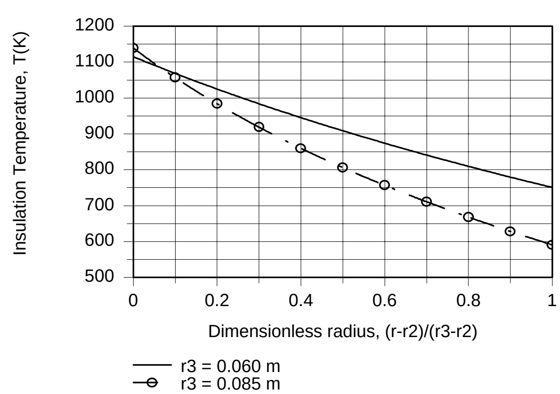
$$R_{tot} = \frac{\ln(r_3/r_2)}{2\pi L k_i} + \frac{1}{2\pi r_3 L h_2}$$

or, for a unit area on surface 2,

$$R''_{tot,2} = (2\pi r_2 L) R_{tot} = \frac{r_2 \ln(r_3/r_2)}{k_i} + \frac{r_2}{r_3 h_2}$$

Again using the capabilities of IHT (hollow cylinder; convection at inner surface and heat transfer from outer surface through $R''_{tot,2}$), the following temperature distributions were determined for the tube and insulation.





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PROBLEM 3.89 (Cont.)

Heat losses through the insulation, $q'(r_2)$, are 4250 and 3890 W/m for δ = 25 and 50 mm, respectively, with corresponding values of $q'(r_1)$ equal to -10,990 and -11,350 W/m. Comparing the tube temperature distributions with those predicted for an adiabatic outer surface, it is evident that the losses reduce tube wall temperatures predicted for the adiabatic surface and also shift the maximum temperature from r = 0.035 m to r ≈ 0.033 m. Although the tube outer and insulation inner surface temperatures, $T_{s,2} = T(r_2)$, increase with increasing insulation thickness, Fig. (c), the insulation outer surface temperature decreases.

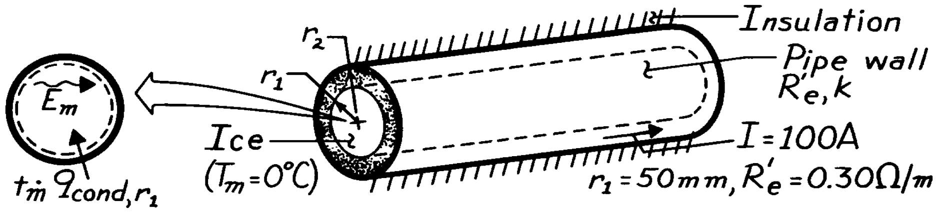
COMMENTS: If the intent is to maximize heat transfer to the airflow, heat losses to the ambient should be reduced by selecting an insulation material with a significantly smaller thermal conductivity.

PROBLEM 3.90

KNOWN: Electric current I is passed through a pipe of resistance R'_e to melt ice under steady-state conditions.

FIND: (a) Temperature distribution in the pipe wall, (b) Time to completely melt the ice.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation in the pipe wall, (5) Outer surface of the pipe is adiabatic, (6) Inner surface is at a constant temperature, T_m.

PROPERTIES: *Table A-3,* Ice (273K): ρ = 920 kg/m^3; *Handbook Chem. & Physics,* Ice: Latent heat of fusion, $h_{sf} = 3.34\times10^5$ J/kg.

ANALYSIS: (a) The appropriate form of the heat equation is Eq. 3.49, and the general solution, Eq. 3.51 is

$$T(r) = -\frac{\dot{q}}{4k}r^2 + C_1 \ln r + C_2$$

where

$$\dot{q} = \frac{I^2 R'_e}{\pi\left(r_2^2 - r_1^2\right)}.$$

Applying the boundary condition $\left(dT/dr\right)_{r_2} = 0,$ it follows that

$$0 = \frac{\dot{q} r_2}{2k} + \frac{C_1}{r_2}$$

Hence $\qquad C_1 = \frac{\dot{q} r_2^2}{2k}$

and $\qquad T(r) = -\frac{\dot{q}}{4k}r^2 + \frac{\dot{q} r_2^2}{2k}\ln r + C_2.$

Continued …..

Applying the second boundary condition, $T(r_1) = T_m$, it follows that

$$T_m = -\frac{\dot{q}}{4k} r_1^2 + \frac{\dot{q} r_2^2}{2k} \ln r_1 + C_2.$$

Solving for C_2 and substituting into the expression for T(r), find

$$T(r) = T_m + \frac{\dot{q} r_2^2}{2k} \ln \frac{r}{r_1} - \frac{\dot{q}}{4k}\left(r^2 - r_1^2\right). \qquad <$$

(b) Conservation of energy dictates that the energy required to completely melt the ice, E_m, must equal the energy which reaches the inner surface of the pipe by conduction through the wall during the melt period. Hence from Eq. 1.11b

$$\Delta E_{st} = E_{in} - E_{out} + E_{gen}$$

$$\Delta E_{st} = E_m = t_m \cdot q_{cond,r_1}$$

or, for a unit length of pipe,

$$\rho\left(\pi\, r_1^2\right) h_{sf} = t_m \left[-k\left(2\pi\, r_1\right) \left[\frac{dT}{dr}\right]_{r_1} \right]$$

$$\rho\left(\pi\, r_1^2\right) h_{sf} = -2\pi r_1 k t_m \left[\frac{\dot{q} r_2^2}{2k r_1} - \frac{\dot{q} r_1}{2k} \right]$$

$$\rho\left(\pi\, r_1^2\right) h_{sf} = -t_m \dot{q} \pi \left(r_2^2 - r_1^2\right).$$

Dropping the minus sign, which simply results from the fact that conduction is in the negative r direction, it follows that

$$t_m = \frac{\rho h_{sf} r_1^2}{\dot{q}\left(r_2^2 - r_1^2\right)} = \frac{\rho h_{sf} \pi\, r_1^2}{I^2 R'_e}.$$

With r_1 = 0.05m, I = 100 A and $R'_e = 0.30$ Ω/m, it follows that

$$t_m = \frac{920 \text{kg/m}^3 \times 3.34 \times 10^5 \text{J/kg} \times \pi \times (0.05\text{m})^2}{(100\text{A})^2 \times 0.30\Omega/\text{m}}$$

or $\quad t_m = 804\text{s}. \qquad <$

COMMENTS: The foregoing expression for t_m could also be obtained by recognizing that all of the energy which is generated by electrical heating in the pipe wall must be transferred to the ice. Hence,

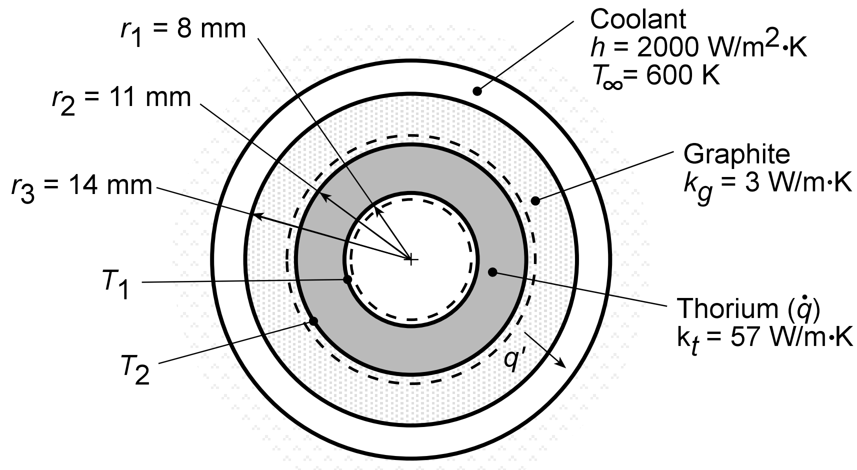
$$I^2 R'_e t_m = \rho h_{sf} \pi\, r_1^2.$$

PROBLEM 3.91

KNOWN: Materials, dimensions, properties and operating conditions of a gas-cooled nuclear reactor.

FIND: (a) Inner and outer surface temperatures of fuel element, (b) Temperature distributions for different heat generation rates and maximum allowable generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation.

PROPERTIES: *Table A.1*, Thoriun: $T_{mp} \approx 2000$ K; *Table A.2*, Graphite: $T_{mp} \approx 2300$ K.

ANALYSIS: (a) The outer surface temperature of the fuel, T_2, may be determined from the rate equation

$$q' = \frac{T_2 - T_\infty}{R'_{tot}}$$

where

$$R'_{tot} = \frac{\ln(r_3/r_2)}{2\pi\, k_g} + \frac{1}{2\pi r_3 h} = \frac{\ln(14/11)}{2\pi\,(3\,\text{W/m}\cdot\text{K})} + \frac{1}{2\pi\,(0.014\,\text{m})\left(2000\,\text{W/m}^2\cdot\text{K}\right)} = 0.0185\,\text{m}\cdot\text{K/W}$$

and the heat rate per unit length may be determined by applying an energy balance to a control surface about the fuel element. Since the interior surface of the element is essentially adiabatic, it follows that

$$q' = \dot{q}\pi\left(r_2^2 - r_1^2\right) = 10^8\,\text{W/m}^3 \times \pi\left(0.011^2 - 0.008^2\right)\text{m}^2 = 17{,}907\,\text{W/m}$$

Hence,

$$T_2 = q'R'_{tot} + T_\infty = 17{,}907\,\text{W/m}\,(0.0185\,\text{m}\cdot\text{K/W}) + 600\,\text{K} = 931\,\text{K}$$ <

With zero heat flux at the inner surface of the fuel element, Eq. C.14 yields

$$T_1 = T_2 + \frac{\dot{q}r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) - \frac{\dot{q}r_1^2}{2k_t}\ln\left(\frac{r_2}{r_1}\right)$$

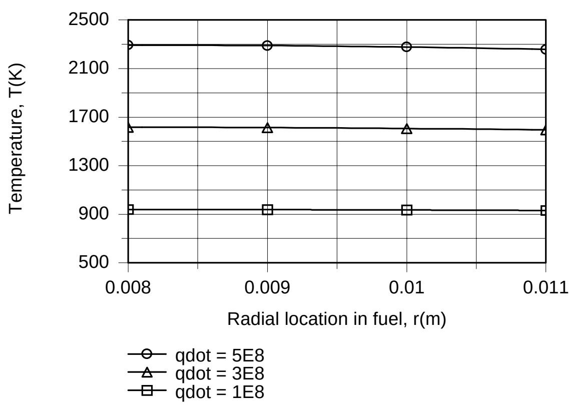
$$T_1 = 931\,\text{K} + \frac{10^8\,\text{W/m}^3\,(0.011\,\text{m})^2}{4\times 57\,\text{W/m}\cdot\text{K}}\left[1 - \left(\frac{0.008}{0.011}\right)^2\right] - \frac{10^8\,\text{W/m}^3\,(0.008\,\text{m})^2}{2\times 57\,\text{W/m}\cdot\text{K}}\ln\left(\frac{0.011}{0.008}\right)$$

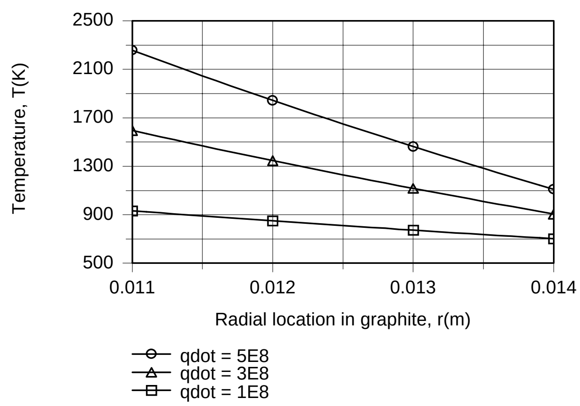
Continued...

PROBLEM 3.91 (Cont.)

$$T_1 = 931\,K + 25\,K - 18\,K = 938\,K$$ <

(b) The temperature distributions may be obtained by using the IHT model for one-dimensional, steady-state conduction in a hollow tube. For the fuel element ($\dot{q} > 0$), an adiabatic surface condition is prescribed at r_1, while heat transfer from the outer surface at r_2 to the coolant is governed by the thermal resistance $R''_{tot,2} = 2\pi r_2 R'_{tot} = 2\pi(0.011\ m)0.0185\ m{\cdot}K/W = 0.00128\ m^2{\cdot}K/W$. For the graphite ($\dot{q}$ = 0), the value of T_2 obtained from the foregoing solution is prescribed as an inner boundary condition at r_2, while a convection condition is prescribed at the outer surface (r_3). For $1 \times 10^8 \le \dot{q} \le 5 \times 10^8\ W/m^3$, the following distributions are obtained.





The comparatively large value of k_t yields small temperature variations across the fuel element, while the small value of k_g results in large temperature variations across the graphite. Operation at $\dot{q} = 5 \times 10^8\ W/m^3$ is clearly unacceptable, since the melting points of thorium and graphite are exceeded and approached, respectively. To prevent softening of the materials, which would occur below their melting points, the reactor should not be operated much above $\dot{q} = 3 \times 10^8\ W/m^3$.

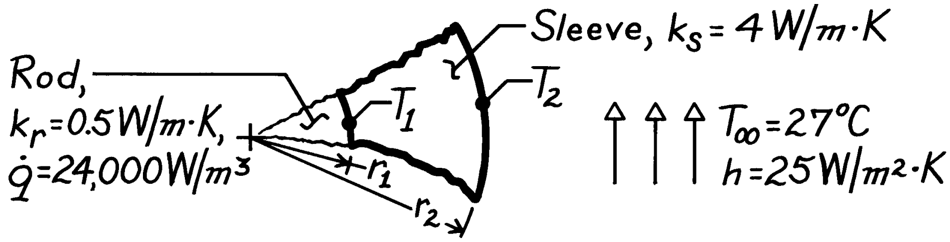
COMMENTS: A contact resistance at the thorium/graphite interface would increase temperatures in the fuel element, thereby reducing the maximum allowable value of $\dot{q}$.

PROBLEM 3.92

KNOWN: Long rod experiencing uniform volumetric generation encapsulated by a circular sleeve exposed to convection.

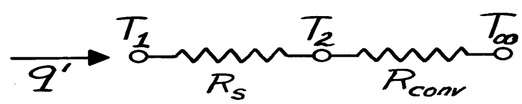
FIND: (a) Temperature at the interface between rod and sleeve and on the outer surface, (b) Temperature at center of rod.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional radial conduction in rod and sleeve, (2) Steady-state conditions, (3) Uniform volumetric generation in rod, (4) Negligible contact resistance between rod and sleeve.

ANALYSIS: (a) Construct a thermal circuit for the sleeve,



where

$$q' = \dot{E}'_{gen} = \dot{q}\pi\ D_1^2/4 = 24{,}000\ W/m^3 \times \pi \times (0.20\ m)^2/4 = 754.0\ W/m$$

$$R'_s = \frac{\ln(r_2/r_1)}{2\pi\ k_s} = \frac{\ln(400/200)}{2\pi \times 4\ W/m\cdot K} = 2.758\times10^{-2}\,m\cdot\ K/W$$

$$R_{conv} = \frac{1}{h\pi\ D_2} = \frac{1}{25\ W/m^2\cdot K\times\pi\times0.400\ m} = 3.183\times10^{-2}\,m\cdot\ K/W$$

The rate equation can be written as

$$q' = \frac{T_1 - T_\infty}{R'_s + R'_{conv}} = \frac{T_2 - T_\infty}{R'_{conv}}$$

$$T_1 = T_\infty + q'(R'_s + R'_{conv}) = 27^\circ C + 754\ W/m\left(2.758\times10^{-2} + 3.183\times10^{-2}\right)K/W\cdot m = 71.8^\circ C \quad <$$

$$T_2 = T_\infty + q'R'_{conv} = 27^\circ C + 754\ W/m \times 3.183\times10^{-2} m\cdot\ K/W = 51.0^\circ C. \quad <$$

(b) The temperature at the center of the rod is

$$T(0) = T_o = \frac{\dot{q}r_1^2}{4k_r} + T_1 = \frac{24{,}000\ W/m^3\,(0.100\ m)^2}{4\times0.5\ W/m\cdot K} + 71.8^\circ C = 192^\circ C. \quad <$$

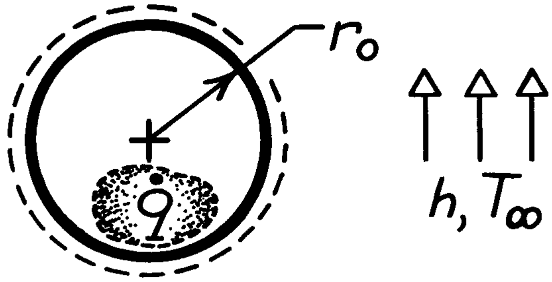
COMMENTS: The thermal resistances due to conduction in the sleeve and convection are comparable. Will increasing the sleeve outer diameter cause the surface temperature T_2 to increase or decrease?

PROBLEM 3.93

KNOWN: Radius, thermal conductivity, heat generation and convection conditions associated with a solid sphere.

FIND: Temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Uniform heat generation.

ANALYSIS: Integrating the appropriate form of the heat diffusion equation,

$$\frac{1}{r^2}\frac{d}{dr}\left[kr^2\frac{dT}{dr}\right]+\dot{q}=0 \qquad \text{or} \qquad \frac{d}{dr}\left[r^2\frac{dT}{dr}\right]=-\frac{\dot{q}r^2}{k}$$

$$r^2\frac{dT}{dr}=-\frac{\dot{q}r^3}{3k}+C_1 \qquad \frac{dT}{dr}=-\frac{\dot{q}r}{3k}+\frac{C_1}{r^2}$$

$$T(r)=-\frac{\dot{q}r^2}{6k}-\frac{C_1}{r}+C_2.$$

The boundary conditions are: $\left.\frac{dT}{dr}\right]_{r=0}=0$ hence $C_1=0,$ and

$$-k\left.\frac{dT}{dr}\right]_{r_o}=h\left[T(r_o)-T_\infty\right].$$

Substituting into the second boundary condition ($r = r_o$), find

$$\frac{\dot{q}r_o}{3}=h\left[-\frac{\dot{q}r_o^2}{6k}+C_2-T_\infty\right] \qquad C_2=\frac{\dot{q}r_o}{3h}+\frac{\dot{q}r_o^2}{6k}+T_\infty.$$

The temperature distribution has the form

$$T(r)=\frac{\dot{q}}{6k}\left(r_o^2-r^2\right)+\frac{\dot{q}r_o}{3h}+T_\infty. \qquad <$$

COMMENTS: To verify the above result, obtain $T(r_o) = T_s$,

$$T_s=\frac{\dot{q}r_o}{3h}+T_\infty$$

Applying energy balance to the control volume about the sphere,

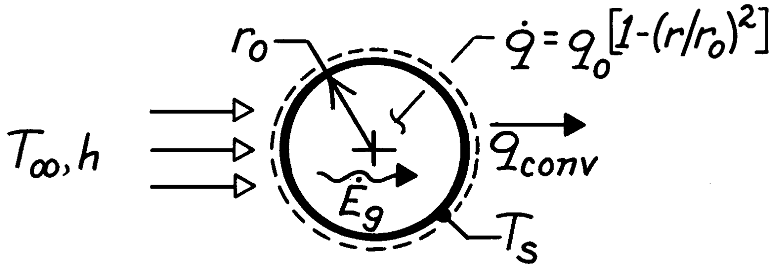
$$\dot{q}\left[\frac{4}{3}\pi\, r_o^3\right]=h4\pi\, r_o^2\left(T_s-T_\infty\right) \qquad \text{find} \qquad T_s=\frac{\dot{q}r_o}{3h}+T_\infty.$$

PROBLEM 3.94

KNOWN: Radial distribution of heat dissipation of a spherical container of radioactive wastes. Surface convection conditions.

FIND: Radial temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible temperature drop across container wall.

ANALYSIS: The appropriate form of the heat equation is

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = -\frac{\dot{q}}{k} = -\frac{\dot{q}_o}{k}\left[1-\left(\frac{r}{r_o}\right)^2\right].$$

Hence $$r^2\frac{dT}{dr} = -\frac{\dot{q}_o}{k}\left(\frac{r^3}{3}-\frac{r^5}{5r_o^2}\right)+C_1$$

$$T = -\frac{\dot{q}_o}{k}\left(\frac{r^2}{6}-\frac{r^4}{20r_o^2}\right)-\frac{C_1}{r}+C_2.$$

From the boundary conditions,

$$dT/dr\,|_{r=0} = 0 \quad \text{and} \quad -k\,dT/dr\,|_{r=r_o} = h\left[T(r_o)-T_\infty\right]$$

it follows that $C_1 = 0$ and

$$\dot{q}_o\left(\frac{r_o}{3}-\frac{r_o}{5}\right) = h\left[-\frac{\dot{q}_o}{k}\left(\frac{r_o^2}{6}-\frac{r_o^2}{20}\right)+C_2-T_\infty\right]$$

$$C_2 = \frac{2r_o\dot{q}_o}{15h}+\frac{7\dot{q}_o r_o^2}{60k}+T_\infty.$$

Hence $$T(r) = T_\infty+\frac{2r_o\dot{q}_o}{15h}+\frac{\dot{q}r_o^2}{k}\left[\frac{7}{60}-\frac{1}{6}\left(\frac{r}{r_o}\right)^2+\frac{1}{20}\left(\frac{r}{r_o}\right)^4\right].$$ <

COMMENTS: Applying the above result at r_o yields

$$T_s = T(r_o) = T_\infty + (2r_o\dot{q}_o/15h).$$

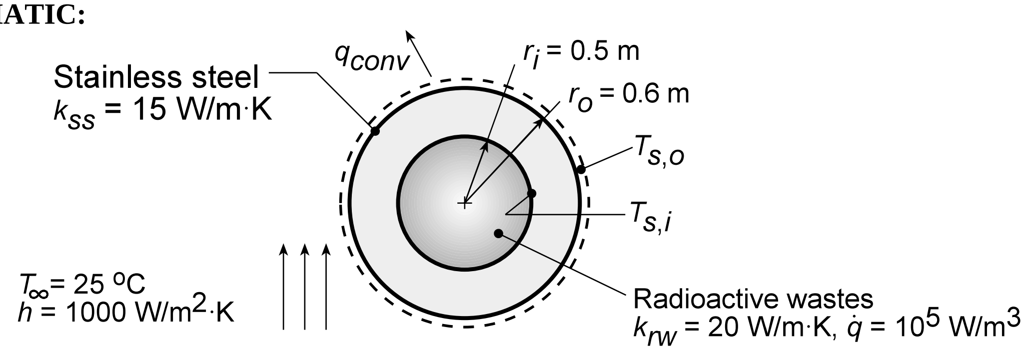
The same result may be obtained by applying an energy balance to a control surface about the container, where $\dot{E}_g = q_{conv}$. The maximum temperature exists at r = 0.

PROBLEM 3.95

KNOWN: Dimensions and thermal conductivity of a spherical container. Thermal conductivity and volumetric energy generation within the container. Outer convection conditions.

FIND: (a) Outer surface temperature, (b) Container inner surface temperature, (c) Temperature distribution within and center temperature of the wastes, (d) Feasibility of operating at twice the energy generation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional radial conduction.

ANALYSIS: (a) For a control volume which includes the container, conservation of energy yields $\dot{E}_g - \dot{E}_{out} = 0$, or $\dot{q}V - q_{conv} = 0$. Hence

$$\dot{q}(4/3)\left(\pi r_i^3\right) = h 4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

and with $\dot{q} = 10^5$ W/m³,

$$T_{s,o} = T_\infty + \frac{\dot{q} r_i^3}{3hr_o^2} = 25^\circ C + \frac{10^5\, W/m^2\,(0.5\,m)^3}{3000\, W/m^2\cdot K\,(0.6\,m)^2} = 36.6^\circ C. \quad <$$

(b) Performing a surface energy balance at the outer surface, $\dot{E}_{in} - \dot{E}_{out} = 0$ or $q_{cond} - q_{conv} = 0$. Hence

$$\frac{4\pi k_{ss}\left(T_{s,i} - T_{s,o}\right)}{(1/r_i) - (1/r_o)} = h 4\pi r_o^2\left(T_{s,o} - T_\infty\right)$$

$$T_{s,i} = T_{s,o} + \frac{h}{k_{ss}}\left(\frac{r_o}{r_i} - 1\right) r_o\left(T_{s,o} - T_\infty\right) = 36.6^\circ C + \frac{1000\, W/m^2\cdot K}{15\, W/m\cdot K}(0.2)0.6\,m\left(11.6^\circ C\right) = 129.4^\circ C. \quad <$$

(c) The heat equation in spherical coordinates is

$$k_{rw}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \dot{q}r^2 = 0.$$

Solving,

$$r^2\frac{dT}{dr} = -\frac{\dot{q}r^3}{3k_{rw}} + C_1 \qquad \text{and} \qquad T(r) = -\frac{\dot{q}r^2}{6k_{rw}} - \frac{C_1}{r} + C_2$$

Applying the boundary conditions,

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \qquad \text{and} \qquad T(r_i) = T_{s,i}$$

$$C_1 = 0 \qquad \text{and} \qquad C_2 = T_{s,i} + \dot{q}r_i^2/6k_{rw}.$$

Continued...

Hence

$$T(r) = T_{s,i} + \frac{\dot{q}}{6k_{rw}}\left(r_i^2 - r^2\right) \quad <$$

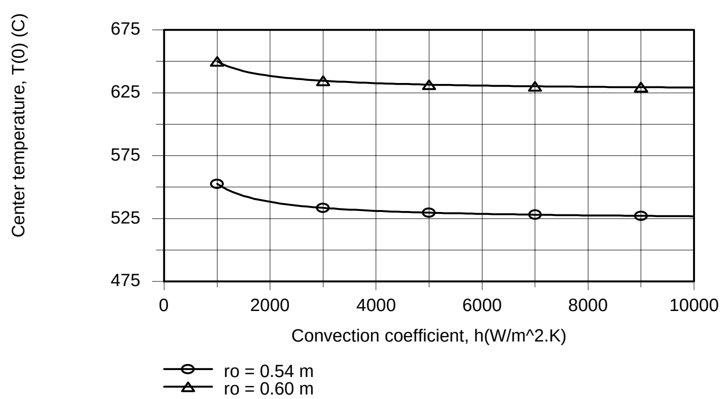
At r = 0,

$$T(0) = T_{s,i} + \frac{\dot{q}r_i^2}{6k_{rw}} = 129.4^\circ C + \frac{10^5\ W/m^3\,(0.5\,m)^2}{6(20\,W/m\cdot K)} = 337.7^\circ C \quad <$$

(d) The feasibility assessment may be performed by using the IHT model for one-dimensional, steady-state conduction in a solid sphere, with the surface boundary condition prescribed in terms of the total thermal resistance

$$R''_{tot,i} = \left(4\pi r_i^2\right)R_{tot} = R''_{cnd,i} + R''_{cnv,i} = \frac{r_i^2\left[(1/r_i) - (1/r_o)\right]}{k_{ss}} + \frac{1}{h}\left(\frac{r_i}{r_o}\right)^2$$

where, for r_o = 0.6 m and h = 1000 W/m^2·K, $R''_{cnd,i}$ = 5.56 × 10^{-3} m^2·K/W, $R''_{cnv,i}$ = 6.94 × 10^{-4} m^2·K/W, and $R''_{tot,i}$ = 6.25 × 10^{-3} m^2·K/W. Results for the center temperature are shown below.



Clearly, even with r_o = 0.54 m = $r_{o,min}$ and h = 10,000 W/m^2·K (a practical upper limit), T(0) > 475°C and the desired condition can not be met. The corresponding resistances are $R''_{cnd,i}$ = 2.47 × 10^{-3} m^2·K/W, $R''_{cnv,i}$ = 8.57 × 10^{-5} m^2·K/W, and $R''_{tot,i}$ = 2.56 × 10^{-3} m^2·K/W. The conduction resistance remains dominant, and the effect of reducing $R''_{cnv,i}$ by increasing h is small. *The proposed extension is not feasible.*

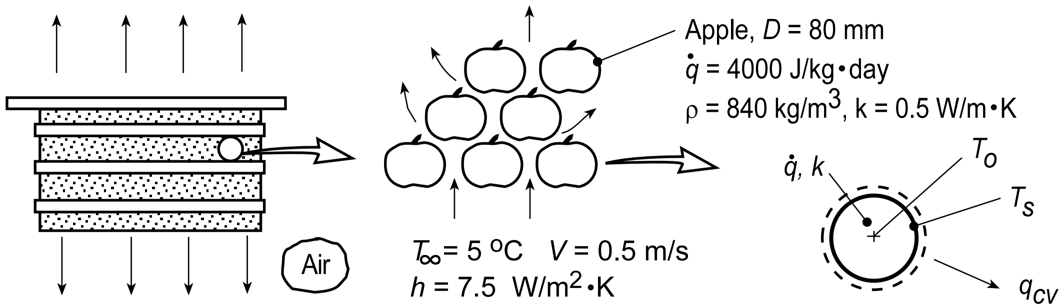
COMMENTS: A value of $\dot{q}$ = 1.79 × 10^5 W/m^3 would allow for operation at T(0) = 475°C with r_o = 0.54 m and h = 10,000 W/m^2·K.

PROBLEM 3.96

KNOWN: Carton of apples, modeled as 80-mm diameter spheres, ventilated with air at 5°C and experiencing internal volumetric heat generation at a rate of 4000 J/kg·day.

FIND: (a) The apple center and surface temperatures when the convection coefficient is 7.5 $W/m^2 \cdot K$, and (b) Compute and plot the apple temperatures as a function of air velocity, V, for the range $0.1 \le V \le 1$ m/s, when the convection coefficient has the form $h = C_1 V^{0.425}$, where $C_1 = 10.1\ W/m^2 \cdot K \cdot (m/s)^{0.425}$.

SCHEMATIC:



ASSUMPTIONS: (1) Apples can be modeled as spheres, (2) Each apple experiences flow of ventilation air at $T_\infty = 5°C$, (3) One-dimensional radial conduction, (4) Constant properties and (5) Uniform heat generation.

ANALYSIS: (a) From Eq. C.24, the temperature distribution in a solid sphere (apple) with uniform generation is

$$T(r) = \frac{\dot{q} r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s \qquad (1)$$

To determine T_s, perform an energy balance on the apple as shown in the sketch above, with volume $V = 4/3 \pi r_o^3$,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad\qquad -q_{cv} + \dot{q}\forall = 0$$

$$-h\left(4\pi r_o^2\right)\left(T_s - T_\infty\right) + \dot{q}\left(4/3\pi r_o^3\right) = 0 \qquad (2)$$

$$-7.5\,W/m^2 \cdot K\left(4\pi \times 0.040^2 m^2\right)\left(T_s - 5^\circ C\right) + 38.9\,W/m^3\left(4/3\pi \times 0.040^3 m^3\right) = 0$$

where the volumetric generation rate is

$$\dot{q} = 4000\,J/kg \cdot day$$

$$\dot{q} = 4000\,J/kg \cdot day \times 840\,kg/m^3 \times \left(1\,day/24\,hr\right) \times \left(1\,hr/3600\,s\right)$$

$$\dot{q} = 38.9\,W/m^3$$

and solving for T_s, find

$$T_s = 5.14^\circ C$$ <

From Eq. (1), at r = 0, with T_s, find

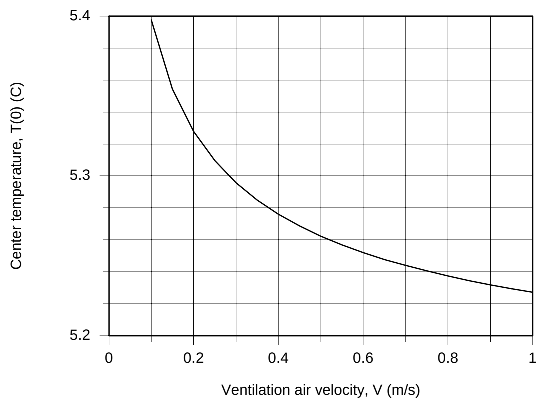
$$T(0) = \frac{38.9\,W/m^3 \times 0.040^2 m^2}{6 \times 0.5\,W/m \cdot K} + 5.14^\circ C = 0.12^\circ C + 5.14^\circ C = 5.26^\circ C$$ <

Continued...

(b) With the convection coefficient depending upon velocity,

$$h = C_1 V^{0.425}$$

with C_1 = 10.1 W/m^2·K·(m/s)$^{0.425}$, and using the energy balance of Eq. (2), calculate and plot T_s as a function of ventilation air velocity V. With very low velocities, the center temperature is nearly 0.5°C higher than the air. From our earlier calculation we know that T(0) - T_s = 0.12°C and is independent of V.



COMMENTS: (1) While the temperature within the apple is nearly isothermal, the center temperature will track the ventilation air temperature which will increase as it passes through stacks of cartons.

(2) The IHT Workspace used to determine T_s for the base condition and generate the above plot is shown below.

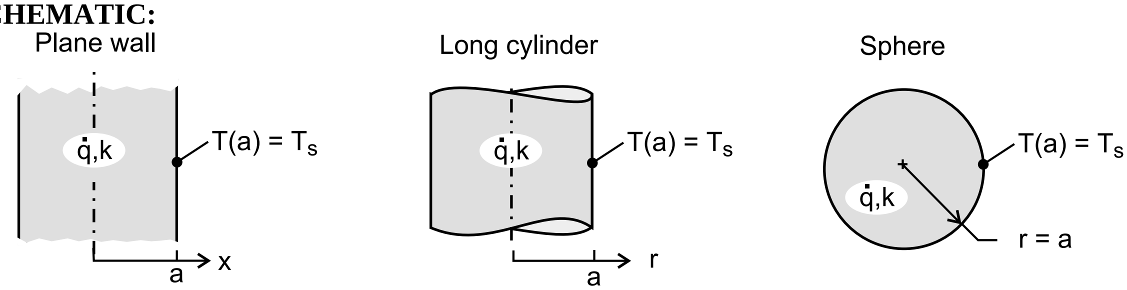
```
// The temperature distribution, Eq (1),
T_r = qdot * ro^2 / (4 * k) * ( 1- r^2/ro^2 )  +  Ts

// Energy balance on the apple, Eq (2)
- qcv + qdot * Vol = 0
Vol = 4 / 3 * pi *  ro ^3

// Convection rate equation:
qcv = h* As * ( Ts - Tinf )
As = 4 * pi * ro^2

// Generation rate:
qdot = qdotm * (1/24) * (1/3600) * rho          // Generation rate, W/m^3; Conversions: days/h and h/sec

// Assigned variables:
ro = 0.080                  // Radius of apple, m
k = 0.5                     // Thermal conductivity, W/m.K
qdotm = 4000                // Generation rate, J/kg.K
rho = 840                   // Specific heat, J/kg.K
r = 0                       // Center, m; location for T(0)
h =  7.5                    // Convection coefficient, W/m^2.K; base case, V = 0.5 m/s
//h = C1 * V^0.425          // Correlation
//C1 = 10.1
//V = 0.5                   // Air velocity, m/s; range 0.1 to 1 m/s
Tinf = 5                    // Air temperature, C
```

PROBLEM 3.97

KNOWN: Plane wall, long cylinder and sphere, each with characteristic length a, thermal conductivity k and uniform volumetric energy generation rate $\dot{q}$.

FIND: (a) On the same graph, plot the dimensionless temperature, $[T(x \text{ or } r) - T(a)]/[\dot{q}a^2/2k]$, *vs.* the dimensionless characteristic length, x/a or r/a, for each shape; (b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area; and (c) Which shape would be preferred for use as a nuclear fuel element? Explain why?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties and (4) Uniform volumetric generation.

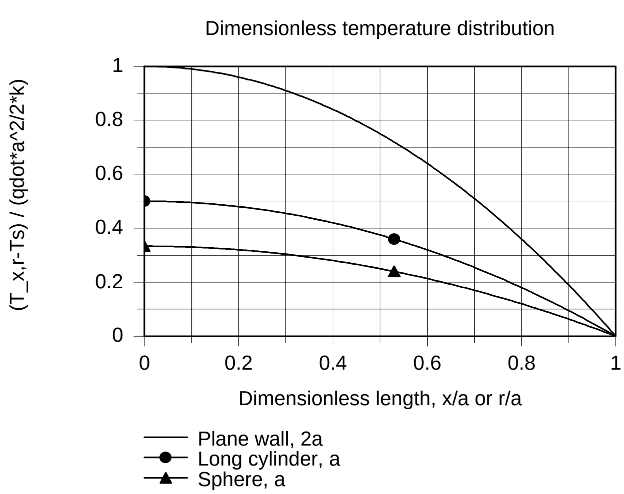
ANALYSIS: (a) For each of the shapes, with $T(a) = T_s$, the dimensionless temperature distributions can be written by inspection from results in Appendix C.3.

Plane wall, Eq. C.22
$$\frac{T(x)-T_s}{\dot{q}a^2/2k} = 1-\left(\frac{x}{a}\right)^2$$

Long cylinder, Eq. C.23
$$\frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{2}\left[1-\left(\frac{r}{a}\right)^2\right]$$

Sphere, Eq. C.24
$$\frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{3}\left[1-\left(\frac{r}{a}\right)^2\right]$$

The dimensionless temperature distributions using the foregoing expressions are shown in the graph below.



Continued …..

(b) The sphere shape has the smallest temperature difference between the center and surface, T(0) – T(a). The ratio of volume-to-surface-area, $\forall/A_s$, for each of the shapes is

Plane wall $$\frac{\forall}{A_s} = \frac{a(1\times1)}{(1\times1)} = a$$

Long cylinder $$\frac{\forall}{A_s} = \frac{\pi a^2 \times 1}{2\pi a \times 1} = \frac{a}{2}$$

Sphere $$\frac{\forall}{A_s} = \frac{4\pi a^3/3}{4\pi a^2} = \frac{a}{3}$$

The smaller the $\forall/A_s$ ratio, the smaller the temperature difference, T(0) – T(a).

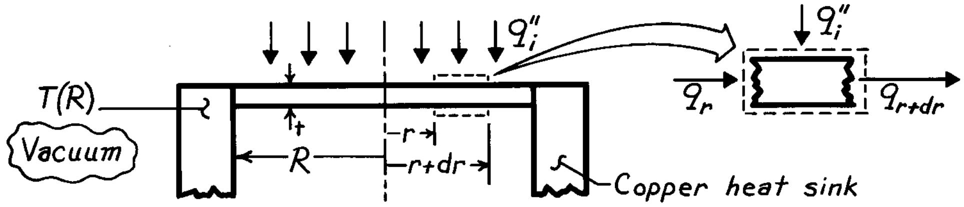
(c) The sphere would be the preferred element shape since, for a given $\forall/A_s$ ratio, which controls the generation and transfer rates, the sphere will operate at the lowest temperature.

PROBLEM 3.98

KNOWN: Radius, thickness, and incident flux for a radiation heat gauge.

FIND: Expression relating incident flux to temperature difference between center and edge of gauge.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ rdr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ rdr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i''r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i''r^2}{4kt} + C_1\ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i''R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i''R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T. \qquad <$$

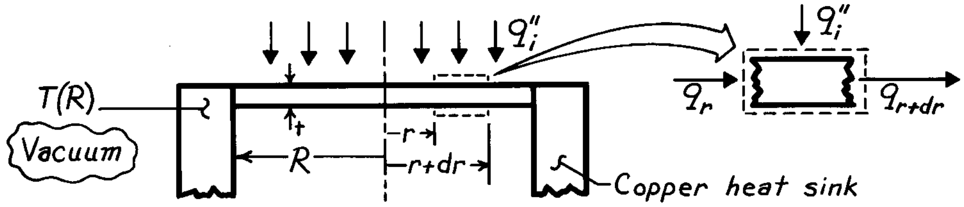
COMMENTS: This technique allows for determination of a radiation flux from measurement of a temperature difference. It becomes inaccurate if emission from the foil becomes significant.

PROBLEM 3.98

KNOWN: Radius, thickness, and incident flux for a radiation heat gauge.

FIND: Expression relating incident flux to temperature difference between center and edge of gauge.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in r (negligible temperature drop across foil thickness), (3) Constant properties, (4) Uniform incident flux, (5) Negligible heat loss from foil due to radiation exchange with enclosure wall, (6) Negligible contact resistance between foil and heat sink.

ANALYSIS: Applying energy conservation to a circular ring extending from r to r + dr,

$$q_r + q_i''(2\pi\ rdr) = q_{r+dr}, \qquad q_r = -k(2\pi\ rt)\frac{dT}{dr}, \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr.$$

Rearranging, find that

$$q_i''(2\pi\ rdr) = \frac{d}{dr}\left[(-k2\pi\ rt)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_i''}{kt}r.$$

Integrating,

$$r\frac{dT}{dr} = -\frac{q_i'' r^2}{2kt} + C_1 \qquad \text{and} \qquad T(r) = -\frac{q_i'' r^2}{4kt} + C_1 \ln r + C_2.$$

With $dT/dr|_{r=0} = 0$, $C_1 = 0$ and with $T(r = R) = T(R)$,

$$T(R) = -\frac{q_i'' R^2}{4kt} + C_2 \qquad \text{or} \qquad C_2 = T(R) + \frac{q_i'' R^2}{4kt}.$$

Hence, the temperature distribution is

$$T(r) = \frac{q_i''}{4kt}\left(R^2 - r^2\right) + T(R).$$

Applying this result at r = 0, it follows that

$$q_i'' = \frac{4kt}{R^2}\left[T(0) - T(R)\right] = \frac{4kt}{R^2}\Delta T.$$ <

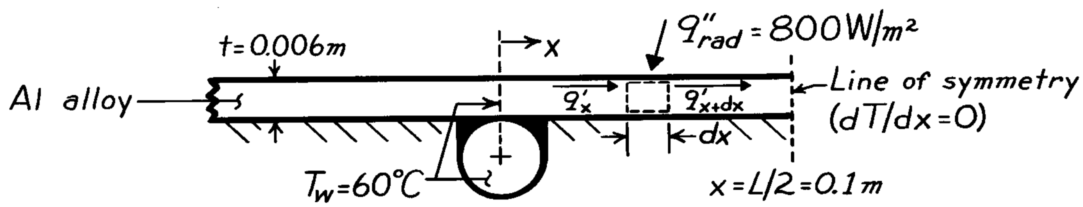
COMMENTS: This technique allows for determination of a radiation flux from measurement of a temperature difference. It becomes inaccurate if emission from the foil becomes significant.

PROBLEM 3.99

KNOWN: Net radiative flux to absorber plate.

FIND: (a) Maximum absorber plate temperature, (b) Rate of energy collected per tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional (x) conduction along absorber plate, (3) Uniform radiation absorption at plate surface, (4) Negligible losses by conduction through insulation, (5) Negligible losses by convection at absorber plate surface, (6) Temperature of absorber plate at x = 0 is approximately that of the water.

PROPERTIES: *Table A-1,* Aluminum alloy (2024-T6): k ≈ 180 W/m·K.

ANALYSIS: The absorber plate acts as an extended surface (a conduction-radiation system), and a differential equation which governs its temperature distribution may be obtained by applying Eq.1.11a to a differential control volume. For a unit length of tube

$$q'_x + q''_{rad}(dx) - q'_{x+dx} = 0.$$

With $$q'_{x+dx} = q'_x + \frac{dq'_x}{dx}dx$$

and $$q'_x = -kt\frac{dT}{dx}$$

it follows that,

$$q''_{rad} - \frac{d}{dx}\left[-kt\frac{dT}{dx}\right] = 0$$

$$\frac{d^2T}{dx^2} + \frac{q''_{rad}}{kt} = 0$$

Integrating twice it follows that, the general solution for the temperature distribution has the form,

$$T(x) = -\frac{q''_{rad}}{2kt}x^2 + C_1x + C_2.$$

Continued …..

The boundary conditions are:

$$T(0) = T_w \qquad C_2 = T_w$$

$$\left.\frac{dT}{dx}\right]_{x=L/2} = 0 \qquad C_1 = \frac{q''_{rad} L}{2kt}$$

Hence,

$$T(x) = \frac{q''_{rad}}{2kt}\, x(L - x) + T_w.$$

The maximum absorber plate temperature, which is at x = L/2, is therefore

$$T_{max} = T(L/2) = \frac{q''_{rad} L^2}{8kt} + T_w.$$

The rate of energy collection per tube may be obtained by applying Fourier's law at x = 0. That is, energy is transferred to the tubes via conduction through the absorber plate. Hence,

$$q' = 2\left[-k\,t\left.\frac{dT}{dx}\right]_{x=0}\right]$$

where the factor of two arises due to heat transfer from both sides of the tube. Hence,

$$q' = -Lq''_{rad}.$$

Hence $$T_{max} = \frac{800\frac{W}{m^2}(0.2m)^2}{8\left[180\frac{W}{m\cdot K}\right](0.006m)} + 60^\circ C$$

or $$T_{max} = 63.7^\circ C$$ <

and $$q' = -0.2m \times 800\ W/m^2$$

or $$q' = -160\ W/m.$$ <

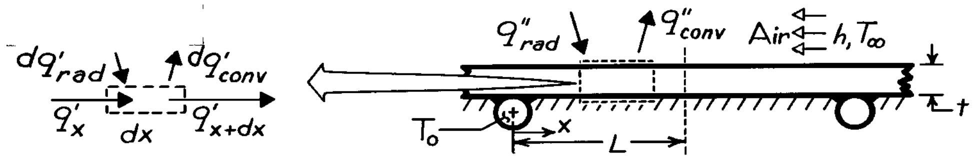
COMMENTS: Convection losses in the typical flat plate collector, which is not evacuated, would reduce the value of q'.

PROBLEM 3.100

KNOWN: Surface conditions and thickness of a solar collector absorber plate. Temperature of working fluid.

FIND: (a) Differential equation which governs plate temperature distribution, (b) Form of the temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Adiabatic bottom surface, (4) Uniform radiation flux and convection coefficient at top, (5) Temperature of absorber plate at x = 0 corresponds to that of working fluid.

ANALYSIS: (a) Performing an energy balance on the differential control volume,

$$q'_x + dq'_{rad} = q'_{x+dx} + dq'_{conv}$$

where
$$q'_{x+dx} = q'_x + \left(dq'_x / dx\right)dx$$
$$dq'_{rad} = q''_{rad} \cdot dx$$
$$dq'_{conv} = h\left(T - T_\infty\right) \cdot dx$$

Hence, $$q''_{rad}dx = \left(dq'_x / dx\right)dx + h\left(T - T_\infty\right)dx.$$

From Fourier's law, the conduction heat rate per unit width is

$$q'_x = -k\,t\,dT/dx \qquad \frac{d^2T}{dx^2} - \frac{h}{kT}\left(T - T_\infty\right) + \frac{q''_{rad}}{kt} = 0. \qquad <$$

(b) Defining $\theta = T - T_\infty, d^2T/dx^2 = d^2\theta / dx^2$ and the differential equation becomes,

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q''_{rad}}{kt} = 0.$$

It is a second-order, differential equation with constant coefficients and a source term, and its general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2$$

where $\lambda = \left(h/kt\right)^{1/2}$, $\quad S = q''_{rad} / kt.$

Appropriate boundary conditions are:

$$\theta\left(0\right) = T_o - T_\infty \equiv \theta_0, \qquad d\theta/dx)_{x=L} = 0.$$

Hence, $$\theta_o = C_1 + C_2 + S/\lambda^2$$

$$d\theta/dx)_{x=L} = C_1\ \lambda e^{+\lambda L} - C_2\ \lambda e^{-\lambda L} = 0 \qquad C_2 = C_1\ e^{2\lambda L}$$

Hence, $$C_1 = \left(\theta_0 - S/\lambda^2\right) / \left(1 + e^{2\lambda L}\right) \qquad C_2 = \left(\theta_0 - S/\lambda^2\right) / \left(1 + e^{-2\lambda L}\right)$$

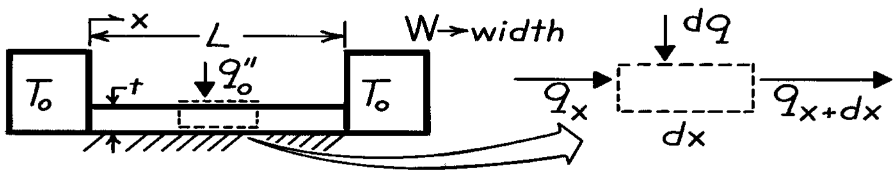
$$\theta = \left(\theta_0 - S/\lambda^2\right) \left[\frac{e^{\lambda x}}{1+e^{2\lambda L}} + \frac{e^{-\lambda x}}{1+e^{-2\lambda L}}\right] + S/\lambda^2. \qquad <$$

PROBLEM 3.101

KNOWN: Dimensions of a plate insulated on its bottom and thermally joined to heat sinks at its ends. Net heat flux at top surface.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and heat loss to heat sinks.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x (W,L>>t), (3) Constant properties, (4) Uniform surface heat flux, (5) Adiabatic bottom, (6) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume, $q_x + dq = q_{x+dx}$, where $q_{x+dx} = q_x + (dq_x/dx)\,dx$ and $dq = q_o''(W \cdot dx)$. Hence, $(dq_x/dx) - q_o'' W = 0$. From Fourier's law, $q_x = -k(t \cdot W)\,dT/dx$. Hence, the differential equation for the temperature distribution is

$$-\frac{d}{dx}\left[ktW\frac{dT}{dx}\right] - q_o'' W = 0 \qquad \frac{d^2T}{dx^2} + \frac{q_o''}{kt} = 0.$$ <

(b) Integrating twice, the general solution is,

$$T(x) = -\frac{q_o''}{2kt}x^2 + C_1 x + C_2$$

and appropriate boundary conditions are $T(0) = T_o$, and $T(L) = T_o$. Hence, $T_o = C_2$, and

$$T_o = -\frac{q_o''}{2kt}L^2 + C_1 L + C_2 \qquad \text{and} \qquad C_1 = \frac{q_o'' L}{2kt}.$$

Hence, the temperature distribution is

$$T(x) = -\frac{q_o'' L}{2kt}\left(x^2 - Lx\right) + T_o.$$ <

Applying Fourier's law at x = 0, and at x = L,

$$q(0) = -k(Wt)\,dT/dx)_{x=0} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=0} = -\frac{q_o'' WL}{2}$$

$$q(L) = -k(Wt)dT/dx)_{x=L} = -kWt\left[-\frac{q_o''}{kt}\right]\left[x - \frac{L}{2}\right]\Bigg|_{x=L} = +\frac{q_o'' WL}{2}$$

Hence the heat loss from the plates is $q = 2(q_o'' WL/2) = q_o'' WL$. <

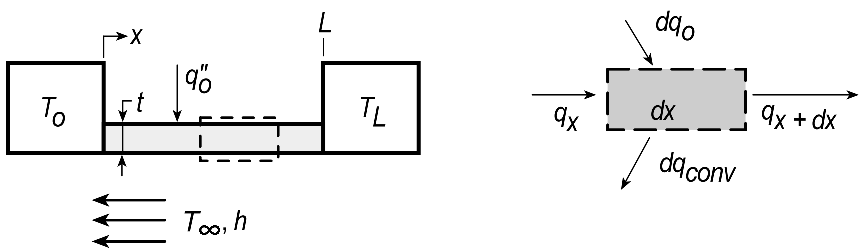
COMMENTS: (1) Note signs associated with q(0) and q(L). (2) Note symmetry about x = L/2. Alternative boundary conditions are $T(0) = T_o$ and $dT/dx)_{x=L/2} = 0$.

PROBLEM 3.102

KNOWN: Dimensions and surface conditions of a plate thermally joined at its ends to heat sinks at different temperatures.

FIND: (a) Differential equation which determines temperature distribution in plate, (b) Temperature distribution and an expression for the heat rate from the plate to the sinks, and (c) Compute and plot temperature distribution and heat rates corresponding to changes in different parameters.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x (W,L >> t), (3) Constant properties, (4) Uniform surface heat flux and convection coefficient, (5) Negligible contact resistance.

ANALYSIS: (a) Applying conservation of energy to the differential control volume

$$q_x + dq_o = q_{x+dx} + dq_{conv}$$

where

$$q_{x+dx} = q_x + (dq_x/dx)dx \qquad dq_{conv} = h(T - T_\infty)(W \cdot dx)$$

Hence,

$$q_x + q''_o(W \cdot dx) = q_x + (dq_x/dx)dx + h(T - T_\infty)(W \cdot dx) \qquad \frac{dq_x}{dx} + hW(T - T_\infty) = q''_o W.$$

Using Fourier's law, $q_x = -k(t \cdot W)dT/dx$,

$$-ktW\frac{d^2T}{dx^2} + hW(T - T_\infty) = q''_o \qquad \frac{d^2T}{dx^2} - \frac{h}{kt}(T - T_\infty) + \frac{q''_o}{kt} = 0.$$ <

(b) Introducing $\theta \equiv T - T_\infty$, the differential equation becomes

$$\frac{d^2\theta}{dx^2} - \frac{h}{kt}\theta + \frac{q''_o}{kt} = 0.$$

This differential equation is of second order with constant coefficients and a source term. With $\lambda^2 \equiv h/kt$ and $S \equiv q''_o/kt$, it follows that the general solution is of the form

$$\theta = C_1 e^{+\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2. \qquad (1)$$

Appropriate boundary conditions are: $\theta(0) = T_o - T_\infty \equiv \theta_o \qquad \theta(L) = T_L - T_\infty \equiv \theta_L$ (2,3)

Substituting the boundary conditions, Eqs. (2,3) into the general solution, Eq. (1),

$$\theta_o = C_1 e^0 + C_2 e^0 + S/\lambda^2 \qquad \theta_L = C_1 e^{+\lambda L} + C_2 e^{-\lambda L} + S/\lambda^2 \qquad (4,5)$$

To solve for C_2, multiply Eq. (4) by $-e^{+\lambda L}$ and add the result to Eq. (5),

$$-\theta_o e^{+\lambda L} + \theta_L = C_2\left(-e^{+\lambda L} + e^{-\lambda L}\right) + S/\lambda^2\left(-e^{+\lambda L} + 1\right)$$

$$C_2 = \left[\left(\theta_L - \theta_o e^{+\lambda L}\right) - S/\lambda^2\left(-e^{+\lambda L} + 1\right)\right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L}\right) \qquad (6)$$

Continued...

PROBLEM 3.102 (Cont.)

Substituting for C_2 from Eq. (6) into Eq. (4), find

$$C_1 = \theta_o - \left\{ \left[\left(\theta_L - \theta_o e^{+\lambda L} \right) - S/\lambda^2 \left(-e^{+\lambda L} + 1 \right) \right] \Big/ \left(-e^{+\lambda L} + e^{-\lambda L} \right) \right\} - S/\lambda^2 \qquad (7)$$

Using C_1 and C_2 from Eqs. (6,7) and Eq. (1), the temperature distribution can be expressed as

$$\theta(x) = \left[e^{+\lambda x} - \frac{\sinh(\lambda x)}{\sinh(\lambda L)} e^{+\lambda L} \right] \theta_o + \frac{\sinh(\lambda x)}{\sinh(\lambda L)} \theta_L + \left[-\left(1 - e^{+\lambda L}\right) \frac{\sinh(\lambda x)}{\sinh(\lambda L)} + \left(1 - e^{+\lambda L}\right) \right] \frac{S}{\lambda^2} \qquad (8)$$

<

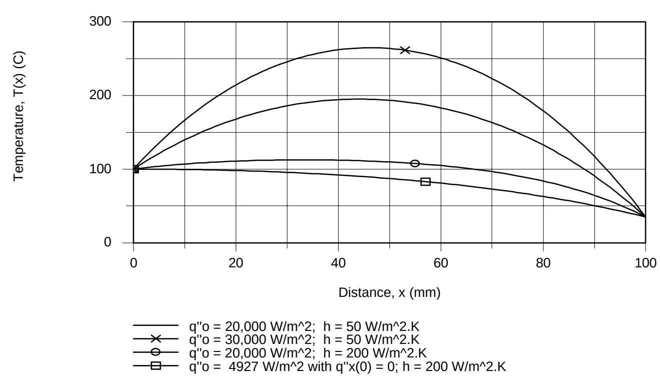
The heat rate from the plate is $q_p = -q_x(0) + q_x(L)$ and using Fourier's law, the conduction heat rates, with $A_c = W \cdot t$, are

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=0} = -kA_c \left\{ \left[\lambda e^0 - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \right] \theta_o + \frac{\lambda}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda - \lambda \right] \frac{S}{\lambda^2} \right\}$$ <

$$q_x(L) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=L} = -kA_c \left\{ \left[\lambda e^{\lambda L} - \frac{e^{\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) \right] \theta_o + \frac{\lambda \cosh(\lambda L)}{\sinh(\lambda L)} \theta_L + \left[-\frac{1 - e^{+\lambda L}}{\sinh(\lambda L)} \lambda \cosh(\lambda L) - \lambda e^{+\lambda L} \right] \frac{S}{\lambda^2} \right\}$$ <

(c) For the prescribed base-case conditions listed below, the temperature distribution (solid line) is shown in the accompanying plot. As expected, the maximum temperature does not occur at the midpoint, but slightly toward the x-origin. The sink heat rates are

$q_x''(0) = -17.22\ \text{W}$ $\qquad$ $q_x''(L) = 23.62\ \text{W}$ <



The additional temperature distributions on the plot correspond to changes in the following parameters, with all the remaining parameters unchanged: (i) q_o'' = 30,000 W/m^2, (ii) h = 200 W/m$^2\cdot$K, (iii) the value of q_o'' for which $q_x''(0) = 0$ with h = 200 W/m$^2\cdot$K. The condition for the last curve is q_o'' = 4927 W/m^2 for which the temperature gradient at x = 0 is zero.

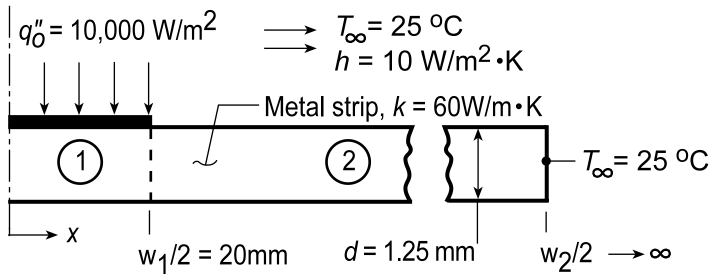
Base case conditions are: q_o'' = 20,000 W/m^2, T_o = 100°C, T_L = 35°C, T_∞ = 25°C, k = 25 W/m·K, h = 50 W/m$^2\cdot$K, L = 100 mm, t = 5 mm, W = 30 mm.

PROBLEM 3.103

KNOWN: Thin plastic film being bonded to a metal strip by laser heating method; strip dimensions and thermophysical properties are prescribed as are laser heating flux and convection conditions.

FIND: (a) Expression for temperature distribution for the region with the plastic strip, $-w_1/2 \le x \le w_1/2$, (b) Temperature at the center (x = 0) and the edge of the plastic strip ($x = \pm w_1/2$) when the laser flux is 10,000 W/m^2; (c) Plot the temperature distribution for the strip and point out special features.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in x-direction only, (3) Plastic film has negligible thermal resistance, (4) Upper and lower surfaces have uniform convection coefficients, (5) Edges of metal strip are at air temperature (T_∞), that is, strip behaves as infinite fin so that $w_2 \to \infty$, (6) All the incident laser heating flux q_o'' is absorbed by the film.

PROPERTIES: Metal strip (given): ρ = 7850 kg/m^3, c_p = 435 J/kg·m^3, k = 60 W/m·K.

ANALYSIS: (a) The strip-plastic film arrangement can be modeled as an infinite fin of uniform cross section a portion of which is exposed to the laser heat flux on the upper surface. The general solutions for the two regions of the strip, in terms of $\theta \equiv T(x) - T_\infty$, are

$$0 \le x \le w_1/2 \qquad \theta_1(x) = C_1 e^{+mx} + C_2 e^{-mx} + M/m^2 \tag{1}$$

$$M = q_o'' P/2kA_c = q_o''/kd \qquad m = (2h/kd)^{1/2} \tag{2,3}$$

$$w_1/2 \le x \le \infty \qquad \theta_2(x) = C_3 e^{+mx} + C_4 e^{-mx}. \tag{4}$$

Four boundary conditions can be identified to evaluate the constants:

At x = 0: $$\frac{d\theta_1}{dx}(0) = 0 = C_1 m e^0 - C_2 m e^{-0} + 0 \quad \rightarrow \quad C_1 = C_2 \tag{5}$$

At x = w$_1$/2: $$\theta(w_1/2) = \theta_2(w_1/2)$$

$$C_1 e^{+mw_1/2} + C_2 e^{-mw_1/2} + M/m^2 = C_3 e^{+mw_1/2} + C_4 e^{-mw_1/2} \tag{6}$$

At x = w$_1$/2: $$d\theta_1(w_1/2)/dx = d\theta_2(w_1/2)/dx$$

$$mC_1 e^{+mw_1/2} - mC_2 e^{-mw_1/2} + 0 = mC_3 e^{+mw_1/2} - mC_4 e^{-mw_1/2} \tag{7}$$

At x → ∞: $$\theta_2(\infty) = 0 = C_3 e^{\infty} + C_4 e^{-\infty} \quad \rightarrow \quad C_3 = 0 \tag{8}$$

With $C_3 = 0$ and $C_1 = C_2$, combine Eqs. (6 and 7) to eliminate C_4 to find

$$C_1 = C_2 = -\frac{M/m^2}{2e^{mw_1/2}}. \tag{9}$$

and using Eq. (6) with Eq. (9) find

$$C_4 = M/m^2 \sinh(mw_1/2) e^{-mx_1/2} \tag{10}$$

Continued...

PROBLEM 3.103 (Cont.)

Hence, the temperature distribution in the region (1) under the plastic film, $0 \le x \le w_1/2$, is

$$\theta_1(x) = -\frac{M/m^2}{2e^{mw_1/w}}\left(e^{+mx} + e^{-mx}\right) + \frac{M}{m^2} = \frac{M}{m^2}\left(1 - e^{-mw_1/2}\cosh mx\right) \qquad (11) \quad <$$

and for the region (2), $x \ge w_1/2$,

$$\theta_2(x) = \frac{M}{m^2}\sinh\left(mw_1/2\right)e^{-mx} \qquad (12)$$

(b) Substituting numerical values into the temperature distribution expression above, $\theta_1(0)$ and $\theta_1(w_1/2)$ can be determined. First evaluate the following parameters:

$$M = 10,000\ \text{W/m}^2 / 60\ \text{W/m}\cdot\text{K} \times 0.00125\ \text{m} = 133,333\ \text{K/m}^2$$

$$m = \left(2 \times 10\ \text{W/m}^2\cdot\text{K} / 60\ \text{W/m}\cdot\text{K} \times 0.00125\ \text{m}\right)^{1/2} = 16.33\ \text{m}^{-1}$$

Hence, for the midpoint x = 0,

$$\theta_1(0) = \frac{133,333\ \text{K/m}^2}{\left(16.33\ \text{m}^{-1}\right)^2}\left[1 - \exp\left(-16.33\ \text{m}^{-1} \times 0.020\ \text{m}\right) \times \cosh(0)\right] = 139.3\ \text{K}$$

$$T_1(0) = \theta_1(0) + T_\infty = 139.3\ \text{K} + 25^\circ\text{C} = 164.3^\circ\text{C}. \quad <$$

For the position $x = w_1/2 = 0.020$ m,

$$\theta_1(w_1/2) = 500.0\left[1 - 0.721\cosh\left(16.33\ \text{m}^{-1} \times 0.020\ \text{m}\right)\right] = 120.1\ \text{K}$$

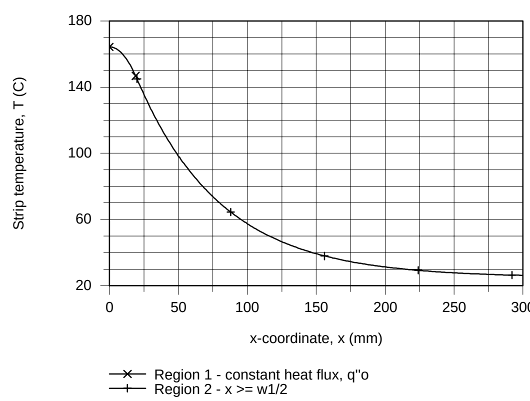
$$T_1(w_1/2) = 120.1\ \text{K} + 25^\circ\text{C} = 145.1^\circ\text{C}. \quad <$$

(c) The temperature distributions, $\theta_1(x)$ and $\theta_2(x)$, are shown in the plot below. Using IHT, Eqs. (11) and (12) were entered into the workspace and a graph created. The special features are noted:

(1) No gradient at midpoint, x = 0; symmetrical distribution.

(2) No discontinuity of gradient at $w_1/2$ (20 mm).

(3) Temperature excess and gradient approach zero with increasing value of x.



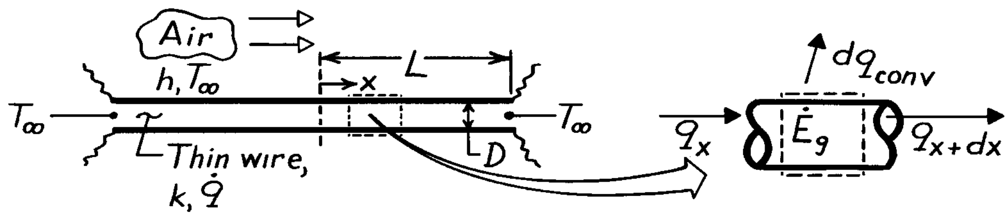
COMMENTS: How wide must the strip be in order to satisfy the infinite fin approximation such that $\theta_2(x \to \infty) = 0$? For x = 200 mm, find θ_2(200 mm) = 6.3°C; this would be a poor approximation. When x = 300 mm, θ_2(300 mm) = 1.2°C; hence when $w_2/2$ = 300 mm, the strip is a reasonable approximation to an infinite fin.

PROBLEM 3.104

KNOWN: Thermal conductivity, diameter and length of a wire which is annealed by passing an electrical current through the wire.

FIND: Steady-state temperature distribution along wire.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the wire, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

ANALYSIS: Applying conservation of energy to a differential control volume,

$$q_x + \dot{E}_g - dq_{conv} - q_{x+dx} = 0$$

$$q_{x+dx} = q_x + \frac{dq_x}{dx}dx \qquad q_x = -k\left(\pi\ D^2/4\right)dT/dx$$

$$dq_{conv} = h(\pi\ D\ dx)\ (T - T_\infty) \qquad \dot{E}_g = \dot{q}\left(\pi\ D^2/4\right)dx.$$

Hence,

$$k\left(\pi\ D^2/4\right)\ \frac{d^2T}{dx^2}dx + \dot{q}\left(\pi\ D^2/4\right)dx - h(\pi\ Ddx)\ (T - T_\infty) = 0$$

or, with $\theta \equiv T - T_\infty$,

$$\frac{d^2\theta}{dx^2} - \frac{4h}{kD}\theta + \frac{\dot{q}}{k} = 0$$

The solution (general and particular) to this nonhomogeneous equation is of the form

$$\theta = C_1\ e^{mx} + C_2\ e^{-mx} + \frac{\dot{q}}{km^2}$$

where $m^2 = (4h/kD)$. The boundary conditions are:

$$\left.\frac{d\theta}{dx}\right]_{x=0} = 0 = m\ C_1\ e^0 - mC_2\ e^0 \rightarrow \qquad C_1 = C_2$$

$$\theta(L) = 0 = C_1\left(e^{mL} + e^{-mL}\right) + \frac{\dot{q}}{km^2} \rightarrow \qquad C_1 = \frac{-\dot{q}/km^2}{e^{mL} + e^{-mL}} = C_2$$

The temperature distribution has the form

$$T = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{e^{mx} + e^{-mx}}{e^{mL} + e^{-mL}} - 1\right] = T_\infty - \frac{\dot{q}}{km^2}\left[\frac{\cosh mx}{\cosh mL} - 1\right].$$ <

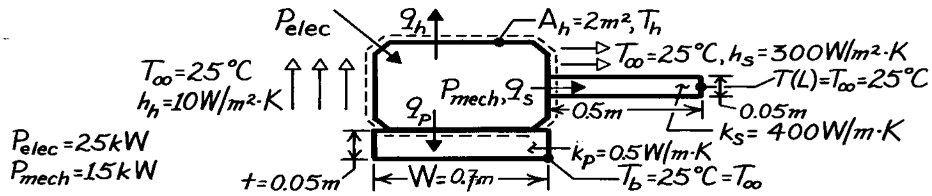
COMMENTS: This process is commonly used to anneal wire and spring products. To check the result, note that $T(L) = T(-L) = T_\infty$.

PROBLEM 3.105

KNOWN: Electric power input and mechanical power output of a motor. Dimensions of housing, mounting pad and connecting shaft needed for heat transfer calculations. Temperature of ambient air, tip of shaft, and base of pad.

FIND: Housing temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in pad and shaft, (3) Constant properties, (4) Negligible radiation.

ANALYSIS: Conservation of energy yields

$$P_{elec} - P_{mech} - q_h - q_p - q_s = 0$$

$$q_h = h_h A_h (T_h - T_\infty), \quad q_p = k_p W^2 \frac{(T_h - T_\infty)}{t}, \quad q_s = M \frac{\cosh mL - \theta_L/\theta_b}{\sinh mL}$$

$$\theta_L = 0, \quad mL = \left(4h_s L^2 / k_s D\right)^{1/2}, \quad M = \left(\frac{\pi^2}{4} D^3 h_s k_s\right)^{1/2} (T_h - T_\infty).$$

Hence

$$q_s = \frac{\left(\left[\pi^2/4\right] D^3 h_s k_s\right)^{1/2} (T_h - T_\infty)}{\tanh\left(4h_s L^2 / k_s D\right)^{1/2}}$$

Substituting, and solving for $(T_h - T_\infty)$,

$$T_h - T_\infty = \frac{P_{elec} - P_{mech}}{h_h A_h + k_p W^2/t + \left(\left(\pi^2/4\right) D^3 h_s k_s\right)^{1/2} / \tanh\left(4h_s L^2/k_s D\right)^{1/2}}$$

$$\left(\left(\pi^2/4\right) D^3 h_s k_s\right)^{1/2} = 6.08 \text{ W/K}, \quad \left(4h_s L^2/k_s D\right)^{1/2} = 3.87, \quad \tanh mL = 0.999$$

$$T_h - T_\infty = \frac{(25-15)\times 10^3 \text{ W}}{\left[10\times 2 + 0.5(0.7)^2/0.05 + 6.08/0.999\right] \text{W/K}} = \frac{10^4 \text{ W}}{(20+4.90+6.15)\text{ W/K}}$$

$$T_h - T_\infty = 322.1\text{K} \qquad T_h = 347.1^\circ\text{C}$$ <

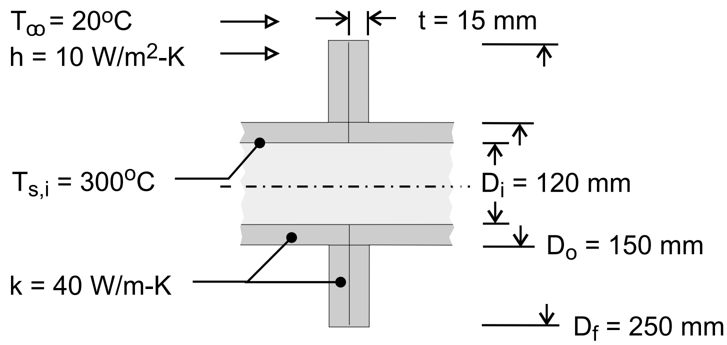
COMMENTS: (1) T_h is large enough to provide significant heat loss by radiation from the housing. Assuming an emissivity of 0.8 and surroundings at 25°C, $q_{rad} = \varepsilon A_h \left(T_h^4 - T_{sur}^4\right)$ = 4347 W, which compares with $q_{conv} = hA_h (T_h - T_\infty) = 5390$ W. Radiation has the effect of decreasing T_h. (2) The infinite fin approximation, q_s = M, is excellent.

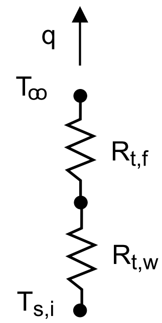
PROBLEM 3.106

KNOWN: Dimensions and thermal conductivity of pipe and flange. Inner surface temperature of pipe. Ambient temperature and convection coefficient.

FIND: Heat loss through flange.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state, (2) One-dimensional radial conduction in pipe and flange, (3) Constant thermal conductivity, (4) Negligible radiation exchange with surroundings.

ANALYSIS: From the thermal circuit, the heat loss through the flanges is

$$q = \frac{T_{s,i} - T_\infty}{R_{t,w} + R_{t,f}} = \frac{T_{s,i} - T_\infty}{\left[\ell n\left(D_o / D_i\right)/4\pi t k\right] + \left(1/hA_f\eta_f\right)}$$

Since convection heat transfer only occurs from one surface of a flange, the connected flanges may be modeled as a single annular fin of thickness $t' = 2t = 30\,\text{mm}$. Hence, $r_{2c} = \left(D_f / 2\right) + t'/2 = 0.140\,\text{m}$,

$A_f = 2\pi\left(r_{2c}^2 - r_1^2\right) = 2\pi\left(r_{2c}^2 - D_o/2\right) = 2\pi\left(0.140^2 - 0.06^2\right)\text{m}^2 = 0.101\,\text{m}^2$, $L_c = L + t'/2 =$

$\left(D_f - D_o\right)/2 + t = 0.065\,\text{m}$, $A_p = L_c\,t' = 0.00195\,\text{m}^2$, $L_c^{2/2}\left(h/kA_p\right)^{1/2} = 0.188$. With $r_{2c}/r_1 =$ $r_{2c}/(D_o/2) = 1.87$, Fig. 3.19 yields $\eta_f = 0.94$. Hence,

$$q = \frac{300°\text{C} - 20°\text{C}}{\left[\ell n(1.25)/4\pi \times 0.03\,\text{m} \times 40\,\text{W/m}\cdot\text{K}\right] + \left(1/10\,\text{W/m}^2\cdot\text{K} \times 0.101\,\text{m}^2 \times 0.94\right)}$$

$$q = \frac{280°\text{C}}{(0.0148 + 1.053)\,\text{K/W}} = 262\,\text{W}$$ <

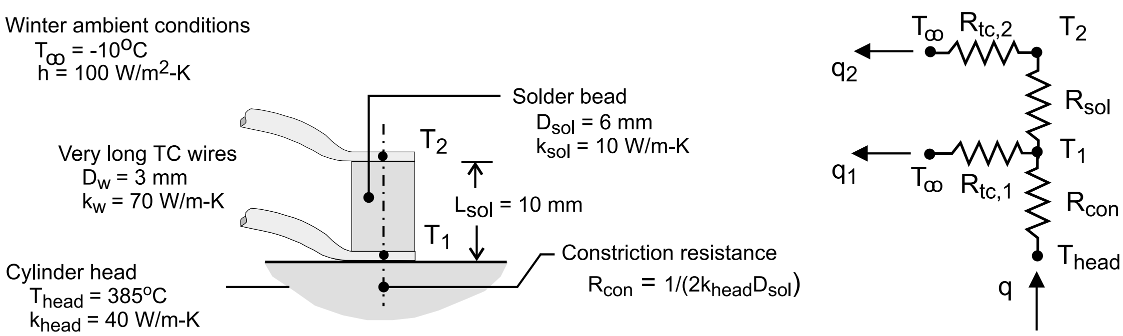
COMMENTS: Without the flange, heat transfer from a section of pipe of width $t' = 2t$ is $q = \left(T_{s,i} - T_\infty\right)/\left(R_{t,w} + R_{t,cnv}\right)$, where $R_{t,cnv} = \left(h \times \pi D_o t'\right)^{-1} = 7.07\,\text{K/W}$. Hence, $q = 39.5\,\text{W}$, and there is significant heat transfer enhancement associated with the extended surfaces afforded by the flanges.

PROBLEM 3.107

KNOWN: TC wire leads attached to the upper and lower surfaces of a cylindrically shaped solder bead. Base of bead attached to cylinder head operating at 350°C. Constriction resistance at base and TC wire convection conditions specified.

FIND: (a) Thermal circuit that can be used to determine the temperature difference between the two intermediate metal TC junctions, $(T_1 - T_2)$; label temperatures, thermal resistances and heat rates; and (b) Evaluate $(T_1 - T_2)$ for the prescribed conditions. Comment on assumptions made in building the model.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in solder bead; no losses from lateral and top surfaces; (3) TC wires behave as infinite fins, (4) Negligible thermal contact resistance between TC wire terminals and bead.

ANALYSIS: (a) The thermal circuit is shown above. Note labels for the temperatures, thermal resistances and the relevant heat fluxes. The thermal resistances are as follows:

Constriction (con) resistance, see Table 4.1, case 10

$$R_{con} = 1/\left(2k_{bead}D_{sol}\right) = 1/\left(2\times 40\ W/m\cdot K\times 0.006\ m\right) = 2.08\ K/W$$

TC (tc) wires, infinitely long fins; Eq. 3.80

$$R_{tc,1} = R_{tc,2} = R_{fin} = \left(hPk_wA_c\right)^{-0.5} \qquad P = \pi D_w,\ A_c = \pi D_w^2/4$$

$$R_{tc} = \left(100\ W/m^2\cdot K\times\pi^2\times\left(0.003\ m\right)^3\times 70\ W/m\cdot K/4\right)^{-0.5} = 46.31\ K/W$$

Solder bead (sol), cylinder D_{sol} *and* L_{sol}

$$R_{sol} = L_{sol}/\left(k_{sol}A_{sol}\right) \qquad A_{sol} = \pi D_{sol}^2/4$$

$$R_{sol} = 0.010\ m/\left(10\ W/m\cdot K\times\pi\left(0.006\ m\right)^2/4\right) = 35.37\ K/W$$

(b) Perform energy balances on the 1- and 2-nodes, solve the equations simultaneously to find T_1 and T_2, from which $(T_1 - T_2)$ can be determined.

Continued …..

Node 1
$$\frac{T_2 - T_1}{R_{sol}} + \frac{T_{head} - T_1}{R_{con}} + \frac{T_\infty - T_1}{R_{tc,1}} = 0$$

Node 2
$$\frac{T_\infty - T_2}{R_{tc,2}} + \frac{T_1 - T_2}{R_{sol}} = 0$$

Substituting numerical values with the equations in the *IHT* Workspace, find

$$T_1 = 359°C \qquad T_2 = 199.2°C \qquad T_1 - T_2 = 160°C$$

COMMENTS: (1) With this arrangement, the TC indicates a systematically low reading of the cylinder head. The size of the solder bead (L_{sol}) needs to be reduced substantially.

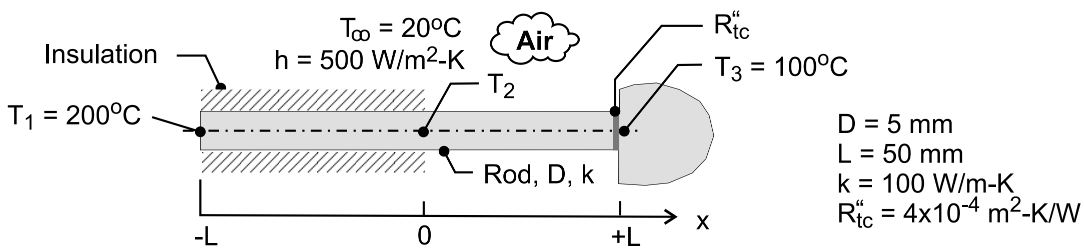
(2) The model neglects heat losses from the top and lateral sides of the solder bead, the effect of which would be to increase our estimate for ($T_1 - T_2$). Constriction resistance is important; note that $T_{head} - T_1$ = 26°C.

PROBLEM 3.108

KNOWN: Rod (D, k, 2L) that is perfectly insulated over the portion of its length $-L \le x \le 0$ and experiences convection (T_∞, h) over the portion $0 \le x \le +L$. One end is maintained at T_1 and the other is separated from a heat sink at T_3 with an interfacial thermal contact resistance R''_{tc}.

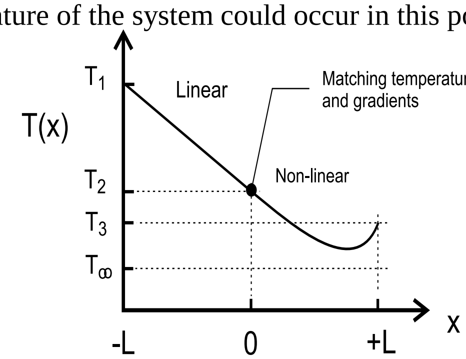
FIND: (a) Sketch the temperature distribution T vs. x and identify key features; assume $T_1 > T_3 > T_2$; (b) Derive an expression for the mid-point temperature T_2 in terms of thermal and geometric parameters of the system, (c) Using, numerical values, calculate T_2 and plot the temperature distribution. Describe key features and compare to your sketch of part (a).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod for $-L \le x \le 0$, (3) Rod behaves as one-dimensional extended surface for $0 \le x \le +L$, (4) Constant properties.

ANALYSIS: (a) The sketch for the temperature distribution is shown below. Over the insulated portion of the rod, the temperature distribution is linear. A temperature drop occurs across the thermal contact resistance at x = +L. The distribution over the exposed portion of the rod is non-linear. The minimum temperature of the system could occur in this portion of the rod.



(b) To derive an expression for T_2, begin with the general solution from the conduction analysis for a fin of uniform cross-sectional area, Eq. 3.66.

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad 0 \le x \le +L \tag{1}$$

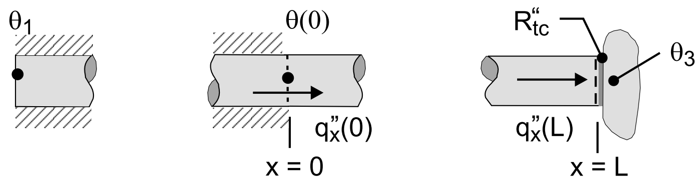
where $m = (hP/kA_c)^{1/2}$ and $\theta = T(x) - T_\infty$. The arbitrary constants are determined from the boundary conditions.

At x = 0, thermal resistance of rod

$$q_x(0) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=0} = kA_c \frac{\theta_1 - \theta(0)}{L} \qquad \theta_1 = T_1 - T_\infty$$

$$m C_1 e^0 - m C_2 e^0 = \frac{1}{L}\left[\theta_1 - \left(C_1 e^0 + C_2 e^0\right)\right] \tag{2}$$

Continued …..



At x=L, thermal contact resistance

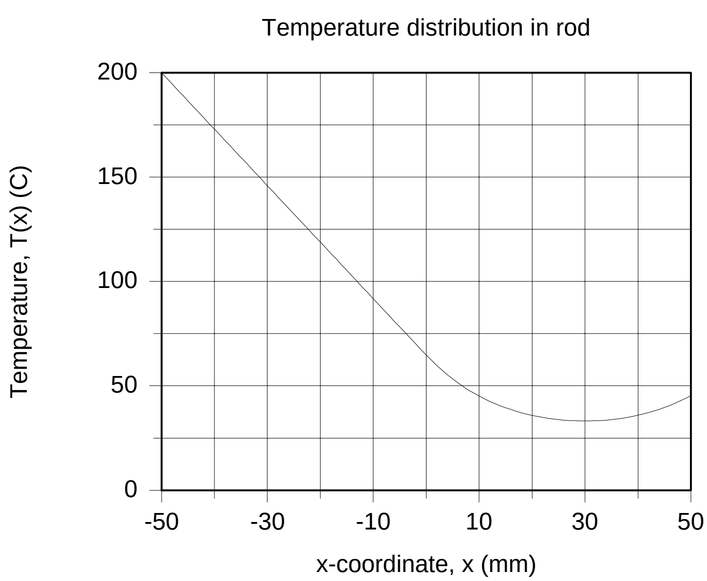
$$q_x(+L) = -kA_c \left.\frac{d\theta}{dx}\right)_{x=L} = \frac{\theta(L) - \theta_3}{R''_{tc}/A_c} \qquad \theta_3 = T_3 - T_\infty$$

$$-k\left[mC_1e^{mL} - mC_2e^{-mL}\right] = \frac{1}{R''_{tc}}\left[C_1e^{mL} + C_2e^{-mL} - \theta_3\right] \tag{3}$$

Eqs. (2) and (3) cannot be rearranged easily to find explicit forms for C_1 and C_2. The constraints will be evaluated numerically in part (c). Knowing C_1 and C_2, Eq. (1) gives

$$\theta_2 = \theta(0) = T_2 - T_\infty = C_1e^0 + C_2e^0 \tag{4}$$

(c) With Eqs. (1-4) in the *IHT Workspace* using numerical values shown in the schematic, find T_2 = 62.1°C. The temperature distribution is shown in the graph below.



COMMENTS: (1) The purpose of asking you to sketch the temperature distribution in part (a) was to give you the opportunity to identify the relevant thermal processes and come to an understanding of the system behavior.

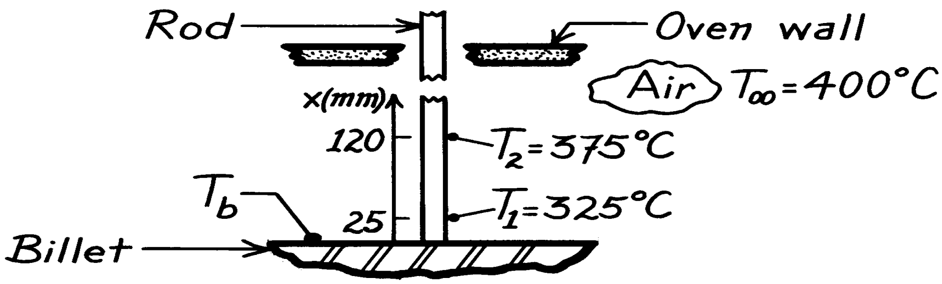
(2) Sketch the temperature distributions for the following conditions and explain their key features: (a) $R''_{tc} = 0$, (b) $R''_{tc} \to \infty$, and (c) the exposed portion of the rod behaves as an infinitely long fin; that is, k is very large.

PROBLEM 3.109

KNOWN: Long rod in oven with air temperature at 400°C has one end firmly pressed against surface of a billet; thermocouples imbedded in rod at locations 25 and 120 mm from the billet indicate 325 and 375°C, respectively.

FIND: The temperature of the billet, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Rod is infinitely long with uniform cross-sectional area, (3) Uniform convection coefficient along rod.

ANALYSIS: For an infinitely long rod of uniform cross-sectional area, the temperature distribution is

$$\theta(x) = \theta_b e^{-mx} \tag{1}$$

where

$$\theta(x) = T(x) - T_\infty \qquad \theta_b = T(0) - T_\infty = T_b - T_\infty.$$

Substituting values for T_1 and T_2 at their respective distances, x_1 and x_2, into Eq. (1), it is possible to evaluate m,

$$\frac{\theta(x_1)}{\theta(x_2)} = \frac{\theta_b e^{-mx_1}}{\theta_b e^{-mx_2}} = e^{-m(x_1 - x_2)}$$

$$\frac{(325-400)^\circ C}{(375\text{-}400)^\circ C} = e^{-m(0.025-0.120)m} \qquad m=11.56.$$

Using the value for m with Eq. (1) at location x_1, it is now possible to determine the rod base or billet temperature,

$$\theta(x_1) = T_1 - T_\infty = (T_b - T_\infty)e^{-mx}$$

$$(325-400)^\circ C = (T_b - 400)^\circ C\ e^{-11.56\times 0.025}$$

$$T_b = 300^\circ C. \qquad <$$

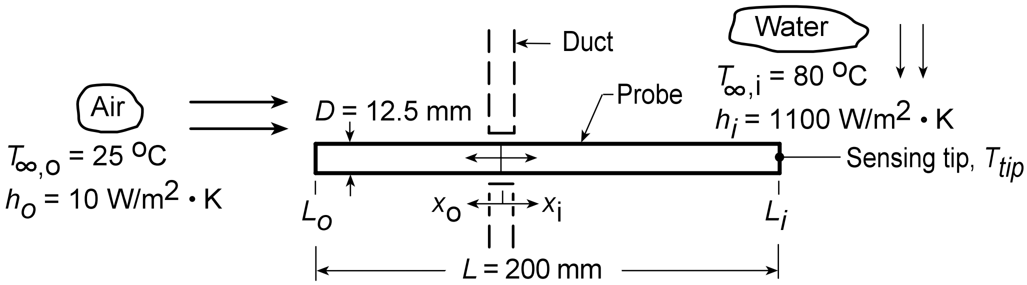
COMMENTS: Using the criteria mL ≥ 2.65 (see Example 3.8) for the infinite fin approximation, the insertion length should be ≥ 229 mm to justify the approximation,

PROBLEM 3.110

KNOWN: Temperature sensing probe of thermal conductivity k, length L and diameter D is mounted on a duct wall; portion of probe L_i is exposed to water stream at $T_{\infty,i}$ while other end is exposed to ambient air at $T_{\infty,o}$; convection coefficients h_i and h_o are prescribed.

FIND: (a) Expression for the measurement error, $\Delta T_{err} = T_{tip} - T_{\infty,i}$, (b) For prescribed $T_{\infty,i}$ and $T_{\infty,o}$, calculate ΔT_{err} for immersion to total length ratios of 0.225, 0.425, and 0.625, (c) Compute and plot the effects of probe thermal conductivity and water velocity (h_i) on ΔT_{err}.

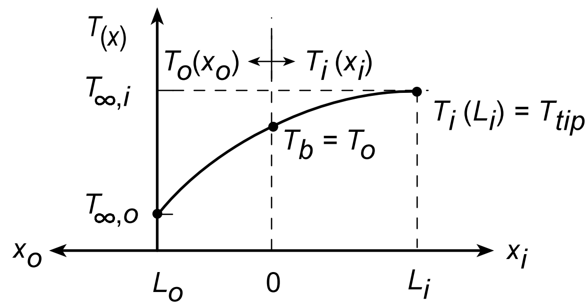
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in probe, (3) Probe is thermally isolated from the duct, (4) Convection coefficients are uniform over their respective regions.

PROPERTIES: Probe material (given): k = 177 W/m·K.

ANALYSIS: (a) To derive an expression for $\Delta T_{err} = T_{tip} - T_{\infty,i}$, we need to determine the temperature distribution in the immersed length of the probe $T_i(x)$. Consider the probe to consist of two regions: $0 \le x_i \le L_i$, the immersed portion, and $0 \le x_o \le (L - L_i)$, the ambient-air portion where the origin corresponds to the location of the duct wall. Use the results for the temperature distribution and fin heat rate of Case A, Table 3.4:



Temperature distribution in region i:

$$\frac{\theta_i}{\theta_{b,i}} = \frac{T_i(x_i) - T_{\infty,i}}{T_o - T_{\infty,i}} = \frac{\cosh\left(m_i(L_i - x_i)\right) + (h_i/m_i k)\sinh(L_i - x_i)}{\cosh(m_i L_i) + (h_i/m_i k)\sinh(m_i L_i)} \qquad (1)$$

and the tip temperature, $T_{tip} = T_i(L_i)$ at $x_i = L_i$, is

$$\frac{T_{tip} - T_{\infty,i}}{T_o - T_{\infty,i}} = A = \frac{\cosh(0) + (h_i/m_i k)\sinh(0)}{\cosh(m_i L_i) + (h_i/m_i k)\sinh(m_i L_i)} \qquad (2)$$

and hence

$$\Delta T_{err} = T_{tip} - T_{\infty,i} = A\left(T_o - T_{\infty,i}\right) \qquad (3)$$ <

where T_o is the temperature at $x_i = x_o = 0$ which at present is unknown, but can be found by setting the fin heat rates equal, that is,

$$q_{f,o} = -q_{f,i} \qquad (4)$$

Continued...

$$\left(h_o P k A_c\right)^{1/2} \theta_{b,o} \cdot B = -\left(h_i P k A_c\right)^{1/2} \theta_{b,i} \cdot C$$

Solving for T_o, find

$$\frac{\theta_{b,o}}{\theta_{b,i}} = \frac{T_o - T_{\infty,o}}{T_o - T_{\infty,i}} = -\left(h_i P k A_c\right)^{1/2} \theta_{b,i} \cdot C$$

$$T_o = \left[T_{\infty,o} + \left(\frac{h_i}{h_o} \right)^{1/2} \frac{C}{B} T_{\infty,i} \right] \Bigg/ \left[1 + \left(\frac{h_i}{h_o} \right)^{1/2} \frac{C}{B} \right] \qquad (5)$$

where the constants B and C are,

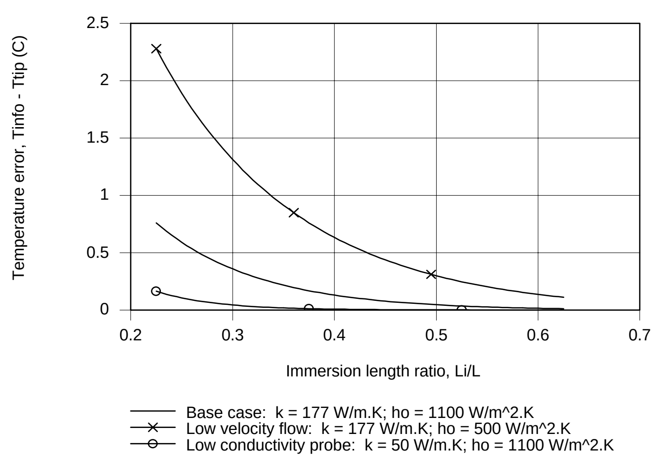
$$B = \frac{\sinh(m_o L_o) + (h_o / m_o k)\cosh(m_o L_o)}{\cosh(m_o L_o) + (h_o / m_o k)\sinh(m_o L_o)} \qquad (6)$$

$$C = \frac{\sinh(m_i L_i) + (h_i / m_i k)\cosh(m_i L_i)}{\cosh(m_i L_i) + (h_i / m_i k)\sinh(m_i L_i)} \qquad (7)$$

(b) To calculate the immersion error for prescribed immersion lengths, L_i/L = 0.225, 0.425 and 0.625, we use Eq. (3) as well as Eqs. (2, 6, 7 and 5) for A, B, C, and T_o, respectively. Results of these calculations are summarized below.

L_i/L	L_o (mm)	L_i (mm)	A	B	C	T_o (°C)	ΔT_{err} (°C)	
0.225	155	45	0.2328	0.5865	0.9731	76.7	-0.76	<
0.425	115	85	0.0396	0.4639	0.992	77.5	-0.10	<
0.625	75	125	0.0067	0.3205	0.9999	78.2	-0.01	<

(c) The probe behaves as a fin having ends exposed to the cool ambient air and the hot ambient water whose temperature is to be measured. If the thermal conductivity is *decreased*, heat transfer along the probe length is likewise decreased, the tip temperature will be closer to the water temperature. If the velocity of the water *decreases*, the convection coefficient will decrease, and the difference between the tip and water temperatures will increase.

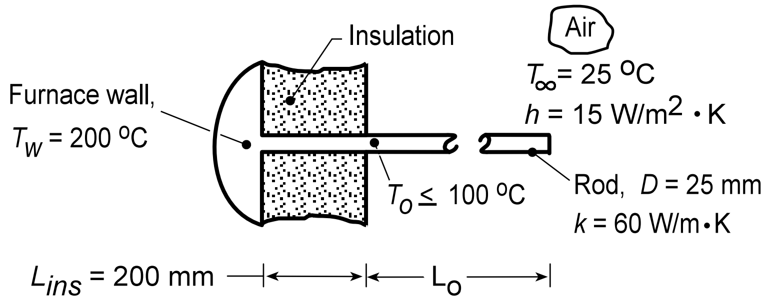


PROBLEM 3.111

KNOWN: Rod protruding normally from a furnace wall covered with insulation of thickness L_{ins} with the length L_o exposed to convection with ambient air.

FIND: (a) An expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. (b) Will a rod of L_o = 100 mm meet the specified operating limit, $T_0 \leq 100°C$? If not, what design parameters would you change?

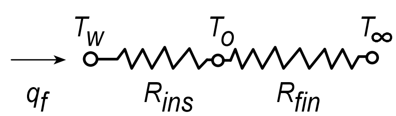
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in rod, (3) Negligible thermal contact resistance between the rod and hot furnace wall, (4) Insulated section of rod, L_{ins}, experiences no lateral heat losses, (5) Convection coefficient uniform over the exposed portion of the rod, L_o, (6) Adiabatic tip condition for the rod and (7) Negligible radiation exchange between rod and its surroundings.

ANALYSIS: (a) The rod can be modeled as a thermal network comprised of two resistances in series: the portion of the rod, L_{ins}, covered by insulation, R_{ins}, and the portion of the rod, L_o, experiencing convection, and behaving as a fin with an adiabatic tip condition, R_{fin}. For the insulated section:

$$R_{ins} = L_{ins}/kA_c \qquad (1)$$



For the fin, Table 3.4, Case B, Eq. 3.76,

$$R_{fin} = \theta_b/q_f = \frac{1}{\left(hPkA_c\right)^{1/2}\tanh\left(mL_o\right)} \qquad (2)$$

$$m = \left(hP/kA_c\right)^{1/2} \qquad A_c = \pi D^2/4 \qquad P = \pi D \qquad (3,4,5)$$

From the thermal network, by inspection,

$$\frac{T_o - T_\infty}{R_{fin}} = \frac{T_w - T_\infty}{R_{ins} + R_{fin}} \qquad T_o = T_\infty + \frac{R_{fin}}{R_{ins} + R_{fin}}\left(T_w - T_\infty\right) \qquad (6)$$ <

(b) Substituting numerical values into Eqs. (1) - (6) with L_o = 200 mm,

$$T_o = 25°C + \frac{6.298}{6.790 + 6.298}\left(200 - 25\right)°C = 109°C$$ <

$$R_{ins} = \frac{0.200\,m}{60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2} = 6.790\,K/W \qquad A_c = \pi\left(0.025\,m\right)^2/4 = 4.909\times10^{-4}\,m^2$$

$$R_{fin} = 1/\left(0.0347\,W^2/K^2\right)^{1/2}\tanh\left(6.324\times0.200\right) = 6.298\,K/W$$

$$\left(hPkA_c\right) = \left(15\,W/m^2\cdot K\times\pi\left(0.025\,m\right)\times60\,W/m\cdot K\times4.909\times10^{-4}\,m^2\right) = 0.0347\,W^2/K^2$$

Continued...

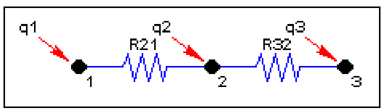
$$m = \left(hP/kA_c\right)^{1/2} = \left(15\,W/m^2\cdot K \times \pi\,(0.025\,m)\Big/60\,W/m\cdot K \times 4.909\times10^{-4}\,m^2\right)^{1/2} = 6.324\,m^{-1}$$

Consider the following design changes aimed at reducing $T_o \leq 100°C$. (1) Increasing length of the fin portions: with L_o = 200 mm, the fin already behaves as an infinitely long fin. Hence, increasing L_o will not result in reducing T_o. (2) Decreasing the thermal conductivity: backsolving the above equation set with T_0 = 100°C, find the required thermal conductivity is k = 14 W/m·K. Hence, we could select a stainless steel alloy; see Table A.1. (3) Increasing the insulation thickness: find that for T_o = 100°C, the required insulation thickness would be L_{ins} = 211 mm. This design solution might be physically and economically unattractive. (4) A very practical solution would be to introduce thermal contact resistance between the rod base and the furnace wall by "tack welding" (rather than a continuous bead around the rod circumference) the rod in two or three places. (5) A less practical solution would be to increase the convection coefficient, since to do so, would require an air handling unit.

COMMENTS: (1) Would replacing the rod by a thick-walled tube provide a practical solution?

(2) The *IHT Thermal Resistance Network Model* and the *Thermal Resistance Tool* for a *fin* with an *adiabatic tip* were used to create a model of the rod. The Workspace is shown below.

// Thermal Resistance Network Model:
// The Network:



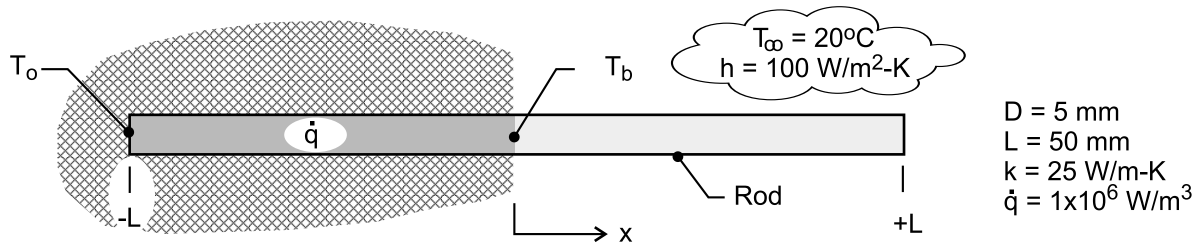

```
// Heat rates into node j,qij, through thermal resistance Rij
q21 = (T2 - T1) / R21
q32 = (T3 - T2) / R32

// Nodal energy balances
q1 + q21 = 0
q2 - q21 + q32 = 0
q3 - q32 = 0

/* Assigned variables list: deselect the qi, Rij and Ti which are unknowns; set qi = 0 for embedded nodal
points at which there is no external source of heat. */
T1 = Tw            // Furnace wall temperature, C
//q1 =             // Heat rate, W
T2 = To            // To, beginning of rod exposed length
q2 =  0            // Heat rate, W; node 2; no external heat source
T3 =  Tinf         // Ambient air temperature, C
//q3 =             // Heat rate, W

// Thermal Resistances:
// Rod - conduction resistance
R21 =  Lins / (k * Ac)          // Conduction resistance, K/W
Ac = pi * D^2 / 4               // Cross sectional area of rod, m^2

// Thermal Resistance Tools - Fin with Adiabatic Tip:
R32 =  Rfin                     //  Resistance of fin, K/W
/* Thermal resistance of a fin of uniform cross sectional area Ac, perimeter P, length L, and thermal
conductivity k with an adiabatic tip condition experiencing convection with a fluid at Tinf and coefficient h, */
Rfin = 1/ ( tanh (m*Lo) * (h * P * k * Ac ) ^ (1/2) )          // Case B, Table 3.4
m = sqrt(h*P / (k*Ac))
P = pi * D                      // Perimeter, m

// Other Assigned Variables:
Tw = 200           // Furnace wall temperature, C
k = 60             // Rod thermal conductivity, W/m.K
Lins = 0.200       // Insulated length, m
D = 0.025          // Rod diameter, m
h = 15             // Convection coefficient, W/m^2.K
Tinf = 25          // Ambient air temperature,C
Lo = 0.200         // Exposed length, m
```

PROBLEM 3.112

KNOWN: Rod (D, k, 2L) inserted into a perfectly insulating wall, exposing one-half of its length to an airstream (T_∞, h). An electromagnetic field induces a uniform volumetric energy generation $(\dot{q})$ in the imbedded portion.

FIND: (a) Derive an expression for T_b at the base of the exposed half of the rod; the exposed region may be approximated as a very long fin; (b) Derive an expression for T_o at the end of the imbedded half of the rod, and (c) Using numerical values, plot the temperature distribution in the rod and describe its key features. Does the rod behave as a very long fin?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in imbedded portion of rod, (3) Imbedded portion of rod is perfectly insulated, (4) Exposed portion of rod behaves as an infinitely long fin, and (5) Constant properties.

ANALYSIS: (a) Since the exposed portion of the rod ($0 \le x \le +L$) behaves as an infinite fin, the fin heat rate using Eq. 3.80 is

$$q_x(0) = q_f = M = (hPkA_c)^{1/2}(T_b - T_\infty) \qquad (1)$$

From an energy balance on the imbedded portion of the rod,

$$q_f = \dot{q} A_c L \qquad (2)$$

Combining Eqs. (1) and (2), with $P = \pi D$ and $A_c = \pi D^2/4$, find

$$T_b = T_\infty + q_f (hPkA_c)^{-1/2} = T_\infty + \dot{q}A_c^{1/2} L (hPk)^{-1/2} \qquad (3) \quad <$$

(b) The imbedded portion of the rod ($-L \le x \le 0$) experiences one-dimensional heat transfer with uniform $\dot{q}$. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \quad <$$

(c) The temperature distribution T(x) for the rod is piecewise parabolic and exponential,

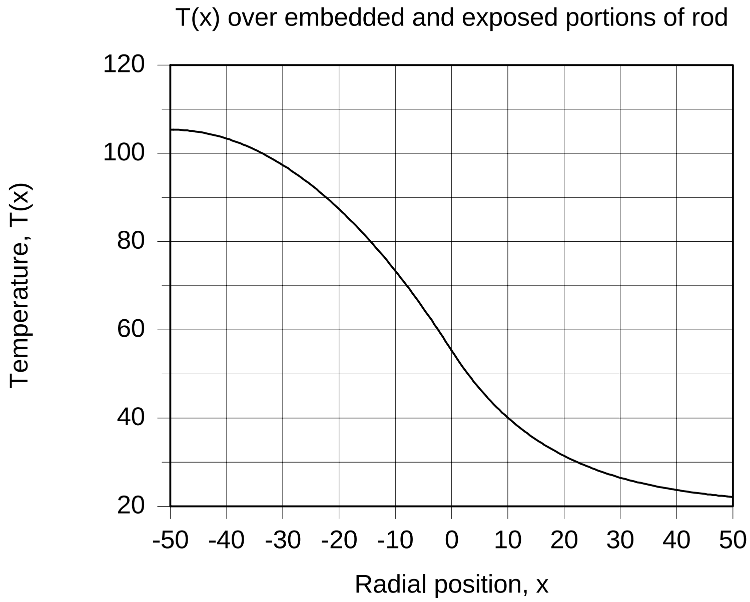
$$T(x) - T_b = \frac{\dot{q}L^2}{2k}\left(\frac{x}{L}\right)^2 \qquad -L \le x \le 0$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \exp(-mx) \qquad 0 \le x \le +L$$

Continued …..

PROBLEM 3.112 (Cont.)

The gradient at x = 0 will be continuous since we used this condition in evaluating T_b. The distribution is shown below with T_o = 105.4°C and T_b = 55.4°C.

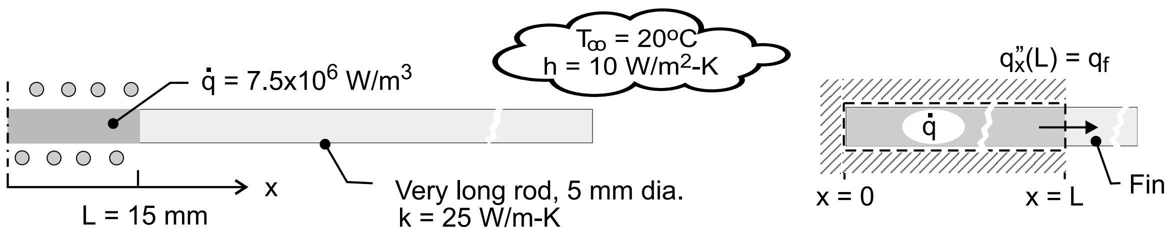


PROBLEM 3.113

KNOWN: Very long rod (D, k) subjected to induction heating experiences uniform volumetric generation $(\dot{q})$ over the center, 30-mm long portion. The unheated portions experience convection (T_∞, h).

FIND: Calculate the temperature of the rod at the mid-point of the heated portion within the coil, T_o, and at the edge of the heated portion, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform $\dot{q}$ in portion of rod within the coil; no convection from lateral surface of rod, (3) Exposed portions of rod behave as infinitely long fins, and (4) Constant properties.

ANALYSIS: The portion of the rod within the coil, $0 \le x \le +L$, experiences one-dimensional conduction with uniform generation. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \tag{1}$$

The portion of the rod beyond the coil, $L \le x \le \infty$, behaves as an infinitely long fin for which the heat rate from Eq. 3.80 is

$$q_f = q_x(L) = (hPkA_c)^{1/2}(T_b - T_\infty) \tag{2}$$

where $P = \pi D$ and $A_c = \pi D^2/4$. From an overall energy balance on the imbedded portion of the rod as illustrated in the schematic above, find the heat rate as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_f + \dot{q}A_cL = 0$$

$$q_f = \dot{q}A_cL \tag{3}$$

Combining Eqs. (1-3),

$$T_b = T_\infty + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \tag{4}$$

$$T_o = T_\infty + \frac{\dot{q}L^2}{2k} + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \tag{5}$$

and substituting numerical values find

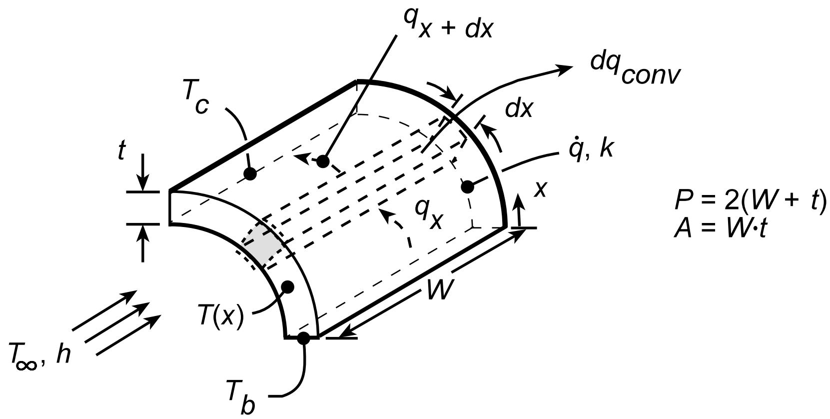
$$T_o = 305°C \qquad\qquad T_b = 272°C$$ <

PROBLEM 3.114

KNOWN: Dimensions, end temperatures and volumetric heating of wire leads. Convection coefficient and ambient temperature.

FIND: (a) Equation governing temperature distribution in the leads, (b) Form of the temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in x, (3) Uniform volumetric heating, (4) Uniform h (both sides), (5) Negligible radiation.

ANALYSIS: (a) Performing an energy balance for the differential control volume,

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \qquad\qquad q_x - q_{x+dx} - dq_{conv} + \dot{q}dV = 0$$

$$-kA_c \frac{dT}{dx} - \left[-kA_c \frac{dT}{dx} - \frac{d}{dx}\left(kA_c \frac{dT}{dx}\right)dx\right] - hPdx\left(T - T_\infty\right) + \dot{q}A_c dx = 0$$

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}\left(T - T_\infty\right) + \frac{\dot{q}}{k} = 0 \qquad <$$

(b) With a *reduced temperature* defined as $\Theta \equiv T - T_\infty - \left(\dot{q}A_c/hP\right)$ and $m^2 \equiv hP/kA_c$, the differential equation may be rendered homogeneous, with a general solution and boundary conditionsas shown

$$\frac{d^2\Theta}{dx^2} - m^2\Theta = 0 \qquad\qquad \Theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

$$\Theta_b = C_1 + C_2 \qquad\qquad \Theta_c = C_1 e^{mL} + C_2 e^{-mL}$$

it follows that

$$C_1 = \frac{\Theta_b e^{-mL} - \Theta_c}{e^{-mL} - e^{mL}} \qquad\qquad C_2 = \frac{\Theta_c - \Theta_b e^{mL}}{e^{-mL} - e^{mL}}$$

$$\Theta(x) = \frac{\left(\Theta_b e^{-mL} - \Theta_c\right)e^{mx} + \left(\Theta_c - \Theta_b e^{mL}\right)e^{-mx}}{e^{-mL} - e^{mL}} \qquad <$$

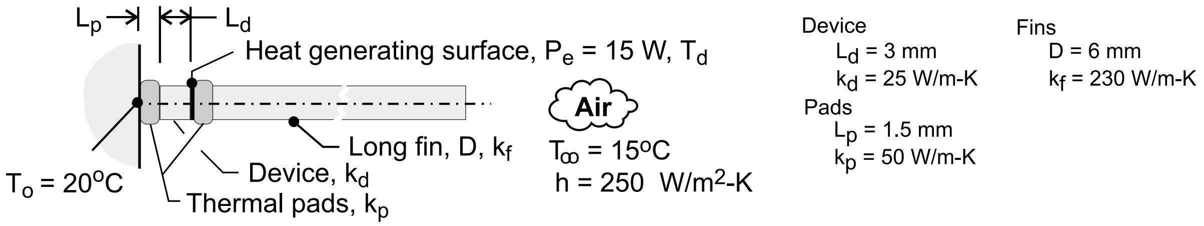
COMMENTS: If $\dot{q}$ is large and h is small, temperatures within the lead may readily exceed the prescribed boundary temperatures.

PROBLEM 3.115

KNOWN: Disk-shaped electronic device (D, L_d, k_d) dissipates electrical power (P_e) at one of its surfaces. Device is bonded to a cooled base (T_o) using a thermal pad (L_p, k_A). Long fin (D, k_f) is bonded to the heat-generating surface using an identical thermal pad. Fin is cooled by convection (T_∞, h).

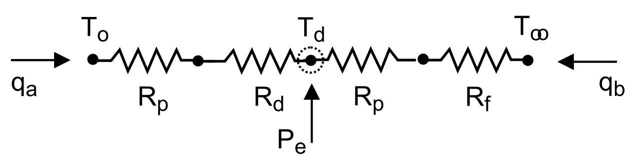
FIND: (a) Construct a thermal circuit of the system, (b) Derive an expression for the temperature of the heat-generating device, T_d, in terms of circuit thermal resistance, T_o and T_∞; write expressions for the thermal resistances; and (c) Calculate T_d for the prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction through thermal pads and device; no losses from lateral surfaces; (3) Fin is infinitely long, (4) Negligible contact resistance between components of the system, and (5) Constant properties.

ANALYSIS: (a) The thermal circuit is shown below with thermal resistances associated with conduction (pads, R_p; device, R_d) and for the long fin, R_f.



(b) To obtain an expression for T_d, perform an energy balance about the d-node

$$\dot{E}_{in} - \dot{E}_{out} = q_a + q_b + P_e = 0 \tag{1}$$

Using the conduction rate equation with the circuit

$$q_a = \frac{T_o - T_d}{R_f + R_d} \qquad q_b = \frac{T_\infty - T_d}{R_p + R_f} \tag{2,3}$$

Combine with Eq. (1), and solve for T_d,

$$T_d = \frac{P_e + T_o/(R_p + R_d) + T_\infty/(R_p + R_f)}{1/(R_p + R_d) + 1/(R_p + R_f)} \tag{4}$$

where the thermal resistances with $P = \pi D$ and $A_c = \pi D^2/4$ are

$$R_p = L_p/k_p A_c \qquad R_d = L_d/k_d A_c \qquad R_f = (hPk_f A_c)^{-1/2} \tag{5,6,7}$$

(c) Substituting numerical values with the foregoing relations, find

$$R_p = 1.061 \text{ K/W} \qquad R_d = 4.244 \text{ K/W} \qquad R_f = 5.712 \text{ K/W}$$

and the device temperature as

$$T_d = 62.4°C$$ <

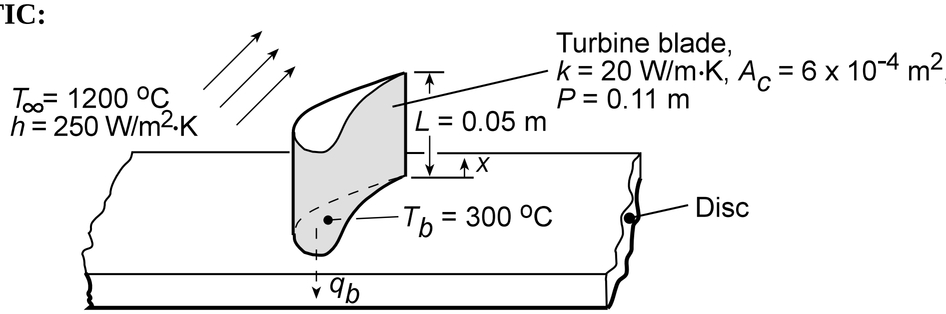
COMMENTS: What fraction of the power dissipated in the device is removed by the fin? Answer: $q_b/P_e = 47\%$.

PROBLEM 3.116

KNOWN: Dimensions and thermal conductivity of a gas turbine blade. Temperature and convection coefficient of gas stream. Temperature of blade base and maximum allowable blade temperature.

FIND: (a) Whether blade operating conditions are acceptable, (b) Heat transfer to blade coolant.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant k, (3) Adiabatic blade tip, (4) Negligible radiation.

ANALYSIS: Conditions in the blade are determined by Case B of Table 3.4.

(a) With the maximum temperature existing at x = L, Eq. 3.75 yields

$$\frac{T(L)-T_\infty}{T_b-T_\infty}=\frac{1}{\cosh mL}$$

$$m=\left(hP/kA_c\right)^{1/2}=\left(250\,W/m^2\cdot K\times 0.11\,m/20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}$$

$m = 47.87\ m^{-1}$ and $mL = 47.87\ m^{-1} \times 0.05\ m = 2.39$

From Table B.1, cosh mL = 5.51. Hence,

$$T(L)=1200^\circ C+(300-1200)^\circ C/5.51=1037^\circ C \qquad <$$

and the operating conditions are acceptable.

(b) With $M=\left(hPkA_c\right)^{1/2}\Theta_b=\left(250\,W/m^2\cdot K\times 0.11\,m\times 20\,W/m\cdot K\times 6\times 10^{-4}\,m^2\right)^{1/2}\left(-900^\circ C\right)=-517\,W$, Eq. 3.76 and Table B.1 yield

$$q_f = M\tanh mL = -517\,W\left(0.983\right)=-508\,W$$

Hence, $q_b=-q_f=508\,W$ $\qquad <$

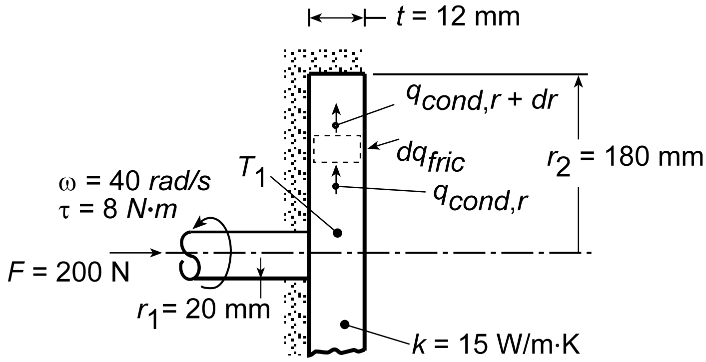
COMMENTS: Radiation losses from the blade surface and convection from the tip will contribute to reducing the blade temperatures.

PROBLEM 3.117

KNOWN: Dimensions of disc/shaft assembly. Applied angular velocity, force, and torque. Thermal conductivity and inner temperature of disc.

FIND: (a) Expression for the friction coefficient μ, (b) Radial temperature distribution in disc, (c) Value of μ for prescribed conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant k, (4) Uniform disc contact pressure p, (5) All frictional heat dissipation is transferred to shaft from base of disc.

ANALYSIS: (a) The normal force acting on a differential ring extending from r to r+dr on the contact surface of the disc may be expressed as $dF_n = p2\pi r dr$. Hence, the tangential force is $dF_t = \mu p 2\pi r dr$, in which case the torque may be expressed as

$$d\tau = 2\pi\mu p r^2 dr$$

For the entire disc, it follows that

$$\tau = 2\pi\mu p \int_o^{r_2} r^2 dr = \frac{2\pi}{3}\mu p r_2^3$$

where $p = F/\pi r_2^2$. Hence,

$$\mu = \frac{3}{2}\frac{\tau}{F r_2}$$ <

(b) Performing an energy balance on a differential control volume in the disc, it follows that

$$q_{cond,r} + dq_{fric} - q_{cond,r+dr} = 0$$

With $dq_{fric} = \omega d\tau = 2\mu F\omega\left(r^2/r_2^2\right)dr$, $q_{cond,r+dr} = q_{cond,r} + \left(dq_{cond,r}/dr\right)dr$, and

$q_{cond,r} = -k(2\pi r t)dT/dr$, it follows that

$$2\mu F\omega\left(r^2/r_2^2\right)dr + 2\pi kt\frac{d(rdT/dr)}{dr}dr = 0$$

or

$$\frac{d(rdT/dr)}{dr} = -\frac{\mu F\omega}{\pi k t r_2^2}r^2$$

Integrating twice,

Continued...

PROBLEM 3.117 (Cont.)

$$\frac{dT}{dr} = -\frac{\mu F \omega}{3\pi k t r_2^2} r^2 + \frac{C_1}{r}$$

$$T = -\frac{\mu F \omega}{9\pi k t r_2^2} r^3 + C_1 \ell n r + C_2$$

Since the disc is well insulated at $r = r_2$, $dT/dr|_{r_2} = 0$ and

$$C_1 = \frac{\mu F \omega r_2}{3\pi k t}$$

With $T(r_1) = T_1$, it also follows that

$$C_2 = T_1 + \frac{\mu F \omega}{9\pi k t r_2^2} r_1^3 - C_1 \ell n r_1$$

Hence,

$$T(r) = T_1 - \frac{\mu F \omega}{9\pi k t r_2^2}\left(r^3 - r_1^3\right) + \frac{\mu F \omega r_2}{3\pi k t} \ell n \frac{r}{r_1}$$ <

(c) For the prescribed conditions,

$$\mu = \frac{3}{2}\frac{8 N \cdot m}{200 N (0.18 m)} = 0.333$$ <

Since the maximum temperature occurs at r = r_2,

$$T_{max} = T(r_2) = T_1 - \frac{\mu F \omega r_2}{9\pi k t}\left[1 - \left(\frac{r_1}{r_2}\right)^3\right] + \frac{\mu F \omega r_2}{3\pi k t} \ell n\left(\frac{r_2}{r_1}\right)$$

With $(\mu F \omega r_2 / 3\pi k t) = (0.333 \times 200 N \times 40 rad/s \times 0.18 m / 3\pi \times 15 W/m \cdot K \times 0.012 m) = 282.7^\circ C$,

$$T_{max} = 80^\circ C - \frac{282.7^\circ C}{3}\left[1 - \left(\frac{0.02}{0.18}\right)^3\right] + 282.7^\circ C \ell n\left(\frac{0.18}{0.02}\right)$$

$$T_{max} = 80^\circ C - 94.1^\circ C + 621.1^\circ C = 607^\circ C$$ <

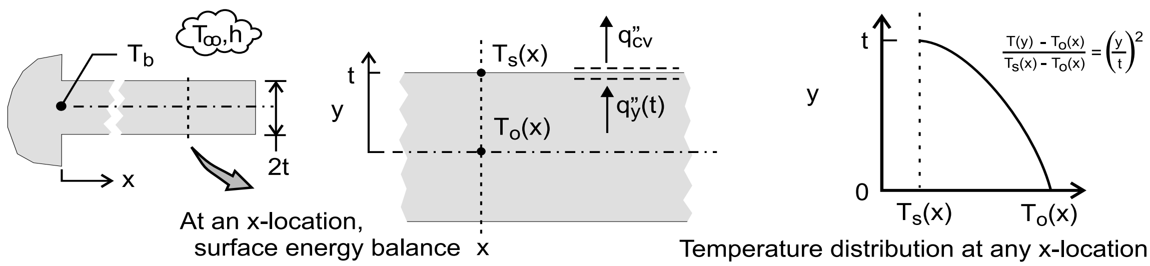
COMMENTS: The maximum temperature is excessive, and the disks should be actively cooled (by convection) at their outer surfaces.

PROBLEM 3.118

KNOWN: Extended surface of rectangular cross-section with heat flow in the longitudinal direction.

FIND: Determine the conditions for which the transverse (y-direction) temperature gradient is negligible compared to the longitudinal gradient, such that the 1-D analysis of Section 3.6.1 is valid by finding: (a) An expression for the conduction heat flux at the surface, $q''_y(t)$, in terms of T_s and T_o, assuming the transverse temperature distribution is parabolic, (b) An expression for the convection heat flux at the surface for the x-location; equate the two expressions, and identify the parameter that determines the ratio $(T_o - T_s)/(T_s - T_\infty)$; and (c) Developing a criterion for establishing the validity of the 1-D assumption used to model an extended surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform convection coefficient and (3) Constant properties.

ANALYSIS: (a) Referring to the schematics above, the conduction heat flux at the surface y = t at any x-location follows from Fourier's law using the parabolic transverse temperature distribution.

$$q''_y(t) = -k\left.\frac{\partial T}{\partial y}\right)_{y=t} = -k\left(\left[T_s(x) - T_o(x)\right]\frac{2y}{t^2}\right)_{y=t} = -\frac{2k}{t}\left[T_s(x) - T_o(x)\right] \qquad (1)$$

(b) The convection heat flux at the surface of any x-location follows from the rate equation

$$q''_{cv} = h\left[T_s(x) - T_\infty\right] \qquad (2)$$

Performing a surface energy balance as represented schematically above, equating Eqs. (1) and (2) provides

$$q''_y(t) = q''_{cv}$$

$$-\frac{2k}{t}\left[T_s(x) - T_o(x)\right] = h\left[T_s(x) - T_\infty\right]$$

$$\frac{T_s(x) - T_o(x)}{T_s(x) - T_\infty(x)} = -0.5\frac{ht}{k} = -0.5\ \text{Bi} \qquad (3)$$

where Bi = ht/k, the Biot number, represents the ratio of the convection to the conduction thermal resistances,

$$\text{Bi} = \frac{R''_{cd}}{R''_{cv}} = \frac{t/k}{1/h} \qquad (4)$$

(c) The transverse gradient (heat flow) will be negligible compared to the longitudinal gradient when Bi << 1, say, 0.1, an order of magnitude smaller. This is the criterion to validate the one-dimensional assumption used to model extended surfaces.

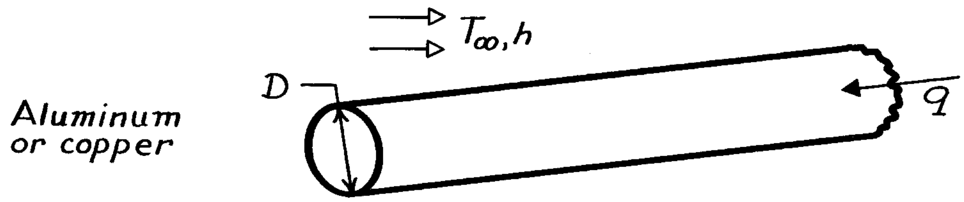
COMMENTS: The coefficient 0.5 in Eq. (3) is a consequence of the parabolic distribution assumption. This distribution represents the simplest polynomial expression that could approximate the real distribution.

PROBLEM 3.119

KNOWN: Long, aluminum cylinder acts as an extended surface.

FIND: (a) Increase in heat transfer if diameter is tripled and (b) Increase in heat transfer if copper is used in place of aluminum.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Uniform convection coefficient, (5) Rod is infinitely long.

PROPERTIES: *Table A-1,* Aluminum (pure): k = 240 W/m·K; *Table A-1,* Copper (pure): k = 400 W/m·K.

ANALYSIS: (a) For an infinitely long fin, the fin heat rate from Table 3.4 is

$$q_f = M = \left(hPkA_c\right)^{1/2}\theta_b$$

$$q_f = \left(h\,\pi\,D\,k\,\pi\,D^2/4\right)^{1/2}\theta_b = \frac{\pi}{2}\left(hk\right)^{1/2}D^{3/2}\,\theta_b.$$

where $P = \pi D$ and $A_c = \pi D^2/4$ for the circular cross-section. Note that $q_f \propto D^{3/2}$. Hence, if the diameter is tripled,

$$\frac{q_f(3D)}{q_f(D)} = 3^{3/2} = 5.2$$

and there is a 420% increase in heat transfer. <

(b) In changing from aluminum to copper, since $q_f \propto k^{1/2}$, it follows that

$$\frac{q_f(Cu)}{q_f(A1)} = \left[\frac{k_{Cu}}{k_{A1}}\right]^{1/2} = \left[\frac{400}{240}\right]^{1/2} = 1.29$$

and there is a 29% increase in the heat transfer rate. <

COMMENTS: (1) Because fin effectiveness is enhanced by maximizing $P/A_c = 4/D$, the use of a larger number of small diameter fins is preferred to a single large diameter fin.

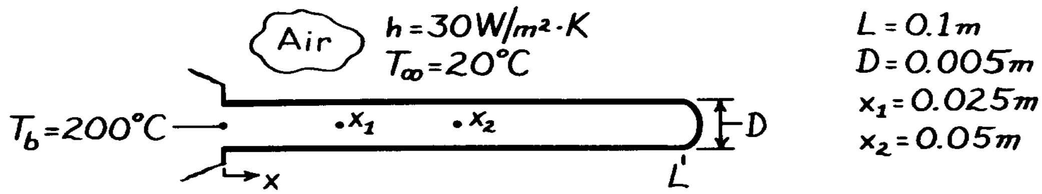
(2) From the standpoint of cost and weight, aluminum is preferred over copper.

PROBLEM 3.120

KNOWN: Length, diameter, base temperature and environmental conditions associated with a brass rod.

FIND: Temperature at specified distances along the rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient h.

PROPERTIES: *Table A-1*, Brass $\left(\bar{T} = 110^\circ C\right)$: $k = 133\ W/m \cdot K$.

ANALYSIS: Evaluate first the fin parameter

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{h\pi\ D}{k\pi\ D^2/4}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4 \times 30\ W/m^2 \cdot K}{133\ W/m \cdot K \times 0.005m}\right]^{1/2}$$

$$m = 13.43\ m^{-1}.$$

Hence, m L = (13.43)×0.1 = 1.34 and from the results of Example 3.8, it is advisable not to make the infinite rod approximation. Thus from Table 3.4, the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}\theta_b$$

Evaluating the hyperbolic functions, cosh mL = 2.04 and sinh mL = 1.78, and the parameter

$$\frac{h}{mk} = \frac{30\ W/m^2 \cdot K}{13.43m^{-1}\ (133\ W/m \cdot K)} = 0.0168,$$

with θ_b = 180°C the temperature distribution has the form

$$\theta = \frac{\cosh m(L-x) + 0.0168\ \sinh m(L-x)}{2.07}\left(180^\circ C\right).$$

The temperatures at the prescribed location are tabulated below.

x(m)	cosh m(L-x)	sinh m(L-x)	θ	T(°C)	
x_1 = 0.025	1.55	1.19	136.5	156.5	<
x_2 = 0.05	1.24	0.725	108.9	128.9	<
L = 0.10	1.00	0.00	87.0	107.0	<

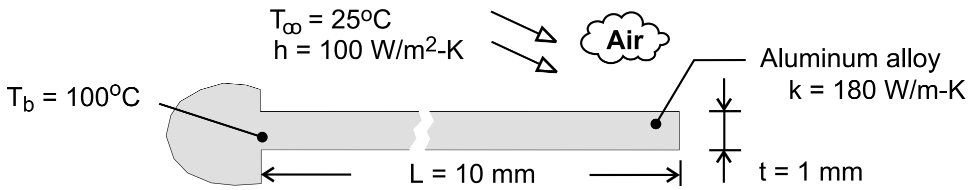
COMMENTS: If the rod were approximated as infinitely long: $T(x_1)$ = 148.7°C, $T(x_2)$ = 112.0°C, and T(L) = 67.0°C. The assumption would therefore result in significant underestimates of the rod temperature.

PROBLEM 3.121

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of convection coefficient and thermal conductivity on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.72), (3.76) and (3.80), where $M \approx (2\,hw^2tk)^{1/2}\,(T_b - T_\infty) = (2 \times 100\ W/m^2\cdot K \times 0.001m \times 180\ W/m\cdot K)^{1/2}\,(75°C)\ w = 450\ w\ W$, $m \approx (2h/kt)^{1/2} = (200\ W/m^2\cdot K/180\ W/m\cdot K \times 0.001m)^{1/2} = 33.3m^{-1}$, $mL \approx 33.3m^{-1} \times 0.010m = 0.333$, and $(h/mk) \approx (100\ W/m^2\cdot K/33.3m^{-1} \times 180\ W/m\cdot K) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.86), (3.81) and (3.83) yield

$$\eta_f \approx \frac{q_f'}{h(2L+t)\theta_b}, \quad \varepsilon_f \approx \frac{q_f'}{ht\,\theta_b}, \quad R_{t,f}' = \frac{\theta_b}{q_f'}$$

Case A: From Eq. (3.72), (3.86), (3.81), (3.83) and (3.70),

$$q_f' = \frac{M}{w}\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ W/m\,\frac{0.340 + 0.0167\times 1.057}{1.057 + 0.0167\times 0.340} = 151\ W/m \qquad <$$

$$\eta_f = \frac{151\ W/m}{100\ W/m^2\cdot K(0.021m)75°C} = 0.96 \qquad <$$

$$\varepsilon_f = \frac{151\ W/m}{100\ W/m^2\cdot K(0.001m)75°C} = 20.1,\ R_{t,f}' = \frac{75°C}{151\ W/m} = 0.50\ m\cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°C + \frac{75°C}{1.057 + (0.0167)0.340} = 95.6°C \qquad <$$

Case B: From Eqs. (3.76), (3.86), (3.81), (3.83) and (3.75)

$$q_f' = \frac{M}{w}\tanh mL = 450\ W/m(0.321) = 144\ W/m \qquad <$$

$$\eta_f = 0.92,\ \varepsilon_f = 19.2,\ R_{t,f}' = 0.52\ m\cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°C + \frac{75°C}{1.057} = 96.0°C \qquad <$$

Continued …..

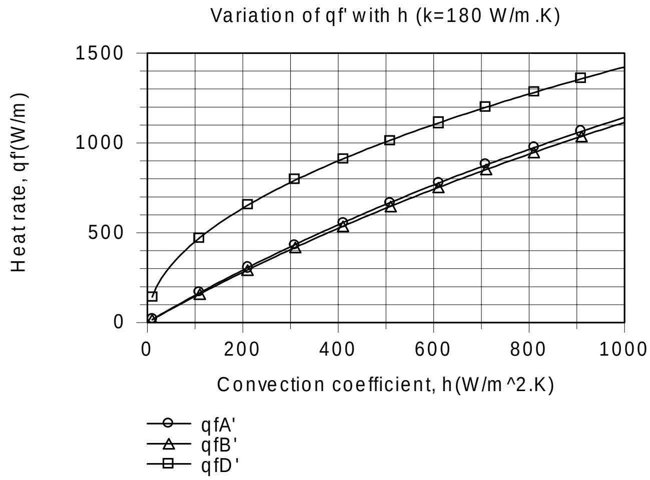
PROBLEM 3.121 (Cont.)

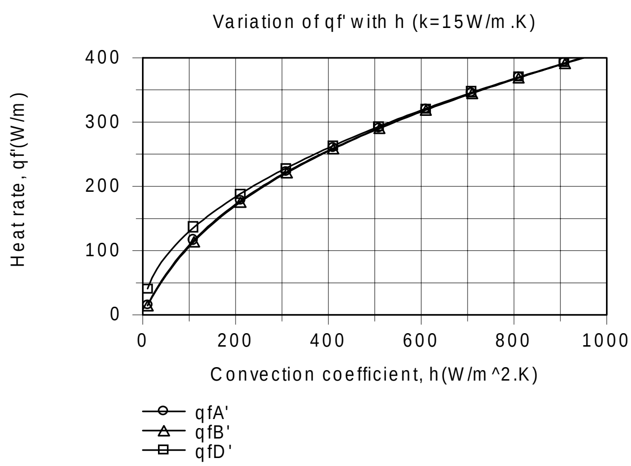
Case D (L → ∞): From Eqs. (3.80), (3.86), (3.81), (3.83) and (3.79)

$$q_f' = \frac{M}{w} = 450\,\text{W/m}$$ <

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R_{t,f}' = 0.167\,\text{m}\cdot\text{K/W},\ T(L) = T_\infty = 25°\text{C}$$ <

(b) The effect of h on the heat rate is shown below for the aluminum and stainless steel fins.





For both materials, there is little difference between the Case A and B results over the entire range of h. The difference (percentage) increases with decreasing h and increasing k, but even for the worst case condition (h = 10 $W/m^2\cdot K$, k = 180 W/m·K), the heat rate for Case A (15.7 W/m) is only slightly larger than that for Case B (14.9 W/m). For aluminum, the heat rate is significantly over-predicted by the infinite fin approximation over the entire range of h. For stainless steel, it is over-predicted for small values of h, but results for all three cases are within 1% for h > 500 $W/m^2\cdot K$.

COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q_f', η_f and ε_f, as well as a larger value of $R_{t,f}'$) associated with insulating the tip.

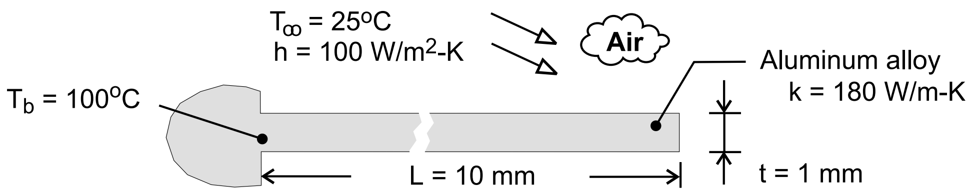
Although $\eta_f = 0$ for the infinite fin, q_f' and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.

PROBLEM 3.122

KNOWN: Thickness, length, thermal conductivity, and base temperature of a rectangular fin. Fluid temperature and convection coefficient.

FIND: (a) Heat rate per unit width, efficiency, effectiveness, thermal resistance, and tip temperature for different tip conditions, (b) Effect of fin length and thermal conductivity on the heat rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along fin, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Fin width is much longer than thickness (w >> t).

ANALYSIS: (a) The fin heat transfer rate for Cases A, B and D are given by Eqs. (3.72), (3.76) and (3.80), where $M \approx (2\,hw^2tk)^{1/2}\,(T_b - T_\infty) = (2 \times 100\ W/m^2{\cdot}K \times 0.001m \times 180\ W/m{\cdot}K)^{1/2}\ (75°C)\ w$ = 450 w W, $m \approx (2h/kt)^{1/2} = (200\ W/m^2{\cdot}K/180\ W/m{\cdot}K \times 0.001m)^{1/2} = 33.3m^{-1}$, $mL \approx 33.3m^{-1} \times 0.010m$ = 0.333, and $(h/mk) \approx (100\ W/m^2{\cdot}K/33.3m^{-1} \times 180\ W/m{\cdot}K) = 0.0167$. From Table B-1, it follows that sinh mL ≈ 0.340, cosh mL ≈ 1.057, and tanh mL ≈ 0.321. From knowledge of q_f, Eqs. (3.86), (3.81) and (3.83) yield

$$\eta_f \approx \frac{q_f'}{h(2L+t)\theta_b},\quad \varepsilon_f \approx \frac{q_f'}{ht\,\theta_b},\quad R_{t,f}' = \frac{\theta_b}{q_f'}$$

Case A: From Eq. (3.72), (3.86), (3.81), (3.83) and (3.70),

$$q_f' = \frac{M}{w}\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 450\ W/m\,\frac{0.340 + 0.0167 \times 1.057}{1.057 + 0.0167 \times 0.340} = 151\ W/m \qquad <$$

$$\eta_f = \frac{151\ W/m}{100\ W/m^2 \cdot K\,(0.021m)\,75°C} = 0.96 \qquad <$$

$$\varepsilon_f = \frac{151\ W/m}{100\ W/m^2 \cdot K\,(0.001m)\,75°C} = 20.1,\quad R_{t,f}' = \frac{75°C}{151\ W/m} = 0.50\ m \cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL + (h/mk)\sinh mL} = 25°C + \frac{75°C}{1.057 + (0.0167)0.340} = 95.6°C \qquad <$$

Case B: From Eqs. (3.76), (3.86), (3.81), (3.83) and (3.75)

$$q_f' = \frac{M}{w}\tanh mL = 450\ W/m\,(0.321) = 144\ W/m \qquad <$$

$$\eta_f = 0.92,\ \varepsilon_f = 19.2,\ R_{t,f}' = 0.52\ m \cdot K/W \qquad <$$

$$T(L) = T_\infty + \frac{\theta_b}{\cosh mL} = 25°C + \frac{75°C}{1.057} = 96.0°C \qquad <$$

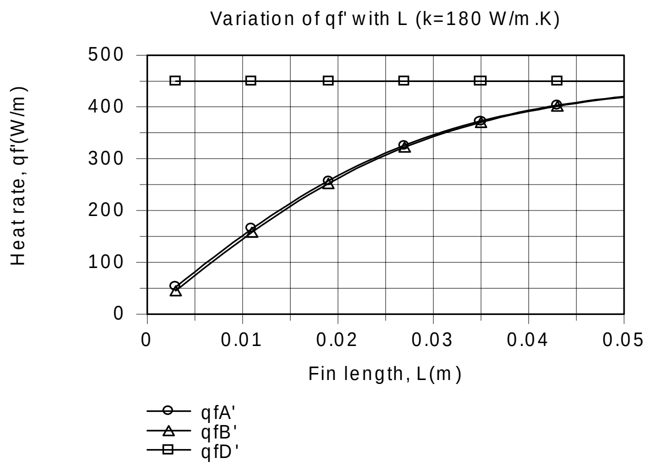
Continued …..

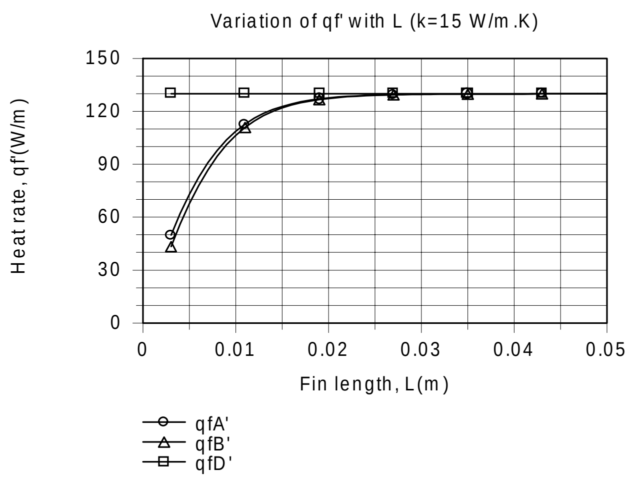
Case D (L → ∞): From Eqs. (3.80), (3.86), (3.81), (3.83) and (3.79)

$$q'_f = \frac{M}{w} = 450\,W/m \qquad <$$

$$\eta_f = 0,\ \varepsilon_f = 60.0,\ R'_{t,f} = 0.167\,m\cdot K/W,\ T(L) = T_\infty = 25°C \qquad <$$

(b) The effect of L on the heat rate is shown below for the aluminum and stainless steel fins.





For both materials, differences between the Case A and B results diminish with increasing L and are within 1% of each other at L ≈ 27 mm and L ≈ 13 mm for the aluminum and steel, respectively. At L = 3 mm, results differ by 14% and 13% for the aluminum and steel, respectively. The Case A and B results approach those of the infinite fin approximation more quickly for stainless steel due to the larger temperature gradients, |dT/dx|, for the smaller value of k.

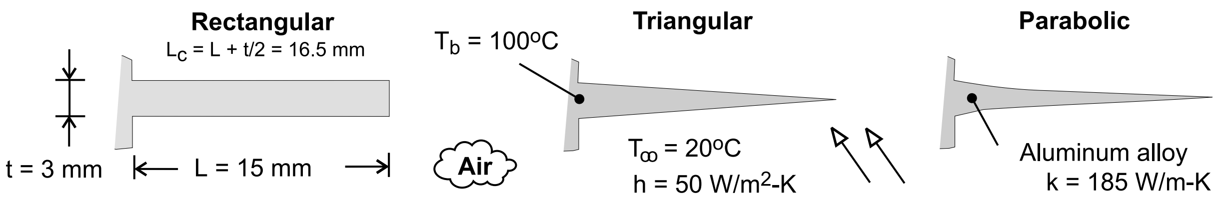
COMMENTS: From the results of Part (a), we see there is a slight reduction in performance (smaller values of q'_f, η_f and ε_f, as well as a larger value of $R'_{t,f}$) associated with insulating the tip. Although $\eta_f = 0$ for the infinite fin, q'_f and ε_f are substantially larger than results for L = 10 mm, indicating that performance may be significantly improved by increasing L.

PROBLEM 3.123

KNOWN: Length, thickness and temperature of straight fins of rectangular, triangular and parabolic profiles. Ambient air temperature and convection coefficient.

FIND: Heat rate per unit width, efficiency and volume of each fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

ANALYSIS: For each fin,

$$q'_f = q'_{max} = \eta_f \, hA'_f \, \theta_b, \qquad V' = A_p$$

where η_f depends on the value of $m = (2h/kt)^{1/2} = (100 \text{ W/m}^2\cdot\text{K}/185 \text{ W/m}\cdot\text{K} \times 0.003\text{m})^{1/2} = 13.4\text{m}^{-1}$ and the product $mL = 13.4\text{m}^{-1} \times 0.015\text{m} = 0.201$ or $mL_c = 0.222$. Expressions for η_f, A'_f and A_p are obtained from Table 3-5.

Rectangular Fin:

$$\eta_f = \frac{\tanh \, mL_c}{mL_c} = \frac{0.218}{0.222} = 0.982, \; A'_f = 2L_c = 0.033\text{m} \qquad <$$

$$q' = 0.982\left(50\,\text{W/m}^2\cdot\text{K}\right)0.033\text{m}\left(80°\text{C}\right) = 129.6\,\text{W/m}, \; V' = tL = 4.5\times10^{-5}\;\text{m}^2 \qquad <$$

Triangular Fin:

$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} = \frac{0.205}{(0.201)1.042} = 0.978, \; A'_f = 2\left[L^2 + (t/2)^2\right]^{1/2} = 0.030\text{m} \qquad <$$

$$q' = 0.978\left(50\,\text{W/m}^2\cdot\text{K}\right)0.030\text{m}\left(80°\text{C}\right) = 117.3\,\text{W/m}, \; V' = (t/2)L = 2.25\times10^{-5}\text{m}^2 \qquad <$$

Parabolic Fin:

$$\eta_f = \frac{2}{\left[4(mL)^2 + 1\right]^{1/2} + 1} = 0.963, \; A'_f = \left[C_1 L + \left(L^2/t\right)\ln\left(t/L + C_1\right)\right] = 0.030\text{m} \qquad <$$

$$q'_f = 0.963\left(50\,\text{W/m}^2\cdot\text{K}\right)0.030\text{m}\left(80°\text{C}\right) = 115.6\,\text{W/m}, \; V' = (t/3)L = 1.5\times10^{-5}\text{m}^2 \qquad <$$

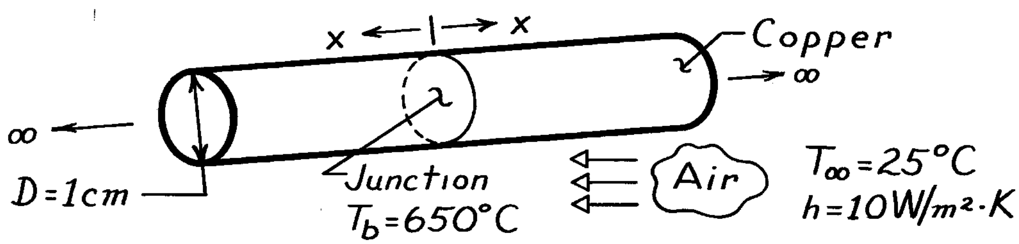
COMMENTS: Although the heat rate is slightly larger (~10%) for the rectangular fin than for the triangular or parabolic fins, the heat rate per unit volume (or mass) is larger and largest for the triangular and parabolic fins, respectively.

PROBLEM 3.124

KNOWN: Melting point of solder used to join two long copper rods.

FIND: Minimum power needed to solder the rods.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction along the rods, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation exchange with surroundings, (6) Uniform h, and (7) Infinitely long rods.

PROPERTIES: *Table A-1:* Copper $\bar{T} = (650 + 25)^\circ C \approx 600K$: $k = 379$ W/m·K.

ANALYSIS: The junction must be maintained at 650°C while energy is transferred by conduction from the junction (along both rods). The minimum power is twice the fin heat rate for an infinitely long fin,

$$q_{min} = 2q_f = 2\left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right).$$

Substituting numerical values,

$$q_{min} = 2\left[10\frac{W}{m^2 \cdot K}(\pi \times 0.01m)\left[379\frac{W}{m \cdot K}\right]\frac{\pi}{4}(0.01m)^2\right]^{1/2}(650-25)^\circ C.$$

Therefore,

$$q_{min} = 120.9 \text{ W.}$$ <

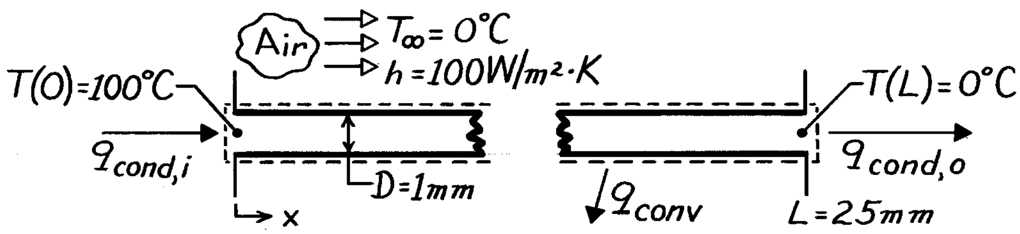
COMMENTS: Radiation losses from the rods may be significant, particularly near the junction, thereby requiring a larger power input to maintain the junction at 650°C.

PROBLEM 3.125

KNOWN: Dimensions and end temperatures of pin fins.

FIND: (a) Heat transfer by convection from a single fin and (b) Total heat transfer from a 1 m^2 surface with fins mounted on 4mm centers.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rod, (3) Constant properties, (4) No internal heat generation, (5) Negligible radiation.

PROPERTIES: *Table A-1*, Copper, pure (323K): k ≈ 400 W/m·K.

ANALYSIS: (a) By applying conservation of energy to the fin, it follows that

$$q_{conv} = q_{cond,i} - q_{cond,o}$$

where the conduction rates may be evaluated from knowledge of the temperature distribution. The general solution for the temperature distribution is

$$\theta(x) = C_1\, e^{mx} + C_2\, e^{-mx} \qquad \theta \equiv T - T_\infty .$$

The boundary conditions are $\theta(0) \equiv \theta_o$ = 100°C and $\theta(L) = 0$. Hence

$$\theta_o = C_1 + C_2$$

$$0 = C_1\, e^{mL} + C_2\, e^{-mL}$$

Therefore, $\quad C_2 = C_1\, e^{2mL}$

$$C_1 = \frac{\theta_o}{1-e^{2mL}}, \qquad C_2 = -\frac{\theta_o\, e^{2mL}}{1-e^{2mL}}$$

and the temperature distribution has the form

$$\theta = \frac{\theta_o}{1-e^{2mL}}\left[e^{mx} - e^{2mL\text{-}mx}\right].$$

The conduction heat rate can be evaluated by Fourier's law,

$$q_{cond} = -kA_c\frac{d\theta}{dx} = -\frac{kA_c\theta_o}{1-e^{2mL}}\, m\left[e^{mx} + e^{2mL\text{-}mx}\right]$$

or, with $m = \left(hP/kA_c\right)^{1/2}$,

$$q_{cond} = -\frac{\theta_o\left(hPkA_c\right)^{1/2}}{1-e^{2mL}}\left[e^{mx} + e^{2mL\text{-}mx}\right].$$

Continued …..

Hence at x = 0,

$$q_{cond,i} = -\frac{\theta_o (hPkA_c)^{1/2}}{1-e^{2mL}}\left(1+e^{2mL}\right)$$

at x = L

$$q_{cond,o} = -\frac{\theta_o (hPkA_c)^{1/2}}{1-e^{2mL}}\left(2e^{mL}\right)$$

Evaluating the fin parameters:

$$m = \left[\frac{hP}{kA_c}\right]^{1/2} = \left[\frac{4h}{kD}\right]^{1/2} = \left[\frac{4\times 100\ W/m^2\cdot K}{400\ W/m\cdot K\times 0.001m}\right]^{1/2} = 31.62\ m^{-1}$$

$$(hPkA_c)^{1/2} = \left[\frac{\pi^2}{4}D^3hk\right]^{1/2} = \left[\frac{\pi^2}{4}\times(0.001m)^3\times 100\frac{W}{m^2\cdot K}\times 400\frac{W}{m\cdot K}\right]^{1/2} = 9.93\times 10^{-3}\ \frac{W}{K}$$

$$mL = 31.62\ m^{-1}\times 0.025m = 0.791, \qquad e^{mL} = 2.204, \qquad e^{2mL} = 4.865$$

The conduction heat rates are

$$q_{cond,i} = \frac{-100K\left(9.93\times 10^{-3}\ W/K\right)}{-3.865}\times 5.865 = 1.507\ W$$

$$q_{cond,o} = \frac{-100K\left(9.93\times 10^{-3}\ W/K\right)}{-3.865}\times 4.408 = 1.133\ W$$

and from the conservation relation,

$$q_{conv} = 1.507\ W - 1.133\ W = 0.374\ W. \qquad <$$

(b) The total heat transfer rate is the heat transfer from N = 250×250 = 62,500 rods and the heat transfer from the remaining (bare) surface ($A = 1m^2 - NA_c$). Hence,

$$q = N\ q_{cond,i} + hA\theta_o = 62,500\ (1.507\ W) + 100 W/m^2\cdot K\left(0.951\ m^2\right)\ 100K$$

$$q = 9.42\times 10^4\ W + 0.95\times 10^4\ W = 1.037\times 10^5\ W.$$

COMMENTS: (1) The fins, which cover only 5% of the surface area, provide for more than 90% of the heat transfer from the surface.

(2) The fin effectiveness, $\varepsilon \equiv q_{cond,i} / hA_c\theta_o$, is ε = 192, and the fin efficiency, $\eta \equiv \left(q_{conv} / h\pi\ DL\theta_o\right)$, is $\eta = 0.48$.

(3) The temperature distribution, $\theta(x)/\theta_o$, and the conduction term, $q_{cond,i}$, could have been obtained directly from Eqs. 3.77and 3.78, respectively.

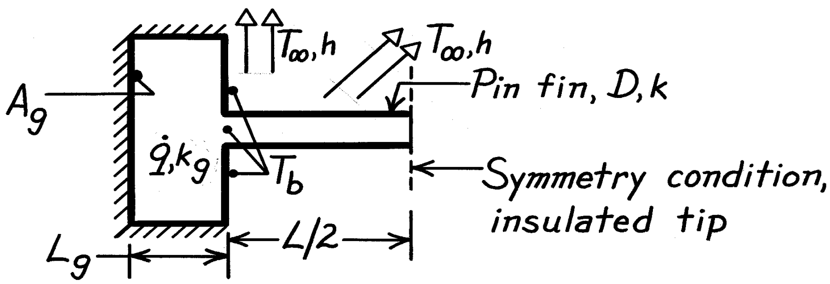
(4) Heat transfer by convection from a single fin could also have been obtained from Eq. 3.73.

PROBLEM 3.126

KNOWN: Pin fin of thermal conductivity k, length L and diameter D connecting two devices (L_g,k_g) experiencing volumetric generation of thermal energy $(\dot{q})$. Convection conditions are prescribed (T_∞, h).

FIND: Expression for the device surface temperature T_b in terms of device, convection and fin parameters.

SCHEMATIC:

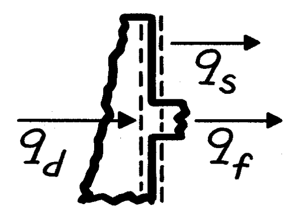


ASSUMPTIONS: (1) Steady-state conditions, (2) Pin fin is of uniform cross-section with constant h, (3) Exposed surface of device is at a uniform temperature T_b, (4) Backside of device is insulated, (5) Device experiences 1-D heat conduction with uniform volumetric generation, (6) Constant properties, and (7) No contact resistance between fin and devices.

ANALYSIS: Recognizing symmetry, the pin fin is modeled as a fin of length L/2 with insulated tip. Perform a surface energy balance,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q_d - q_s - q_f = 0 \qquad (1)$$

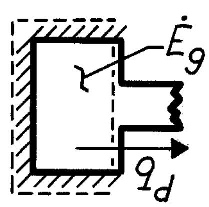


The heat rate q_d can be found from an energy balance on the entire device to find

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-q_d + \dot{q}V = 0$$

$$q_d = \dot{q}A_g L_g \qquad (2)$$



The fin heat rate, q_f, follows from Case B, Table 3.4

$$q_f = M \tanh mL/2 = \left(hPkA_c\right)^{1/2}\left(T_b - T_\infty\right)\tanh\left(mL/2\right), \qquad m = \left(hP/kA_c\right)^{1/2} \qquad (3,4)$$

$$P/A_c = \pi D/\left(\pi D^2/4\right) = 4/D \quad \text{and} \quad PA_c = \pi^2 D^3/4. \qquad (5,6)$$

Hence, the heat rate expression can be written as

$$\dot{q}A_g L_g = h\left(A_g - A_c\right)\left(T_b - T_\infty\right) + \left(hk\left(\pi^2 D^3/4\right)\right)^{1/2}\tanh\left(\left(\frac{4h}{kD}\right)^{1/2}\cdot\frac{L}{2}\right)\left(T_b - T_\infty\right) \qquad (7)$$

Solve now for T_b,

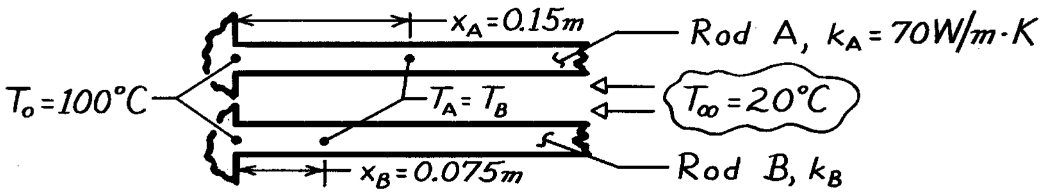
$$T_b = T_\infty + \dot{q}A_g L_g \Big/ \left[h\left(A_g - A_c\right) + \left(hk\left(\pi^2 D^3/4\right)\right)^{1/2}\tanh\left(\left(\frac{4h}{kD}\right)^{1/2}\cdot\frac{L}{2}\right)\right] \qquad (8) <$$

PROBLEM 3.127

KNOWN: Positions of equal temperature on two long rods of the same diameter, but different thermal conductivity, which are exposed to the same base temperature and ambient air conditions.

FIND: Thermal conductivity of rod B, k_B.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Rods are infinitely long fins of uniform cross-sectional area, (3) Uniform heat transfer coefficient, (4) Constant properties.

ANALYSIS: The temperature distribution for the infinite fin has the form

$$\frac{\theta}{\theta_b} = \frac{T(x) - T_\infty}{T_o - T_\infty} = e^{-mx} \qquad m = \left[\frac{hP}{kA_c}\right]^{1/2}. \qquad (1,2)$$

For the two positions prescribed, x_A and x_B, it was observed that

$$T_A(x_A) = T_B(x_B) \qquad \text{or} \qquad \theta_A(x_A) = \theta_B(x_B). \qquad (3)$$

Since θ_b is identical for both rods, Eq. (1) with the equality of Eq. (3) requires that

$$m_A x_A = m_B x_B$$

Substituting for m from Eq. (2) gives

$$\left[\frac{hP}{k_A A_c}\right]^{1/2} x_A = \left[\frac{hP}{k_B A_c}\right]^{1/2} x_B.$$

Recognizing that h, P and A_c are identical for each rod and rearranging,

$$k_B = \left[\frac{x_B}{x_A}\right]^2 k_A$$

$$k_B = \left[\frac{0.075\text{m}}{0.15\text{m}}\right]^2 \times 70 \text{ W/m}\cdot\text{K} = 17.5 \text{ W/m}\cdot\text{K}.$$ <

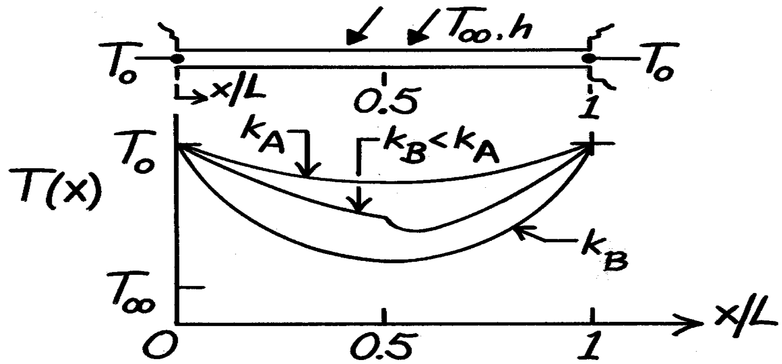
COMMENTS: This approach has been used as a method for determining the thermal conductivity. It has the attractive feature of not requiring power or temperature measurements, assuming of course, a reference material of known thermal conductivity is available.

PROBLEM 3.128

KNOWN: Slender rod of length L with ends maintained at T_o while exposed to convection cooling ($T_\infty < T_o$, h).

FIND: Temperature distribution for three cases, when rod has thermal conductivity (a) k_A, (b) $k_B < k_A$, and (c) k_A for $0 \le x \le L/2$ and k_B for $L/2 \le x \le L$.

SCHEMATIC:



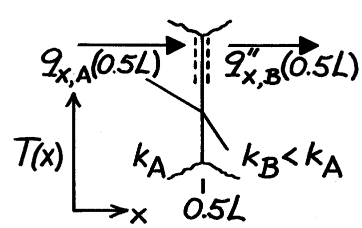
ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, and (4) Negligible thermal resistance between the two materials (A, B) at the mid-span for case (c).

ANALYSIS: (a, b) The effect of thermal conductivity on the temperature distribution when all other conditions (T_o, h, L) remain the same is to reduce the minimum temperature with decreasing thermal conductivity. Hence, as shown in the sketch, the mid-span temperatures are $T_B\,(0.5L) < T_A\,(0.5L)$ for $k_B < k_A$. The temperature distribution is, of course, symmetrical about the mid-span.

(c) For the composite rod, the temperature distribution can be reasoned by considering the boundary condition at the mid-span.

$$q''_{x,A}\left(0.5L\right) = q''_{x,B}\left(0.5L\right)$$

$$-k_A \left.\frac{dT}{dx}\right)_{A,x=0.5L} = -k_B \left.\frac{dT}{dx}\right)_{B,x=0.5L}$$



Since $k_A > k_B$, it follows that

$$\left(\frac{dT}{dx}\right)_{A,x=0.5L} < \left(\frac{dT}{dx}\right)_{B,x=0.5L}.$$

It follows that the minimum temperature in the rod must be in the k_B region, $x > 0.5L$, and the temperature distribution is not symmetrical about the mid-span.

COMMENTS: (1) Recognize that the area under the curve on the T-x coordinates is proportional to the fin heat rate. What conclusions can you draw regarding the relative magnitudes of q_{fin} for cases (a), (b) and (c)?

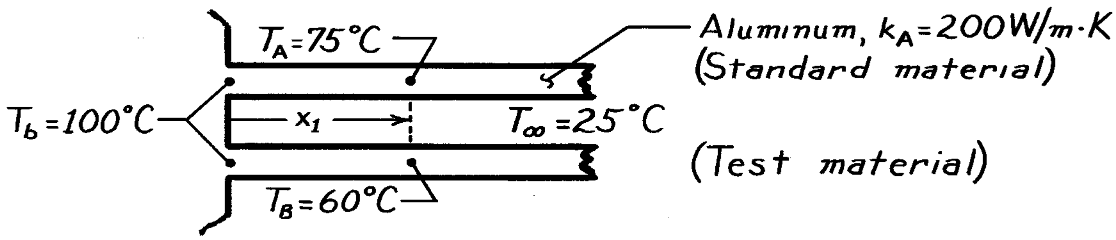
(2) If L is increased substantially, how would the temperature distribution be affected?

PROBLEM 3.129

KNOWN: Base temperature, ambient fluid conditions, and temperatures at a prescribed distance from the base for two long rods, with one of known thermal conductivity.

FIND: Thermal conductivity of other rod.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction along rods, (3) Constant properties, (4) Negligible radiation, (5) Negligible contact resistance at base, (6) Infinitely long rods, (7) Rods are identical except for their thermal conductivity.

ANALYSIS: With the assumption of infinitely long rods, the temperature distribution is

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

or

$$\ln \frac{T - T_\infty}{T_b - T_\infty} = -mx = \left[\frac{hP}{kA}\right]^{1/2} x$$

Hence, for the two rods,

$$\frac{\ln\left[\frac{T_A - T_\infty}{T_b - T_\infty}\right]}{\ln\left[\frac{T_B - T_\infty}{T_b - T_\infty}\right]} = \left[\frac{k_B}{k_A}\right]^{1/2}$$

$$k_B^{1/2} = k_A^{1/2} \frac{\ln\left[\frac{T_A - T_\infty}{T_b - T_\infty}\right]}{\ln\left[\frac{T_B - T_\infty}{T_b - T_\infty}\right]} = (200)^{1/2} \frac{\ln \frac{75-25}{100-25}}{\ln \frac{60-25}{100-25}} = 7.524$$

$$k_B = 56.6 \text{ W/m} \cdot \text{K}.$$ <

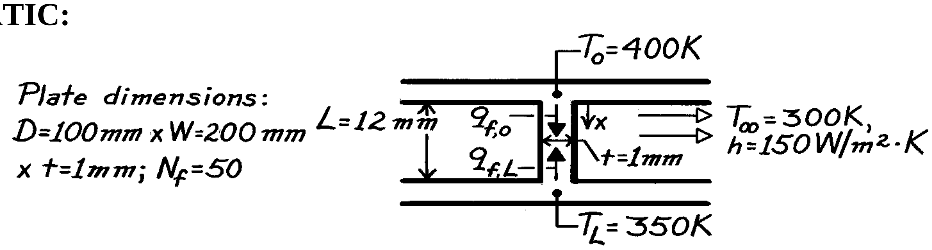
COMMENTS: Providing conditions for the two rods may be maintained nearly identical, the above method provides a convenient means of measuring the thermal conductivity of solids.

PROBLEM 3.130

KNOWN: Arrangement of fins between parallel plates. Temperature and convection coefficient of air flow in finned passages. Maximum allowable plate temperatures.

FIND: (a) Expressions relating fin heat transfer rates to end temperatures, (b) Maximum power dissipation for each plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) All of the heat is dissipated to the air, (6) Uniform h, (7) Negligible variation in T_∞, (8) Negligible contact resistance.

PROPERTIES: *Table A.1,* Aluminum (pure), 375 K: k = 240 W/m·K.

ANALYSIS: (a) The general solution for the temperature distribution in fin is

$$\theta(x) \equiv T(x) - T_\infty = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary conditions: $\theta(0) = \theta_o = T_o - T_\infty$, $\quad \theta(L) = \theta_L = T_L - T_\infty$.

Hence $\quad \theta_o = C_1 + C_2 \qquad \theta_L = C_1 e^{mL} + C_2 e^{-mL}$

$$\theta_L = C_1 e^{mL} + (\theta_o - C_1) e^{-mL}$$

$$C_1 = \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} \qquad C_2 = \theta_o - \frac{\theta_L - \theta_o e^{-mL}}{e^{mL} - e^{-mL}} = \frac{\theta_o e^{mL} - \theta_L}{e^{mL} - e^{-mL}}.$$

Hence
$$\theta(x) = \frac{\theta_L e^{mx} - \theta_o e^{m(x-L)} + \theta_o e^{m(L-x)} - \theta_L e^{-mx}}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \left[e^{m(L-x)} - e^{-m(L-x)}\right] + \theta_L \left(e^{mx} - e^{-mx}\right)}{e^{mL} - e^{-mL}}$$

$$\theta(x) = \frac{\theta_o \sinh m(L-x) + \theta_L \sinh mx}{\sinh mL}.$$

The fin heat transfer rate is then

$$q_f = -kA_c \frac{dT}{dx} = -kDt\left[-\frac{\theta_o m}{\sinh mL}\cosh m(L-x) + \frac{\theta_L m}{\sinh mL}\cosh mx\right].$$

Hence
$$q_{f,o} = kDt\left(\frac{\theta_o m}{\tanh mL} - \frac{\theta_L m}{\sinh mL}\right) \qquad <$$

$$q_{f,L} = kDt\left(\frac{\theta_o m}{\sinh mL} - \frac{\theta_L m}{\tanh mL}\right). \qquad <$$

Continued …..

(b)
$$m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left(\frac{50\ \text{W/m}^2\cdot\text{K}\left(2\times0.1\ \text{m}+2\times0.001\ \text{m}\right)}{240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}}\right)^{1/2} = 35.5\ \text{m}^{-1}$$

$$mL = 35.5\ \text{m}^{-1}\times0.012\ \text{m} = 0.43$$

$$\sinh\ mL = 0.439 \qquad \tanh\ mL = 0.401 \qquad \theta_o = 100\ \text{K} \qquad \theta_L = 50\ \text{K}$$

$$q_{f,o} = 240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}\right)$$

$q_{f,o} = 115.4\ \text{W}$ (*from* the top plate)

$$q_{f,L} = 240\ \text{W/m}\cdot\text{K}\times0.1\ \text{m}\times0.001\ \text{m}\left(\frac{100\ \text{K}\times35.5\ \text{m}^{-1}}{0.439}-\frac{50\ \text{K}\times35.5\ \text{m}^{-1}}{0.401}\right)$$

$q_{f,L} = 87.8\ \text{W.}$ (*into* the bottom plate)

Maximum power dissipations are therefore

$$q_{o,max} = N_f q_{f,o} + \left(W - N_f t\right)Dh\theta_o$$

$$q_{o,max} = 50\times115.4\ \text{W}+\left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times100\ \text{K}$$

$$q_{o,max} = 5770\ \text{W}+225\ \text{W} = 5995\ \text{W} \qquad <$$

$$q_{L,max} = -N_f q_{f,L} + \left(W - N_f t\right)Dh\theta_o$$

$$q_{L,max} = -50\times87.8\text{W} + \left(0.200-50\times0.001\right)\text{m}\times0.1\ \text{m}\times150\ \text{W/m}^2\cdot\text{K}\times50\ \text{K}$$

$$q_{L,max} = -4390\ \text{W}+112\text{W} = -4278\ \text{W.} \qquad <$$

COMMENTS: (1) It is of interest to determine the air velocity needed to prevent excessive heating of the air as it passes between the plates. If the air temperature change is restricted to $\Delta T_\infty = 5$ K, its flowrate must be

$$\dot{m}_{air} = \frac{q_{tot}}{c_p\Delta T_\infty} = \frac{1717\ \text{W}}{1007\ \text{J/kg}\cdot\text{K}\times5\ \text{K}} = 0.34\ \text{kg/s.}$$

Its mean velocity is then

$$V_{air} = \frac{\dot{m}_{air}}{\rho_{air}A_c} = \frac{0.34\ \text{kg/s}}{1.16\ \text{kg/m}^3\times0.012\ \text{m}\left(0.2-50\times0.001\right)\text{m}} = 163\ \text{m/s.}$$

Such a velocity would be impossible to maintain. To reduce it to a reasonable value, e.g. 10 m/s, A_c would have to be increased substantially by increasing W (and hence the space between fins) and by increasing L. The present configuration is impractical from the standpoint that 1717 W could not be transferred to air in such a small volume.

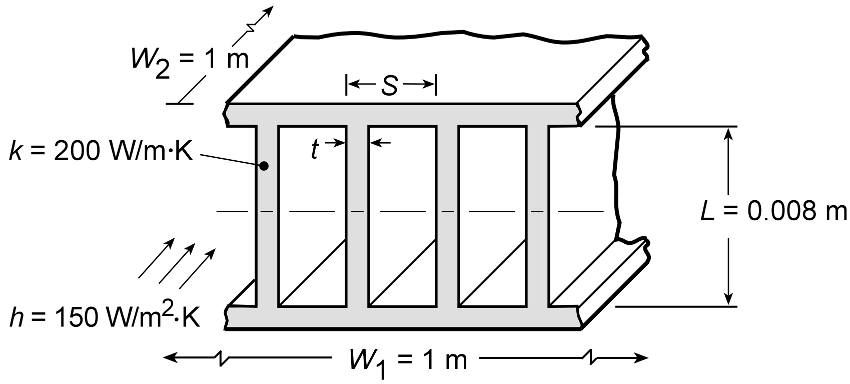
(2) A negative value of $q_{L,max}$ implies that heat must be transferred from the bottom plate to the air to maintain the plate at 350 K.

PROBLEM 3.131

KNOWN: Conditions associated with an array of straight rectangular fins.

FIND: Thermal resistance of the array.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Uniform convection coefficient, (3) Symmetry about midplane.

ANALYSIS: (a) Considering a one-half section of the array, the corresponding resistance is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $A_t = NA_f + A_b$. With S = 4 mm and t = 1 mm, it follows that N = W_1/S = 250, A_f = $2(L/2)W_2 = 0.008\ m^2$, $A_b = W_2(W_1 - Nt) = 0.75\ m^2$, and $A_t = 2.75\ m^2$.

The overall surface efficiency is

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f)$$

where the fin efficiency is

$$\eta_f = \frac{\tanh m(L/2)}{m(L/2)} \qquad \text{and} \qquad m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left[\frac{h(2t + 2W_2)}{ktW_2}\right]^{1/2} \approx \left(\frac{2h}{kt}\right)^{1/2} = 38.7\text{m}^{-1}$$

With m(L/2) = 0.155, it follows that η_f = 0.992 and η_o = 0.994. Hence

$$R_{t,o} = \left(0.994 \times 150\text{W/m}^2 \cdot \text{K} \times 2.75\text{m}^2\right)^{-1} = 2.44 \times 10^{-3}\text{K/W}$$ <

(b) The requirements that t ≥ 0.5 m and (S - t) > 2 mm are based on manufacturing and flow passage restriction constraints. Repeating the foregoing calculations for representative values of t and (S - t), we obtain

S (mm)	N	t (mm)	$R_{t,o}$ (K/W)
2.5	400	0.5	0.00169
3	333	0.5	0.00193
3	333	1	0.00202
4	250	0.5	0.00234
4	250	2	0.00268
5	200	0.5	0.00264
5	200	3	0.00334

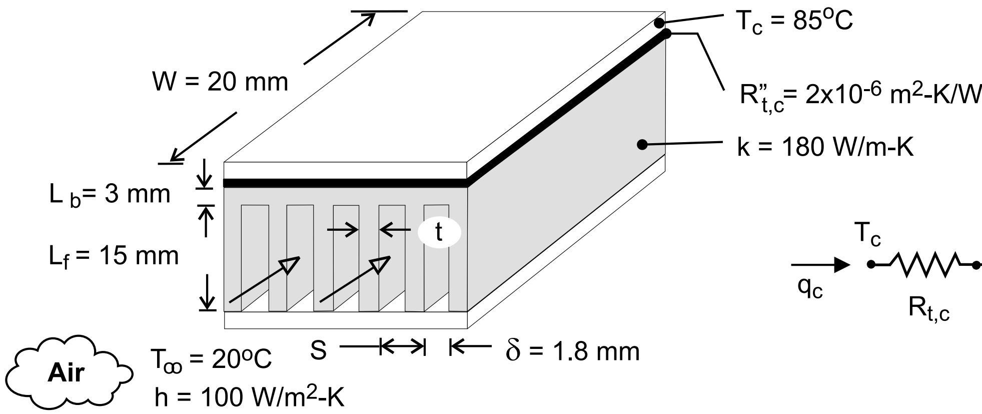
COMMENTS: Clearly, the thermal performance of the fin array improves ($R_{t,o}$ decreases) with increasing N. Because $\eta_f \approx 1$ for the entire range of conditions, there is a slight degradation in performance ($R_{t,o}$ increases) with increasing t and fixed N. The reduced performance is associated with the reduction in surface area of the exposed base. Note that the overall thermal resistance for the entire fin array (top and bottom) is $R_{t,o}/2 = 1.22 \times 10^{-2}$ K/W.

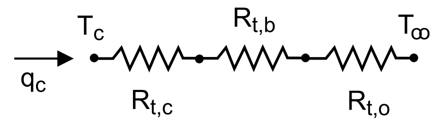
PROBLEM 3.132

KNOWN: Width and maximum allowable temperature of an electronic chip. Thermal contact resistance between chip and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through the heat sink.

FIND: (a) Maximum allowable chip power for heat sink with prescribed number of fins, fin thickness, and fin pitch, and (b) Effect of fin thickness/number and convection coefficient on performance.

SCHEMATIC:





ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surfaces of heat sink, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{tot}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

where $R_{t,c} = R''_{t,c}/W^2 = 2\times10^{-6}\,m^2 \cdot K/W/(0.02m)^2 = 0.005\ K/W$ and $R_{t,b} = L_b/k\left(W^2\right)$

$= 0.003m/180\ W/m\cdot K(0.02m)^2 = 0.042\ K/W$. From Eqs. (3.103), (3.102), and (3.99)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \qquad \eta_o = 1 - \frac{N A_f}{A_t}(1-\eta_f), \qquad A_t = N A_f + A_b$$

where $A_f = 2WL_f = 2 \times 0.02m \times 0.015m = 6 \times 10^{-4}\ m^2$ and $A_b = W^2 - N(tW) = (0.02m)^2 - 11(0.182 \times 10^{-3}\ m \times 0.02m) = 3.6 \times 10^{-4}\ m^2$. With $mL_f = (2h/kt)^{1/2} L_f = (200\ W/m^2 \cdot K/180\ W/m\cdot K \times 0.182 \times 10^{-3} m)^{1/2} (0.015m) = 1.17$, $\tanh mL_f = 0.824$ and Eq. (3.87) yields

$$\eta_f = \frac{\tanh mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

It follows that $A_t = 6.96 \times 10^{-3}\ m^2$, $\eta_o = 0.719$, $R_{t,o} = 2.00$ K/W, and

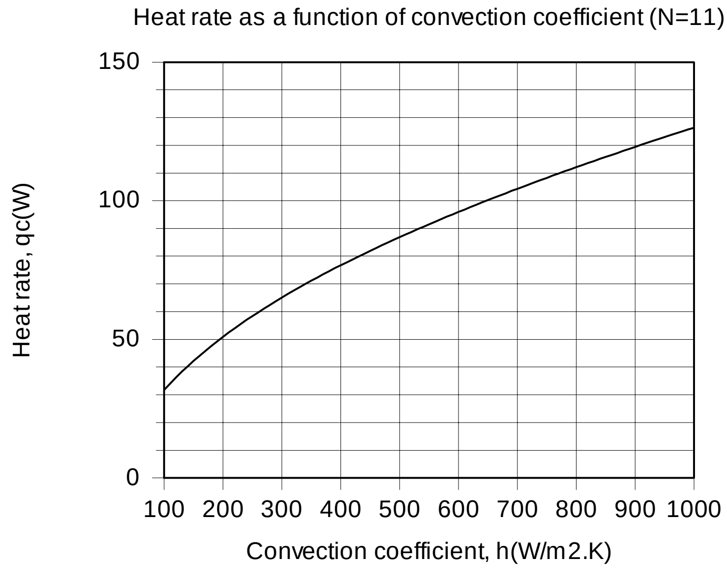
$$q_c = \frac{(85-20)°C}{(0.005+0.042+2.00)K/W} = 31.8\ W \qquad <$$

(b) The following results are obtained from parametric calculations performed to explore the effect of decreasing the number of fins and increasing the fin thickness.

Continued …..

N	t(mm)	η_f	$R_{t,o}$ (K/W)	q_c (W)	A_t (m^2)
6	1.833	0.957	2.76	23.2	0.00378
7	1.314	0.941	2.40	26.6	0.00442
8	0.925	0.919	2.15	29.7	0.00505
9	0.622	0.885	1.97	32.2	0.00569
10	0.380	0.826	1.89	33.5	0.00632
11	0.182	0.704	2.00	31.8	0.00696

Although η_f (and η_o) increases with decreasing N (increasing t), there is a reduction in A_t which yields a minimum in $R_{t,o}$, and hence a maximum value of q_c, for N = 10. For N = 11, the effect of h on the performance of the heat sink is shown below.



With increasing h from 100 to 1000 W/m^2·K, $R_{t,o}$ decreases from 2.00 to 0.47 K/W, despite a decrease in η_f (and η_o) from 0.704 (0.719) to 0.269 (0.309). The corresponding increase in q_c is significant.

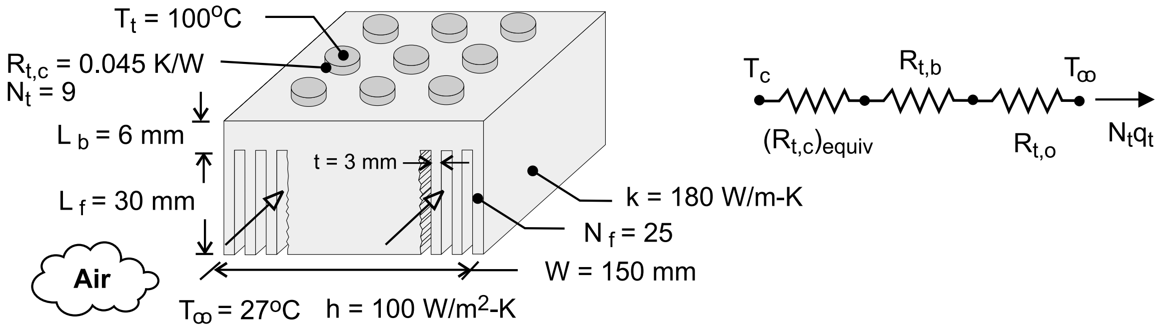
COMMENTS: The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with h = 100 W/m^2·K, R_{tot} = 2.05 K/W from Part (a) would be replaced by $R_{cnv} = 1/hW^2$ = 25 K/W, yielding q_c = 2.60 W.

PROBLEM 3.133

KNOWN: Number and maximum allowable temperature of power transistors. Contact resistance between transistors and heat sink. Dimensions and thermal conductivity of heat sink. Temperature and convection coefficient associated with air flow through and along the sides of the heat sink.

FIND: (a) Maximum allowable power dissipation per transistor, (b) Effect of the convection coefficient and fin length on the transistor power.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal transistors, (4) Negligible heat transfer from top surface of heat sink (all heat transfer is through the heat sink), (5) Negligible temperature rise for the air flow, (6) Uniform convection coefficient, (7) Negligible radiation.

ANALYSIS: (a) From the thermal circuit,

$$N_t\, q_t = \frac{T_t - T_\infty}{\left(R_{t,c}\right)_{equiv} + R_{t,b} + R_{t,o}}$$

For the array of transistors, the corresponding contact resistance is the equivalent resistance associated with the component resistances, in which case,

$$\left(R_{t,c}\right)_{equiv} = \left[N_t\left(1/R_{t,c}\right)\right]^{-1} = \left(9/0.045\,K/W\right)^{-1} = 5\times10^{-3}\,K/W$$

The thermal resistance associated with the base of the heat sink is

$$R_{t,b} = \frac{L_b}{k(W)^2} = \frac{0.006m}{180\;W/m\cdot K\left(0.150m\right)^2} = 1.48\times10^{-3}\;K/W$$

From Eqs. (3.103), (3.102) and (3.99), the thermal resistance associated with the fin array and the corresponding overall efficiency and total surface area are

$$R_{t,o} = \frac{1}{\eta_o\, h\, A_t}, \qquad \eta_o = 1 - \frac{N_f A_f}{A_t}\left(1-\eta_f\right), \qquad A_t = N_f A_f + A_b$$

Each fin has a surface area of $A_f \approx 2\,W\,L_f = 2 \times 0.15m \times 0.03m = 9 \times 10^{-3}\,m^2$, and the area of the exposed base is $A_b = W^2 - N_f\,(tW) = (0.15m)^2 - 25\,(0.003m \times 0.15m) = 1.13 \times 10^{-2}\,m^2$. With $mL_f = (2h/kt)^{1/2}\,L_f = (200\,W/m^2{\cdot}K/180\,W/m{\cdot}K \times 0.003m)^{1/2}\,(0.03m) = 0.577$, $\tanh mL_f = 0.520$ and Eq. (3.87) yields

$$\eta_f = \frac{\tanh\; mL_f}{mL_f} = \frac{0.520}{0.577} = 0.902$$

Hence, with $A_t = [25\,(9 \times 10^{-3}) + 1.13 \times 10^{-2}]m^2 = 0.236m^2$,

Continued …..

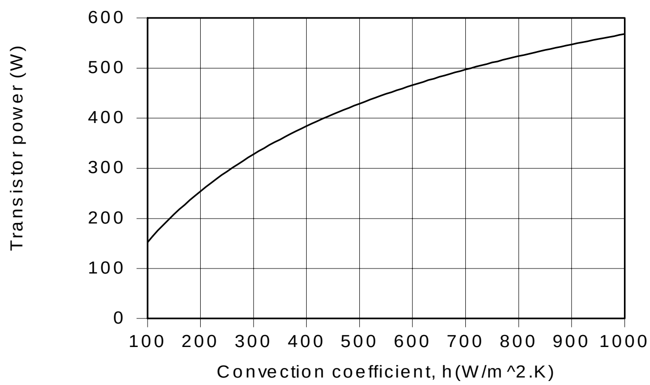
$$\eta_o = 1 - \frac{25\left(0.009\text{m}^2\right)}{0.236\text{m}^2}(1-0.901) = 0.907$$

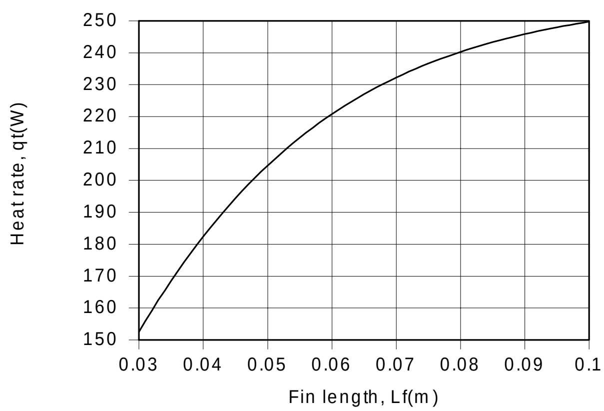
$$R_{t,o} = \left(0.907 \times 100\ \text{W/m}^2 \cdot \text{K} \times 0.236\text{m}^2\right)^{-1} = 0.0467\ \text{K/W}$$

The heat rate per transistor is then

$$q_t = \frac{1}{9}\frac{(100-27)°\text{C}}{(0.0050+0.0015+0.0467)\text{K/W}} = 152\ \text{W}$$ <

(b) As shown below, the transistor power dissipation may be enhanced by increasing h and/or L_f.





However, in each case, the effect of the increase diminishes due to an attendant reduction in η_f. For example, as h increases from 100 to 1000 $\text{W/m}^2\cdot\text{K}$ for L_f = 30 mm, η_f decreases from 0.902 to 0.498.

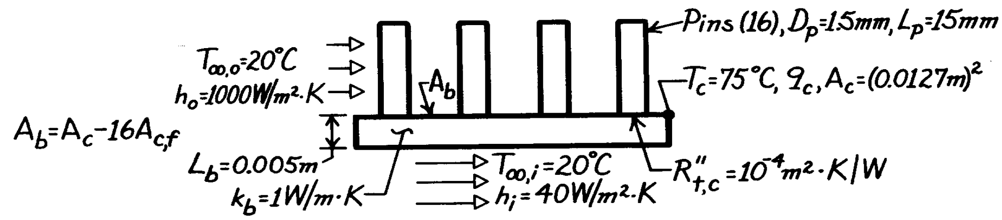
COMMENTS: The heat sink significantly increases the allowable transistor power. If it were not used and heat was simply transferred from a surface of area W^2 = 0.0225 m^2 with h = 100 $\text{W/m}^2\cdot\text{K}$, the corresponding thermal resistance would be $R_{t,cnv} = (hW^2)^{-1}$ K/W = 0.44 and the transistor power would be $q_t = (T_t - T_\infty)/N_t\, R_{t,cnv}$ = 18.4 W.

PROBLEM 3.134

KNOWN: Geometry and cooling arrangement for a chip-circuit board arrangement. Maximum chip temperature.

FIND: (a) Equivalent thermal circuit, (b) Maximum chip heat rate.

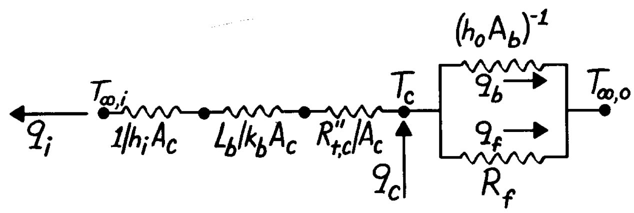
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-board assembly, (3) Negligible pin-chip contact resistance, (4) Constant properties, (5) Negligible chip thermal resistance, (6) Uniform chip temperature.

PROPERTIES: *Table A.1,* Copper (300 K): k ≈ 400 W/m·K.

ANALYSIS: (a) The thermal circuit is



$$R_f = \frac{\theta_b}{16q_f} = \frac{\cosh mL + (h_o/mk)\sinh mL}{16(h_oPkA_{c,f})^{1/2}[\sinh mL + (h_o/mk)\cosh mL]}$$

(b) The maximum chip heat rate is

$$q_c = 16q_f + q_b + q_i.$$

Evaluate these parameters

$$m = \left(\frac{h_oP}{kA_{c,f}}\right)^{1/2} = \left(\frac{4h_o}{kD_p}\right)^{1/2} = \left(\frac{4\times1000\ \text{W/m}^2\cdot\text{K}}{400\ \text{W/m}\cdot\text{K}\times0.0015\ \text{m}}\right)^{1/2} = 81.7\ \text{m}^{-1}$$

$$mL = \left(81.7\ \text{m}^{-1}\times0.015\ \text{m}\right) = 1.23, \quad \sinh mL = 1.57, \quad \cosh mL = 1.86$$

$$(h/mk) = \frac{1000\ \text{W/m}^2\cdot\text{K}}{81.7\ \text{m}^{-1}\times400\ \text{W/m}\cdot\text{K}} = 0.0306$$

$$M = \left(h_o\pi\ D_pk\pi\ D_p^2/4\right)^{1/2}\theta_b$$

$$M = \left[1000\ \text{W/m}^2\cdot\text{K}\left(\pi^2/4\right)(0.0015\ \text{m})^3\ 400\ \text{W/m}\cdot\text{K}\right]^{1/2}\left(55^\circ\text{C}\right) = 3.17\ \text{W}.$$

Continued …..

The fin heat rate is

$$q_f = M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} = 3.17\ \text{W}\frac{1.57 + 0.0306 \times 1.86}{1.86 + 0.0306 \times 1.57}$$

$$q_f = 2.703\ \text{W}.$$

The heat rate from the board by convection is

$$q_b = h_o A_b \theta_b = 1000\ \text{W/m}^2 \cdot \text{K}\left[(0.0127\ \text{m})^2 - (16\pi/4)(0.0015\ \text{m})^2\right]55^\circ\text{C}$$

$$q_b = 7.32\ \text{W}.$$

The convection heat rate is

$$q_i = \frac{T_c - T_{\infty,i}}{\left(1/h_i + R''_{t,c} + L_b/k_b\right)(1/A_c)} = \frac{(0.0127\ \text{m})^2 (55^\circ\text{C})}{\left(1/40 + 10^{-4} + 0.005/1\right)\text{m}^2 \cdot \text{K/W}}$$

$$q_i = 0.29\ \text{W}.$$

Hence, the maximum chip heat rate is

$$q_c = [16(2.703) + 7.32 + 0.29]\ \text{W} = [43.25 + 7.32 + 0.29]\ \text{W}$$

$$q_c = 50.9\ \text{W}.$$ <

COMMENTS: (1) The fins are extremely effective in enhancing heat transfer from the chip (assuming negligible contact resistance). Their effectiveness is $\varepsilon = q_f / \left(\pi\ D_p^2/4\right) h_o \theta_b = 2.703$ W/0.097 W = 27.8

(2) Without the fins, q_c = 1000 W/m^2·K(0.0127 m)^{2}55°C + 0.29 W = 9.16 W. Hence the fins provide for a (50.9 W/9.16 W) × 100% = 555% enhancement of heat transfer.

(3) With the fins, the chip heat flux is 50.9 W/(0.0127 m)^2 or $q''_c = 3.16 \times 10^5\ \text{W/m}^2 = 31.6$ W/cm^2.

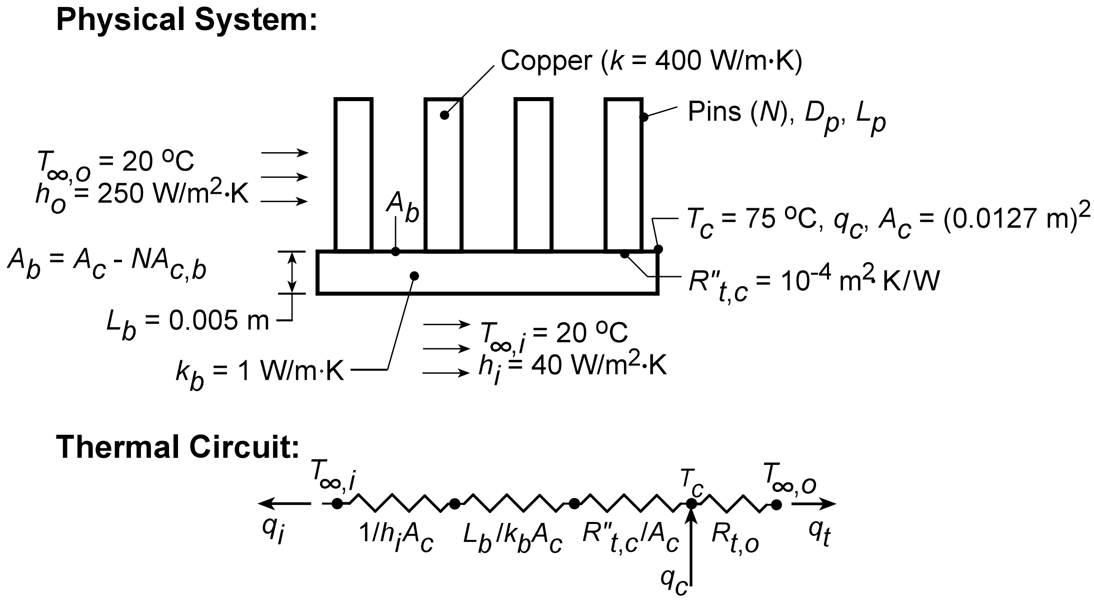
(4) If the infinite fin approximation is made, q_f = M = 3.17 W, and the actual fin heat transfer is overestimated by 17%.

PROBLEM 3.135

KNOWN: Geometry of pin fin array used as heat sink for a computer chip. Array convection and chip substrate conditions.

FIND: Effect of pin diameter, spacing and length on maximum allowable chip power dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-board assembly, (3) Negligible pin-chip contact resistance, (4) Constant properties, (5) Negligible chip thermal resistance, (6) Uniform chip temperature.

ANALYSIS: The total power dissipation is $q_c = q_i + q_t$, where

$$q_i = \frac{T_c - T_{\infty,i}}{\left(1/h_i + R''_{t,c} + L_b/k_b\right)/A_c} = 0.3\text{W}$$

and

$$q_t = \frac{T_c - T_{\infty,o}}{R_{t,o}}$$

The resistance of the pin array is

$$R_{t,o} = \left(\eta_o h_o A_t\right)^{-1}$$

where

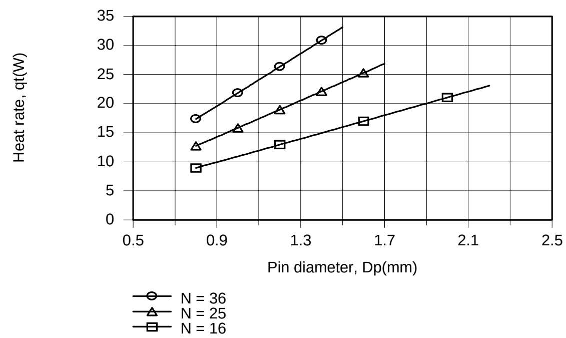
$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right)$$

$$A_t = NA_f + A_b$$

$$A_f = \pi D_p L_c = \pi D_p \left(L_p + D_p/4\right)$$

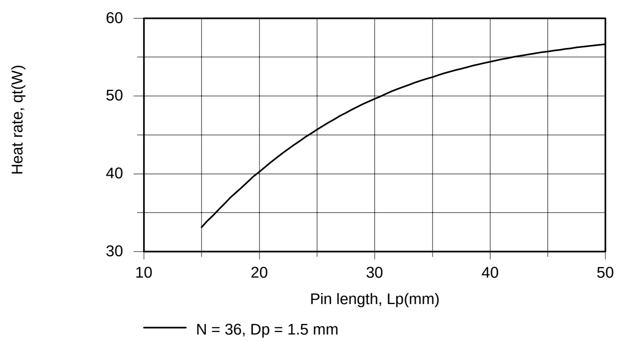
Subject to the constraint that $N^{1/2} D_p \le 9$ mm, the foregoing expressions may be used to compute q_t as a function of D_p for L_p = 15 mm and values of N = 16, 25 and 36. Using the IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array*, we obtain

Continued...



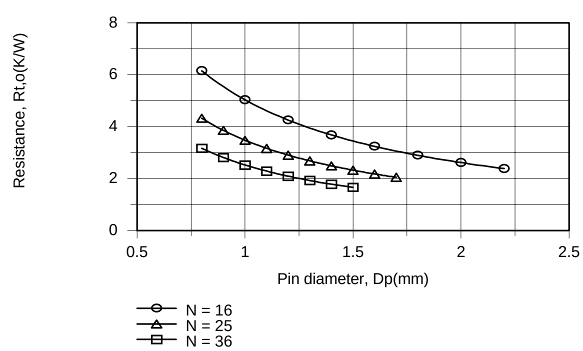
Clearly, it is desirable to maximize the number of pins and the pin diameter, so long as flow passages are not constricted to the point of requiring an excessive pressure drop to maintain the prescribed convection coefficient. The maximum heat rate for the fin array (q_t = 33.1 W) corresponds to N = 36 and D_p = 1.5 mm. Further improvement could be obtained by using N = 49 pins of diameter D_p = 1.286 mm, which yield q_t = 37.7 W.

Exploring the effect of L_p for N = 36 and D_p = 1.5 mm, we obtain



Clearly, there are benefits to increasing L_p, although the effect diminishes due to an attendant reduction in η_f (from η_f = 0.887 for L_p = 15 mm to η_f = 0.471 for L_p = 50 mm). Although a heat dissipation rate of q_t = 56.7 W is obtained for L_p = 50 mm, package volume constraints could preclude such a large fin length.

COMMENTS: By increasing N, D_p and/or L_p, the total surface area of the array, A_t, is increased, thereby reducing the array thermal resistance, $R_{t,o}$. The effects of D_p and N are shown for L_p = 15 mm.

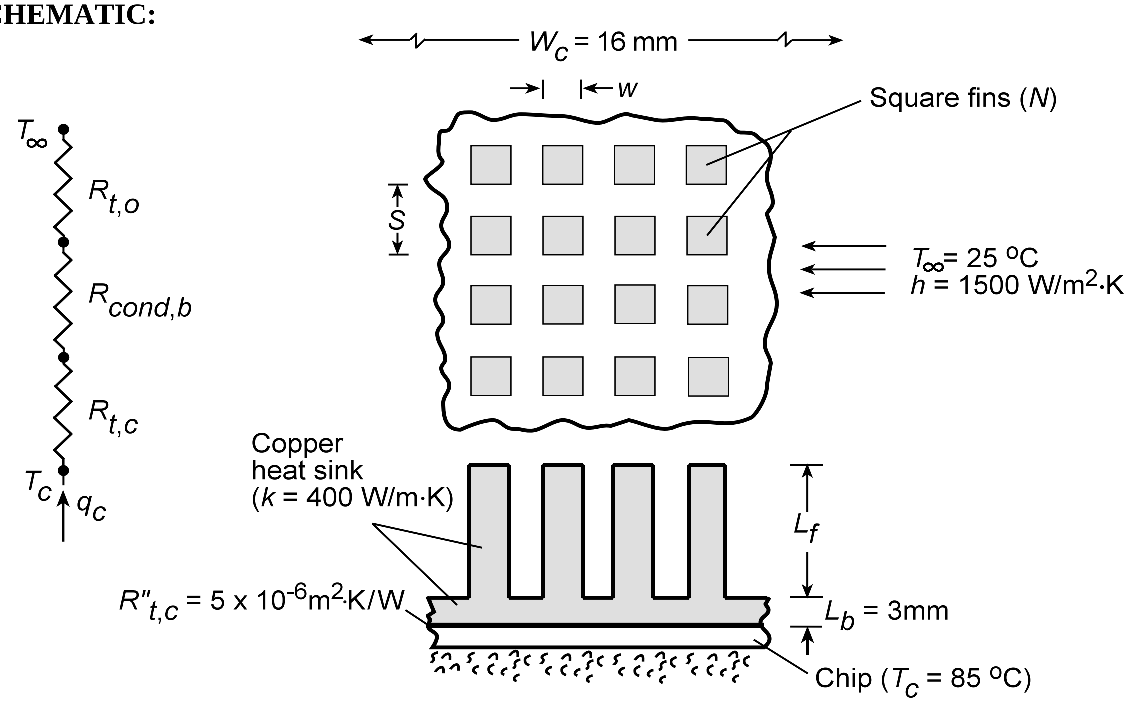


PROBLEM 3.136

KNOWN: Copper heat sink dimensions and convection conditions.

FIND: (a) Maximum allowable heat dissipation for a prescribed chip temperature and interfacial chip/heat-sink contact resistance, (b) Effect of fin length and width on heat dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-heat sink assembly, (3) Constant k, (4) Negligible chip thermal resistance, (5) Negligible heat transfer from back of chip, (6) Uniform chip temperature.

ANALYSIS: (a) For the prescribed system, the chip power dissipation may be expressed as

$$q_c = \frac{T_c - T_\infty}{R_{t,c} + R_{cond,b} + R_{t,o}}$$

where $R_{t,c} = \frac{R''_{t,c}}{W_c^2} = \frac{5\times10^{-6}\,m^2\cdot K/W}{(0.016m)^2} = 0.0195\,K/W$

$$R_{cond,b} = \frac{L_b}{kW_c^2} = \frac{0.003m}{400\,W/m\cdot K\,(0.016m)^2} = 0.0293\,K/W$$

The thermal resistance of the fin array is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$

and $A_t = NA_f + A_b = N(4wL_c) + \left(W_c^2 - Nw^2\right)$

Continued...

With w = 0.25 mm, S = 0.50 mm, L_f = 6 mm, N = 1024, and $L_c \approx L_f + w/4 = 6.063 \times 10^{-3}$ m, it follows that $A_f = 6.06 \times 10^{-6}\,m^2$ and $A_t = 6.40 \times 10^{-3}\,m^2$. The fin efficiency is

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

where $m = (hP/kA_c)^{1/2} = (4h/kw)^{1/2}$ = 245 m^{-1} and mL_c = 1.49. It follows that η_f = 0.608 and η_o = 0.619, in which case

$$R_{t,o} = \left(0.619 \times 1500\,W/m^2 \cdot K \times 6.40 \times 10^{-3}\,m^2\right) = 0.168\,K/W$$

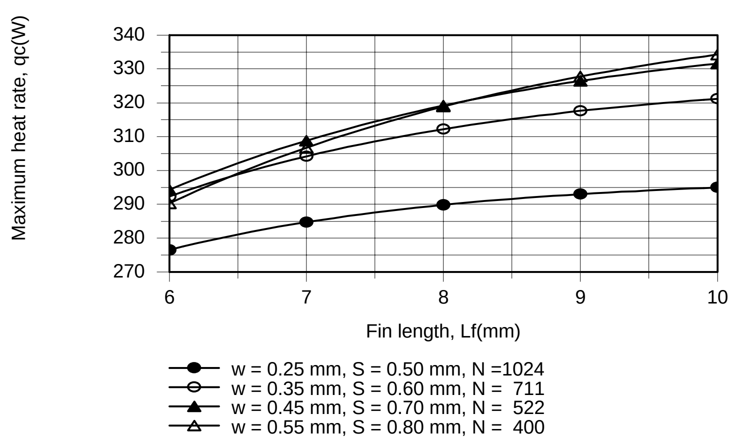
and the maximum allowable heat dissipation is

$$q_c = \frac{(85-25)^\circ C}{(0.0195 + 0.0293 + 0.168)K/W} = 276W$$

<

(b) The IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array* has been used to determine q_c as a function of L_f for four different cases, each of which is characterized by the closest allowable fin spacing of (S - w) = 0.25 mm.

Case	w (mm)	S (mm)	N
A	0.25	0.50	1024
B	0.35	0.60	711
C	0.45	0.70	522
D	0.55	0.80	400



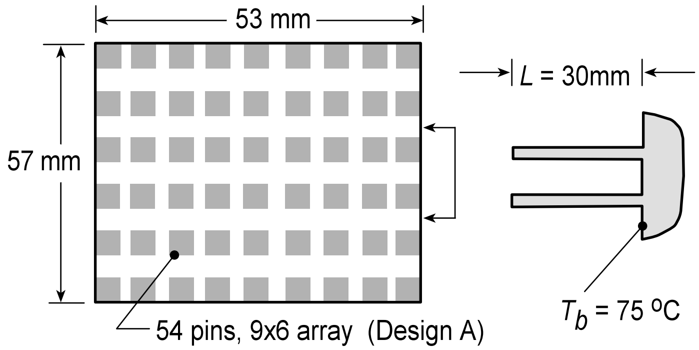
With increasing w and hence decreasing N, there is a reduction in the total area A_t associated with heat transfer from the fin array. However, for Cases A through C, the reduction in A_t is more than balanced by an increase in η_f (and η_o), causing a reduction in $R_{t,o}$ and hence an increase in q_c. As the fin efficiency approaches its limiting value of η_f = 1, reductions in A_t due to increasing w are no longer balanced by increases in η_f, and q_c begins to decrease. Hence there is an optimum value of w, which depends on L_f. For the conditions of this problem, L_f = 10 mm and w = 0.55 mm provide the largest heat dissipation.

Problem 3.137

KNOWN: Two finned heat sinks, Designs A and B, prescribed by the number of fins in the array, N, fin dimensions of square cross-section, w, and length, L, with different convection coefficients, h.

FIND: Determine which fin arrangement is superior. Calculate the heat rate, q_f, efficiency, η_f, and effectiveness, ε_f, of a single fin, as well as, the total heat rate, q_t, and overall efficiency, η_o, of the array. Also, compare the total heat rates per unit volume.

SCHEMATIC:



Design	Fin dimensions Cross section w x w (mm)	Length L (mm)	Number of fins	Convection coefficient ($W/m^2\cdot K$)
A	1 x 1	30	6 x 9	125
B	3 x 3	7	14 x 17	375

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction in fins, (3) Convection coefficient is uniform over fin and prime surfaces, (4) Fin tips experience convection, and (5) Constant properties.

ANALYSIS: Following the treatment of Section 3.6.5, the overall efficiency of the array, Eq. (3.98), is

$$\eta_o = \frac{q_t}{q_{max}} = \frac{q_t}{hA_t\theta_b} \quad (1)$$

where A_t is the total surface area, the sum of the exposed portion of the base (prime area) plus the fin surfaces, Eq. 3.99,

$$A_t = N \cdot A_f + A_b \quad (2)$$

where the surface area of a single fin and the prime area are

$$A_f = 4(L \times W) + w^2 \quad (3)$$

$$A_b = b1 \times b2 - N \cdot A_c \quad (4)$$

Combining Eqs. (1) and (2), the total heat rate for the array is

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \quad (5)$$

where η_f is the efficiency of a single fin. From Table 4.3, Case A, for the tip condition with convection, the single fin efficiency based upon Eq. 3.86,

$$\eta_f = \frac{q_f}{hA_f\theta_b} \quad (6)$$

Continued...

where

$$q_f = M\frac{\sinh(mL) + (h/mk)\cosh(mL)}{\cosh(mL) + (h/mk)\sinh(mL)} \tag{7}$$

$$M = (hPkA_c)^{1/2}\theta_b \qquad m = (hP/kA_c)^{1/2} \qquad P = 4w \qquad A_c = w^2 \qquad (8,9,10)$$

The single fin effectiveness, from Eq. 3.81,

$$\varepsilon_f = \frac{q_f}{hA_c\theta_b} \tag{11}$$

Additionally, we want to compare the performance of the designs with respect to the array volume, vol

$$q_f''' = q_f/\forall = q_f/(b1 \cdot b2 \cdot L) \tag{12}$$

The above analysis was organized for easy treatment with equation-solving software. Solving Eqs. (1) through (11) simultaneously with appropriate numerical values, the results are tabulated below.

Design	q_t (W)	q_f (W)	η_o	η_f	ε_f	q_f''' (W/m^3)
A	113	1.80	0.804	0.779	31.9	1.25×10^6
B	165	0.475	0.909	0.873	25.3	7.81×10^6

COMMENTS: (1) Both designs have good efficiencies and effectiveness. Clearly, Design B is superior because the heat rate is nearly 50% larger than Design A for the same board footprint. Further, the space requirement for Design B is four times less ($\forall = 2.12\times10^{-5}$ vs. 9.06×10^{-5} m^3) and the heat rate per unit volume is 6 times greater.

(2) Design A features 54 fins compared to 238 fins for Design B. Also very significant to the performance comparison is the magnitude of the convection coefficient which is 3 times larger for Design B. Estimating convection coefficients for fin arrays (and tube banks) is discussed in Chapter 7.6. Of concern is how the fins alter the flow past the fins and whether the convection coefficient is uniform over the array.

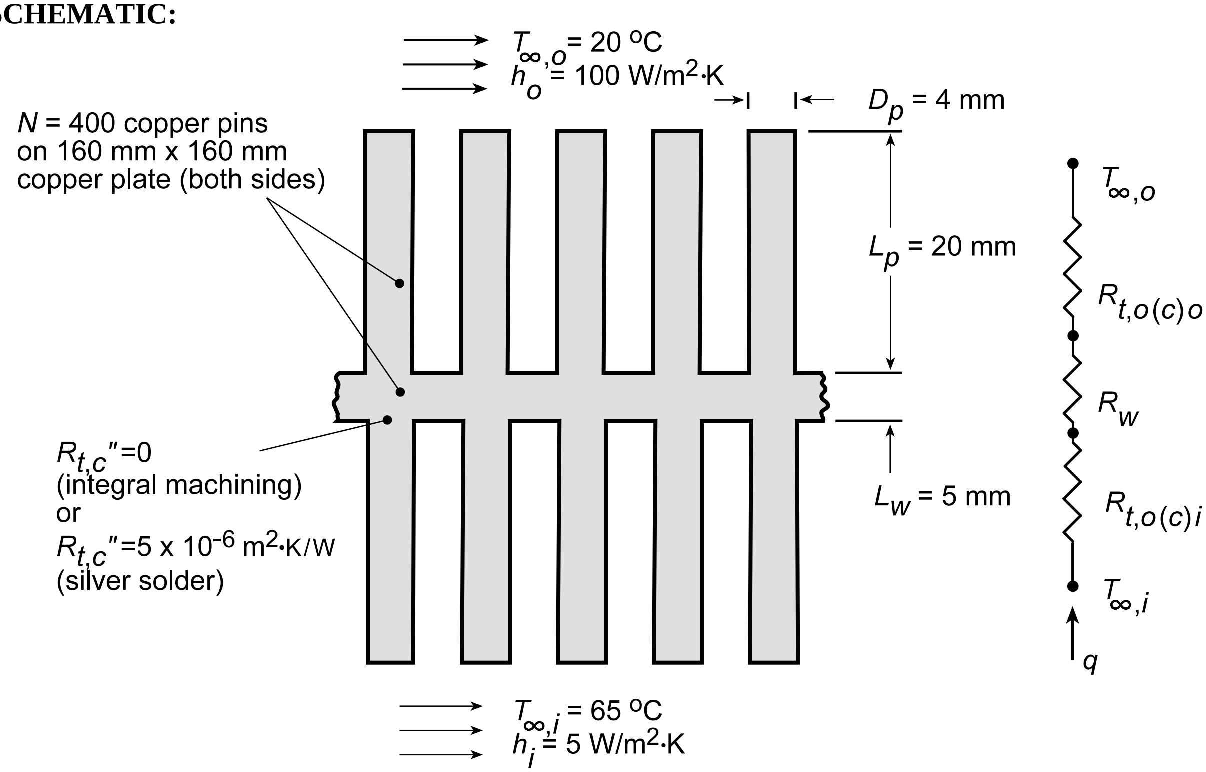
(3) The *IHT Extended Surfaces Model*, for a *Rectangular Pin Fin Array* could have been used to solve this problem.

PROBLEM 3.138

KNOWN: Geometrical characteristics of a plate with pin fin array on both surfaces. Inner and outer convection conditions.

FIND: (a) Heat transfer rate with and without pin fin arrays, (b) Effect of using silver solder to join the pins and the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant k, (3) Negligible radiation.

PROPERTIES: Table A-1: Copper, $\overline{T} \approx 315$ K, k = 400 W/m·K.

ANALYSIS: (a) The heat rate may be expressed as

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{t,o(c),i} + R_w + R_{t,o(c),o}}$$

where

$$R_{t,o(c)} = \left(\eta_{o(c)} h A_t\right)^{-1},$$

$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t}\left(1 - \frac{\eta_f}{C_1}\right),$$

$$A_t = NA_f + A_b,$$

$$A_f = \pi D_p L_c \approx \pi D_p \left(L + D/4\right),$$

$$A_b = W^2 - NA_{c,b} = W^2 - N\left(\pi D_p^2/4\right),$$

$$\eta_f = \frac{\tanh mL_c}{mL_c}, \qquad m = \left(4h/kD_p\right)^{1/2},$$

Continued...

PROBLEM 3.138 (Cont.)

$$C_1 = 1 + \eta_f hA_f \left(R''_{t,c}/A_{c,b}\right),$$

and

$$R_w = \frac{L_w}{W^2 k}.$$

Calculations may be expedited by using the IHT *Performance Calculation, Extended Surface* Model for the *Pin Fin* Array. For $R''_{t,c}$ = 0, C_1 = 1, and with W = 0.160 m, R_w = 0.005 m/(0.160 m)^2 400 W/m·K = 4.88 × 10^{-4} K/W. For the prescribed array geometry, we also obtain $A_{c,b}$ = 1.26 × 10^{-5} m^2, A_f = 2.64 × 10^{-4} m^2, A_b = 2.06 × 10^{-2} m^2, and A_t = 0.126 m^2.

On the outer surface, where h_o = 100 W/m^2·K, m = 15.8 m^{-1}, η_f = 0.965, η_o = 0.970 and $R_{t,o}$ = 0.0817 K/W. On the inner surface, where h_i = 5 W/m^2·K, m = 3.54 m^{-1}, η_f = 0.998, η_o = 0.999 and $R_{t,o}$ = 1.588 K/W.

Hence, the heat rate is

$$q = \frac{(65-20)^\circ \text{C}}{\left(1.588 + 4.88\times10^{-4} + 0.0817\right)\text{K/W}} = 26.94\text{W}$$ <

Without the fins,

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{(1/h_i A_w) + R_w + (1/h_o A_w)} = \frac{(65-20)^\circ \text{C}}{\left(7.81 + 4.88\times10^{-4} + 0.39\right)} = 5.49\text{W}$$ <

Hence, the fin arrays provide nearly a five-fold increase in heat rate.

(b) With use of the silver solder, $\eta_{o(c),o}$ = 0.962 and $R_{t,o(c),o}$ = 0.0824 K/W. Also, $\eta_{o(c),i}$ = 0.998 and $R_{t,o(c),i}$ = 1.589 K/W. Hence

$$q = \frac{(65-20)^\circ \text{C}}{\left(1.589 + 4.88\times10^{-4} + 0.0824\right)\text{K/W}} = 26.92\text{W}$$ <

Hence, the effect of the contact resistance is negligible.

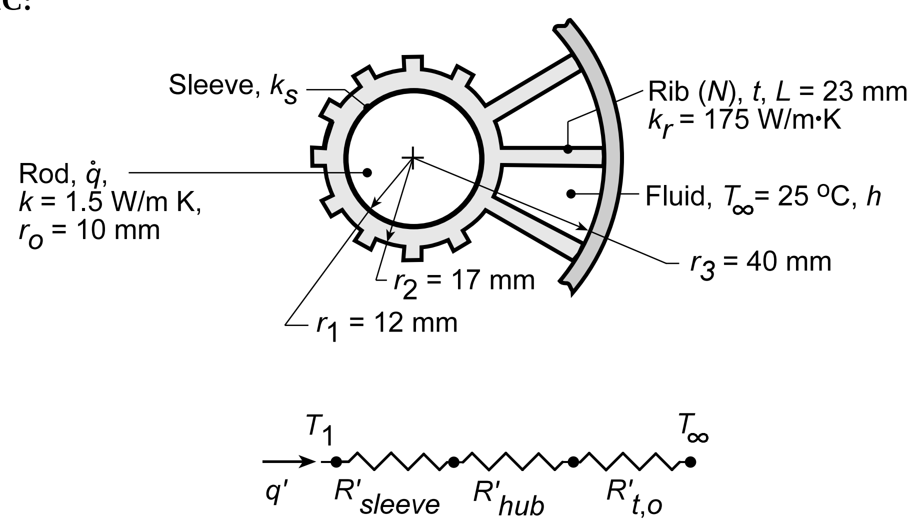
COMMENTS: The dominant contribution to the total thermal resistance is associated with internal conditions. If the heat rate must be increased, it should be done by increasing h_i.

PROBLEM 3.139

KNOWN: Long rod with internal volumetric generation covered by an electrically insulating sleeve and supported with a ribbed spider.

FIND: Combination of convection coefficient, spider design, and sleeve thermal conductivity which enhances volumetric heating subject to a maximum centerline temperature of 100°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial heat transfer in rod, sleeve and hub, (3) Negligible interfacial contact resistances, (4) Constant properties, (5) Adiabatic outer surface.

ANALYSIS: The system heat rate per unit length may be expressed as

$$q' = \dot{q}\left(\pi r_o^2\right) = \frac{T_1 - T_\infty}{R'_{sleeve} + R'_{hub} + R'_{t,o}}$$

where

$$R'_{sleeve} = \frac{\ln\left(r_1/r_o\right)}{2\pi k_s}, \quad R'_{hub} = \frac{\ln\left(r_2/r_1\right)}{2\pi k_r} = 3.168\times10^{-4}\,\text{m}\cdot\text{K/W}, \quad R'_{t,o} = \frac{1}{\eta_o h A'_t},$$

$$\eta_o = 1 - \frac{N A'_f}{A'_t}\left(1-\eta_f\right), \qquad A'_f = 2\left(r_3 - r_2\right), \qquad A'_t = N A'_f + \left(2\pi r_3 - Nt\right),$$

$$\eta_f = \frac{\tanh m\left(r_3 - r_2\right)}{m\left(r_3 - r_2\right)}, \qquad m = \left(2h/k_r t\right)^{1/2}.$$

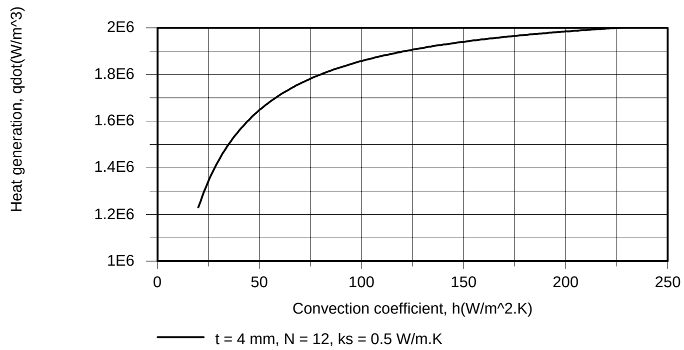
The rod centerline temperature is related to T_1 through

$$T_o = T(0) = T_1 + \frac{\dot{q} r_o^2}{4k}$$

Calculations may be expedited by using the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array*. For base case conditions of k_s = 0.5 W/m·K, h = 20 W/m^2·K, t = 4 mm and N = 12, R'_{sleeve} = 0.0580 m·K/W, $R'_{t,o}$ = 0.0826 m·K/W, η_f = 0.990, q' = 387 W/m, and $\dot{q}$ = 1.23×10^6 W/m^3. As shown below, $\dot{q}$ may be increased by increasing h, where h = 250 W/m^2·K represents a reasonable upper limit for airflow. However, a more than 10-fold increase in h yields only a 63% increase in $\dot{q}$.

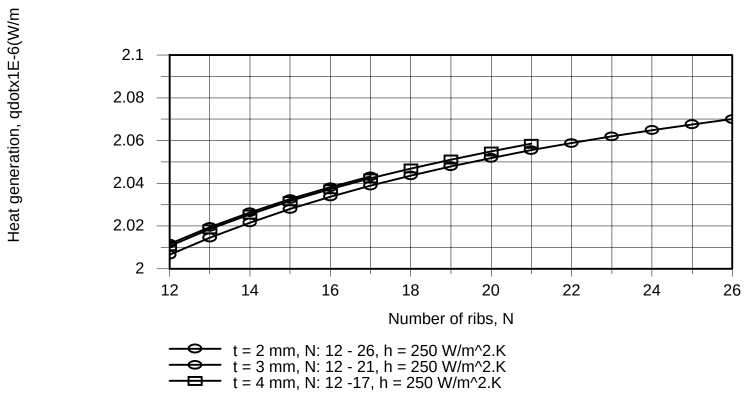
Continued...

PROBLEM 3.139 (Cont.)

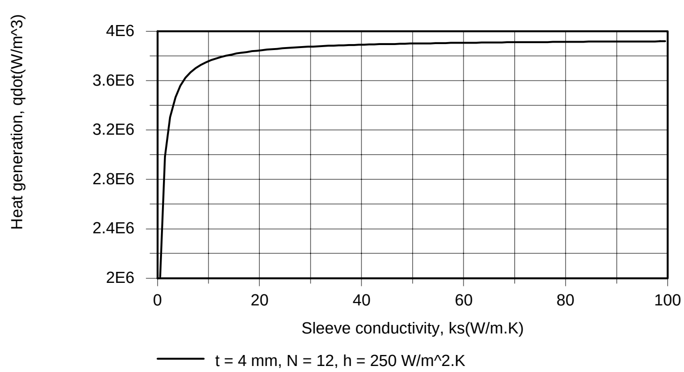


The difficulty is that, by significantly increasing h, the thermal resistance of the fin array is reduced to 0.00727 m·K/W, rendering the sleeve the dominant contributor to the total resistance.

Similar results are obtained when N and t are varied. For values of t = 2, 3 and 4 mm, variations of N in the respective ranges $12 \le N \le 26$, $12 \le N \le 21$ and $12 \le N \le 17$ were considered. The upper limit on N was fixed by requiring that $(S - t) \ge 2$ mm to avoid an excessive resistance to airflow between the ribs. As shown below, the effect of increasing N is small, and there is little difference between results for the three values of t.



In contrast, significant improvement is associated with changing the sleeve material, and it is only necessary to have $k_s \approx 25$ W/m·K (e.g. a boron sleeve) to approach an upper limit to the influence of k_s.



For $h = 250$ W/m^2·K and $k_s = 25$ W/m·K, only a slight improvement is obtained by increasing N. Hence, the recommended conditions are:

$$h = 250\,\mathrm{W/m^2 \cdot K}, \qquad k_s = 25\,\mathrm{W/m \cdot K}, \qquad N = 12, \qquad t = 4\mathrm{mm}$$ <

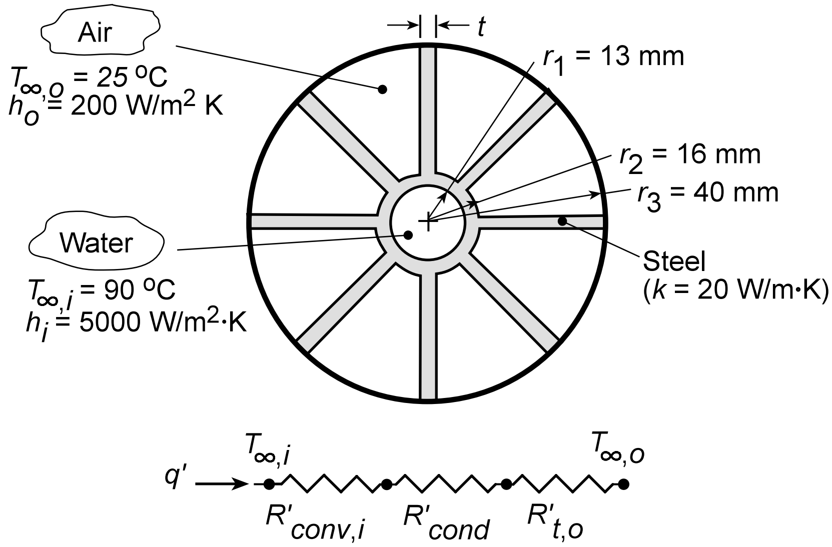
COMMENTS: The upper limit to $\dot{q}$ is reached as the total thermal resistance approaches zero, in which case $T_1 \rightarrow T_\infty$. Hence $\dot{q}_{max} = 4k\left(T_o - T_\infty\right)/r_o^2 = 4.5\times10^6\ \mathrm{W/m^3}$.

PROBLEM 3.140

KNOWN: Geometrical and convection conditions of internally finned, concentric tube air heater.

FIND: (a) Thermal circuit, (b) Heat rate per unit tube length, (c) Effect of changes in fin array.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in radial direction, (3) Constant k, (4) Adiabatic outer surface.

ANALYSIS: (a) For the thermal circuit shown schematically,

$$R'_{conv,i} = \left(h_i 2\pi r_1\right)^{-1}, \qquad R'_{cond} = \ln\left(r_2/r_1\right)/2\pi k \text{, and} \qquad R'_{t,o} = \left(\eta_o h_o A'_t\right)^{-1},$$

where

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1-\eta_f\right), \quad A'_f = 2L = 2\left(r_3 - r_2\right), \quad A'_t = NA'_f + \left(2\pi r_2 - Nt\right), \text{ and } \quad \eta_f = \frac{\tanh mL}{mL}.$$

(b)
$$q' = \frac{\left(T_{\infty,i} - T_{\infty,o}\right)}{R'_{conv,i} + R'_{cond} + R'_{t,o}}$$

Substituting the known conditions, it follows that

$$R'_{conv,i} = \left(5000\,\text{W/m}^2\cdot\text{K}\times 2\pi\times 0.013\text{m}\right)^{-1} = 2.45\times10^{-3}\,\text{m}\cdot\text{K/W}$$

$$R'_{cond} = \ln\left(0.016\text{m}/0.013\text{m}\right)/2\pi\left(20\,\text{W/m}\cdot\text{K}\right) = 1.65\times10^{-3}\,\text{m}\cdot\text{K/W}$$

$$R'_{t,o} = \left(0.575\times 200\,\text{W/m}^2\cdot\text{K}\times 0.461\text{m}\right)^{-1} = 18.86\times10^{-3}\,\text{m}\cdot\text{K/W}$$

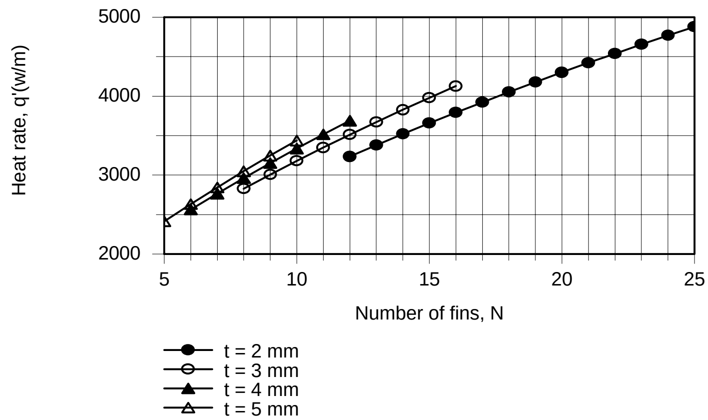
where η_f = 0.490. Hence,

$$q' = \frac{\left(90-25\right)^\circ\text{C}}{\left(2.45+1.65+18.86\right)\times10^{-3}\,\text{m}\cdot\text{K/W}} = 2831\,\text{W/m}$$ <

(c) The small value of η_f suggests that some benefit may be gained by increasing t, as well as by increasing N. With the requirement that Nt ≤ 50 mm, we use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to consider the following range of conditions: t = 2 mm, 12 ≤ N ≤ 25; t = 3 mm, 8 ≤ N ≤ 16; t = 4 mm, 6 ≤ N ≤ 12; t = 5 mm, 5 ≤ N ≤ 10. Calculations based on the foregoing model are plotted as follows.

Continued...

PROBLEM 3.140 (Cont.)



By increasing t from 2 to 5 mm, η_f increases from 0.410 to 0.598. Hence, for fixed N, q' increases with increasing t. However, from the standpoint of maximizing q'_t, it is clearly preferable to use the larger number of thinner fins. Hence, subject to the prescribed constraint, we would choose t = 2 mm and N = 25, for which q' = 4880 W/m.

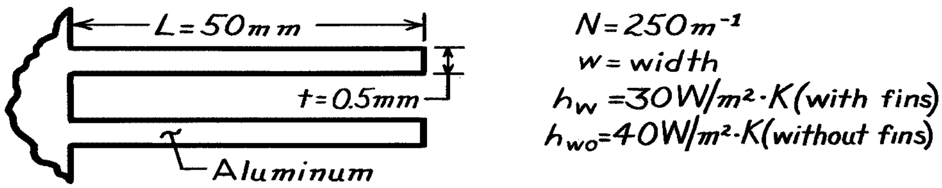
COMMENTS: (1) The air side resistance makes the dominant contribution to the total resistance, and efforts to increase q' by reducing $R'_{t,o}$ are well directed. (2) A fin thickness any smaller than 2 mm would be difficult to manufacture.

PROBLEM 3.141

KNOWN: Dimensions and number of rectangular aluminum fins. Convection coefficient with and without fins.

FIND: Percentage increase in heat transfer resulting from use of fins.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Negligible fin contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure: $k \approx 240$ W/m·K.

ANALYSIS: Evaluate the fin parameters

$$L_c = L + t/2 = 0.05025\text{m}$$

$$A_p = L_c t = 0.05025\text{m} \times 0.5 \times 10^{-3}\text{m} = 25.13 \times 10^{-6}\ \text{m}^2$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = (0.05025\text{m})^{3/2}\left[\frac{30\ \text{W/m}^2 \cdot \text{K}}{240\ \text{W/m}\cdot\text{K} \times 25.13 \times 10^{-6}\text{m}^2}\right]^{1/2}$$

$$L_c^{3/2}\left(h_w / kA_p\right)^{1/2} = 0.794$$

It follows from Fig. 3.18 that $\eta_f \approx 0.72$. Hence,

$$q_f = \eta_f q_{max} = 0.72\ h_w 2wL\ \theta_b$$

$$q_f = 0.72 \times 30\ \text{W/m}^2 \cdot \text{K} \times 2 \times 0.05\text{m} \times (w\ \theta_b) = 2.16\ \text{W/m}\cdot\text{K}\,(w\ \theta_b)$$

With the fins, the heat transfer from the walls is

$$q_w = N\, q_f + (1 - Nt)\, w\, h_w\ \theta_b$$

$$q_w = 250 \times 2.16 \frac{\text{W}}{\text{m}\cdot\text{K}}(w\ \theta_b) + \left(1\text{m} - 250 \times 5 \times 10^{-4}\text{m}\right) \times 30\ \text{W/m}^2 \cdot \text{K}\ (w\ \theta_b)$$

$$q_w = (540 + 26.3)\frac{\text{W}}{\text{m}\cdot\text{K}}(w\ \theta_b) = 566\ w\ \theta_b.$$

Without the fins, $q_{wo} = h_{wo}\ 1\text{m} \times w\ \theta_b = 40\ w\ \theta_b$. Hence the percentage increase in heat transfer is

$$\frac{q_w - q_{wo}}{q_{wo}} = \frac{(566 - 40)\,w\ \theta_b}{40\ w\ \theta_b} = 13.15 = 1315\%$$ <

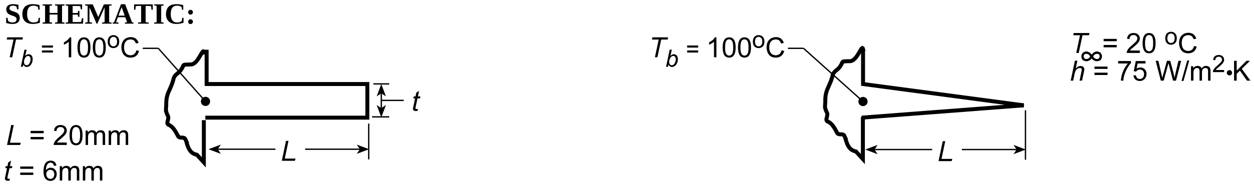
COMMENTS: If the infinite fin approximation is made, it follows that $q_f = (hPkA_c)^{1/2}\ \theta_b = [h_w 2wkwt]^{1/2}\ \theta_b = (30 \times 2 \times 240 \times 5\times10^{-4})^{1/2}\ w\ \theta_b = 2.68\ w\ \theta_b$. Hence, q_f is overestimated.

PROBLEM 3.142

KNOWN: Dimensions, base temperature and environmental conditions associated with rectangular and triangular stainless steel fins.

FIND: Efficiency, heat loss per unit width and effectiveness associated with each fin.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Stainless Steel 304 (T = 333 K): k = 15.3 W/m·K.

ANALYSIS: For the rectangular fin, with $L_c = L + t/2$, evaluate the parameter

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.023\,\text{m})^{3/2}\left[\frac{75\,\text{W/m}^2\cdot\text{K}}{15.3\,\text{W/m}\cdot\text{K}(0.023\,\text{m})(0.006\,\text{m})}\right]^{1/2} = 0.66\,.$$

Hence, from Fig. 3.18, the fin efficiency is

$$\eta_f \approx 0.79$$ <

From Eq. 3.86, the fin heat rate is $q_f = \eta_f hA_f\theta_b = \eta_f hPL_c\theta_b = \eta_f h2wL_c\theta_b$ or, per unit width,

$$q_f' = \frac{q_f}{w} = 0.79\left(75\,\text{W/m}^2\cdot\text{K}\right)2(0.023\,\text{m})80^\circ\text{C} = 218\,\text{W/m}\,.$$ <

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{q_f'\times w}{h(t\times w)\theta_b} = \frac{218\,\text{W/m}}{75\,\text{W/m}^2\cdot\text{K}(0.006\,\text{m})80^\circ\text{C}} = 6.06\,.$$ <

For the triangular fin with

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.02\,\text{m})^{3/2}\left[\frac{75\,\text{W/m}^2\cdot\text{K}}{(15.3\,\text{W/m}\cdot\text{K})(0.020\,\text{m})(0.003\,\text{m})}\right]^{1/2} = 0.81\,,$$

find from Figure 3.18,

$$\eta_f \approx 0.78\,,$$ <

From Eq. 3.86 and Table 3.5 find

$$q_f' = \eta_f hA_f'\theta_b = \eta_f h2\left[L^2 + (t/2)^2\right]^{1/2}\theta_b$$

$$q_f' = 0.78\times 75\,\text{W/m}^2\cdot\text{K}\times 2\left[(0.02)^2 + (0.006/2)^2\right]^{1/2}\text{m}\left(80^\circ\text{C}\right) = 187\,\text{W/m}\,.$$ <

and from Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f'\times w}{h(t\times w)\theta_b} = \frac{187\,\text{W/m}}{75\,\text{W/m}^2\cdot\text{K}(0.006\,\text{m})80^\circ\text{C}} = 5.19$$ <

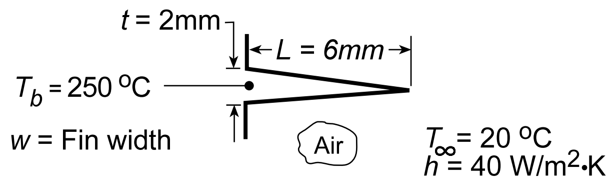
COMMENTS: Although it is 14% less effective, the triangular fin offers a 50% weight savings.

PROBLEM 3.143

KNOWN: Dimensions, base temperature and environmental conditions associated with a triangular, aluminum fin.

FIND: (a) Fin efficiency and effectiveness, (b) Heat dissipation per unit width.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and base contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) With $L_c = L = 0.006$ m, find

$$A_p = Lt/2 = (0.006\,m)(0.002\,m)/2 = 6\times10^{-6}\,m^2,$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.006\,m)^{3/2}\left(\frac{40\,W/m^2\cdot K}{240\,W/m\cdot K\times6\times10^{-6}\,m^2}\right)^{1/2} = 0.077$$

and from Fig. 3.18, the fin efficiency is

$$\eta_f \approx 0.99\,.$$ <

From Eq. 3.86 and Table 3.5, the fin heat rate is

$$q_f = \eta_f\, q_{max} = \eta_f\, hA_{f\,(tri)}\theta_b = 2\eta_f\, hw\left[L^2 + (t/2)^2\right]^{1/2}\theta_b\,.$$

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{2\eta_f\, hw\left[L^2 + (t/2)^2\right]^{1/2}\theta_b}{g(w\cdot t)\theta_b} = \frac{2\eta_f\left[L^2 + (t/2)^2\right]^{1/2}}{t}$$

$$\varepsilon_f = \frac{2\times0.99\left[(0.006)^2 + (0.002/2)^2\right]^{1/2}\,m}{0.002\,m} = 6.02$$ <

(b) The heat dissipation per unit width is

$$q_f' = (q_f/w) = 2\eta_f\, h\left[L^2 + (t/2)^2\right]^{1/2}\theta_b$$

$$q_f' = 2\times0.99\times40\,W/m^2\cdot K\left[(0.006)^2 + (0.002/2)^2\right]^{1/2} m\times(250-20)^\circ C = 110.8\,W/m\,.$$ <

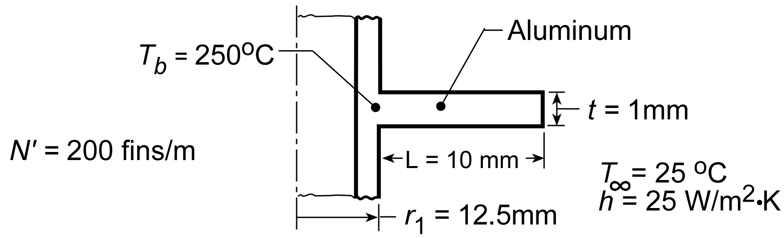
COMMENTS: The triangular profile is known to provide the maximum heat dissipation per unit fin mass.

PROBLEM 3.144

KNOWN: Dimensions and base temperature of an annular, aluminum fin of rectangular profile. Ambient air conditions.

FIND: (a) Fin heat loss, (b) Heat loss per unit length of tube with 200 fins spaced at 5 mm increments.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation and contact resistance, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = (12.5\,\text{mm} + 10\,\text{mm}) + 0.5\,\text{mm} = 23\,\text{mm} = 0.023\,\text{m}$$

$$r_{2c}/r_1 = 1.84 \qquad L_c = L + t/2 = 10.5\,\text{mm} = 0.0105\,\text{m}$$

$$A_p = L_c t = 0.0105\,\text{m} \times 0.001\,\text{m} = 1.05 \times 10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.0105\,\text{m})^{3/2}\left(\frac{25\,\text{W/m}^2\cdot\text{K}}{240\,\text{W/m}\cdot\text{K} \times 1.05\times10^{-5}\,\text{m}^2}\right)^{1/2} = 0.15\,.$$

Hence, the fin effectiveness is $\eta_f \approx 0.97$, and from Eq. 3.86 and Fig. 3.5, the fin heat rate is

$$q_f = \eta_f q_{max} = \eta_f h A_{f\,(ann)}\theta_b = 2\pi\eta_f h\left(r_{2,c}^2 - r_1^2\right)\theta_b$$

$$q_f = 2\pi \times 0.97 \times 25\,\text{W/m}^2\cdot\text{K} \times \left[(0.023\,\text{m})^2 - (0.0125\,\text{m})^2\right]225^\circ\text{C} = 12.8\,\text{W}\,. \qquad <$$

(b) Recognizing that there are N = 200 fins per meter length of the tube, the total heat rate considering contributions due to the fin and base (unfinned surfaces is

$$q' = N'q_f + h\left(1 - N't\right)2\pi r_1\theta_b$$

$$q' = 200\,\text{m}^{-1} \times 12.8\,\text{W} + 25\,\text{W/m}^2\cdot\text{K}\left(1 - 200\,\text{m}^{-1} \times 0.001\,\text{m}\right) \times 2\pi \times (0.0125\,\text{m})\,225^\circ\text{C}$$

$$q' = (2560\,\text{W} + 353\,\text{W})/\text{m} = 2.91\,\text{kW/m}\,. \qquad <$$

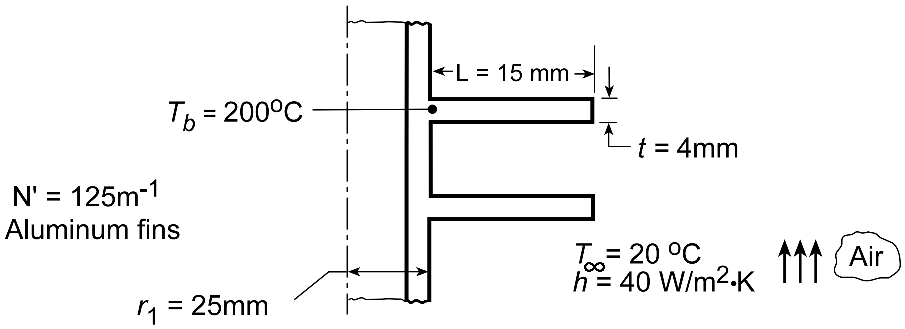
COMMENTS: Note that, while covering only 20% of the tube surface area, the tubes account for more than 85% of the total heat dissipation.

PROBLEM 3.145

KNOWN: Dimensions and base temperature of aluminum fins of rectangular profile. Ambient air conditions.

FIND: (a) Fin efficiency and effectiveness, (b) Rate of heat transfer per unit length of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction in fins, (3) Constant properties, (4) Negligible radiation, (5) Negligible base contact resistance, (6) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 400 K): k = 240 W/m·K.

ANALYSIS: (a) The fin parameters for use with Figure 3.19 are

$$r_{2c} = r_2 + t/2 = 40\,\text{mm} + 2\,\text{mm} = 0.042\,\text{m} \qquad L_c = L + t/2 = 15\,\text{mm} + 2\,\text{mm} = 0.017\,\text{m}$$

$$r_{2c}/r_1 = 0.042\,\text{m}/0.025\,\text{m} = 1.68 \qquad A_p = L_c t = 0.017\,\text{m} \times 0.004\,\text{m} = 6.8\times10^{-5}\,\text{m}^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.017\,\text{m})^{3/2}\left[40\,\text{W/m}^2\cdot\text{K}/240\,\text{W/m}\cdot\text{K}\times6.8\times10^{-5}\,\text{m}^2\right]^{1/2} = 0.11$$

The fin efficiency is $\eta_f \approx 0.97$. From Eq. 3.86 and Fig. 3.5,

$$q_f = \eta_f q_{max} = \eta_f h A_{f\,(ann)}\theta_b = 2\pi\eta_f h\left[r_{2c}^2 - r_1^2\right]\theta_b$$

$$q_f = 2\pi\times0.97\times40\,\text{W/m}^2\cdot\text{K}\left[(0.042)^2 - (0.025)^2\right]\text{m}^2\times180^\circ\text{C} = 50\,\text{W} \qquad <$$

From Eq. 3.81, the fin effectiveness is

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{50\,\text{W}}{40\,\text{W/m}^2\cdot\text{K}\;2\pi(0.025\,\text{m})(0.004\,\text{m})180^\circ\text{C}} = 11.05 \qquad <$$

(b) The rate of heat transfer per unit length is

$$q' = N'q_f + h(1 - N't)(2\pi r_1)\theta_b$$

$$q' = 125\times50\,\text{W/m} + 40\,\text{W/m}^2\cdot\text{K}(1 - 125\times0.004)(2\pi\times0.025\,\text{m})\times180^\circ\text{C}$$

$$q' = (6250 + 565)\,\text{W/m} = 6.82\,\text{kW/m} \qquad <$$

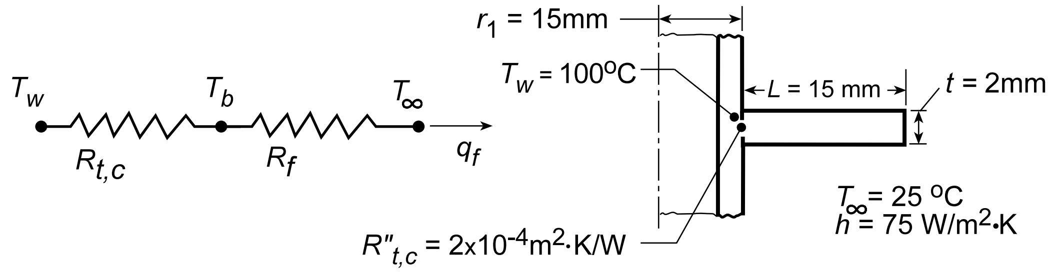
COMMENTS: Note the dominant contribution made by the fins to the total heat transfer.

PROBLEM 3.146

KNOWN: Dimensions, base temperature, and contact resistance for an annular, aluminum fin. Ambient fluid conditions.

FIND: Fin heat transfer with and without base contact resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient.

PROPERTIES: *Table A-1*, Aluminum, pure (T ≈ 350 K): k ≈ 240 W/m·K.

ANALYSIS: With the contact resistance, the fin heat loss is $q_f = \dfrac{T_w - T_\infty}{R_{t,c} + R_f}$ where

$$R_{t,c} = R''_{t,c}/A_b = 2\times10^{-4}\,m^2\cdot K/W\big/2\pi(0.015\,m)(0.002\,m) = 1.06\,K/W\,.$$

From Eqs. 3.83 and 3.86, the fin resistance is

$$R_f = \frac{\theta_b}{q_f} = \frac{\theta_b}{\eta_f q_{max}} = \frac{\theta_b}{\eta_f h A_f \theta_b} = \frac{1}{2\pi h \eta_f\left(r_{2,c}^2 - r_1^2\right)}\,.$$

Evaluating parameters,

$$r_{2,c} = r_2 + t/2 = 30\,mm + 1\,mm = 0.031\,m \qquad L_c = L + t/2 = 0.016\,m$$

$$r_{2c}/r_1 = 0.031/0.015 = 2.07\,\mathbf{Z} \qquad A_p = L_c t = 3.2\times10^{-5}\,m^2$$

$$L_c^{3/2}\left(h/kA_p\right)^{1/2} = (0.016\,m)^{3/2}\left[75\,W/m^2\cdot K\big/240\,W/m\cdot K\times3.2\times10^{-5}\,m^2\right]^{1/2} = 0.20$$

find the fin efficiency from Figure 3.19 as η_f = 0.94. Hence,

$$R_f = \frac{1}{2\pi\left(75\,W/m^2\cdot K\right)0.94\left[(0.031\,m)^2 - (0.015\,m)^2\right]} = 3.07\,K/W$$

$$q_f = \frac{(100-25)^\circ C}{(1.06+3.07)\,K/W} = 18.2\,W\,.$$ <

Without the contact resistance, $T_w = T_b$ and

$$q_f = \frac{\theta_b}{R_f} = \frac{75^\circ C}{3.07\,K/W} = 24.4\,W\,.$$ <

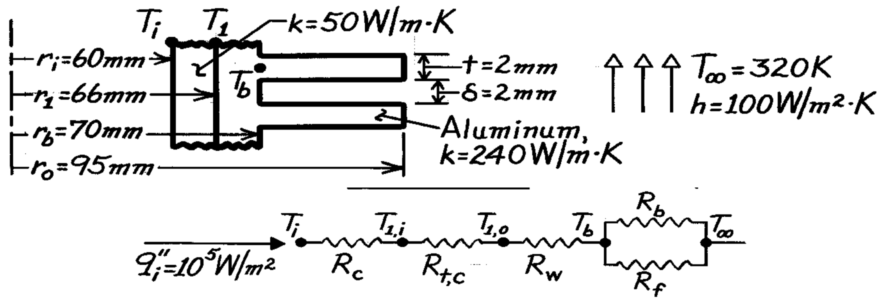
COMMENTS: To maximize fin performance, every effort should be made to minimize contact resistance.

PROBLEM 3.147

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Heat flux and ambient air conditions. Contact resistance.

FIND: Surface and interface temperatures (a) without and (b) with an interface contact resistance.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: The analysis may be performed per unit length of cylinder or for a 4 mm long section. The following calculations are based on a unit length. The inner surface temperature may be obtained from

$$q' = \frac{T_i - T_\infty}{R'_{tot}} = q''_i (2\pi\ r_i) = 10^5\ \text{W/m}^2 \times 2\pi \times 0.06\ \text{m} = 37,700\ \text{W/m}$$

where $R'_{tot} = R'_c + R'_{t,c} + R'_w + R'_{equiv}$; $R'_{equiv} = (1/R'_f + 1/R'_b)^{-1}$.

R'_c, *Conduction resistance of cylinder wall:*

$$R'_c = \frac{\ln(r_1/r_i)}{2\pi\ k} = \frac{\ln(66/60)}{2\pi(50\ \text{W/m}\cdot\text{K})} = 3.034\times10^{-4}\ \text{m}\cdot\text{K/W}$$

$R'_{t,c}$, *Contact resistance:*

$$R'_{t,c} = R''_{t,c}/2\pi\ r_1 = 10^{-4}\ \text{m}^2\cdot\text{K/W}/2\pi\times0.066\ \text{m} = 2.411\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_w, *Conduction resistance of aluminum base:*

$$R'_w = \frac{\ln(r_b/r_1)}{2\pi\ k} = \frac{\ln(70/66)}{2\pi\times240\ \text{W/m}\cdot\text{K}} = 3.902\times10^{-5}\ \text{m}\cdot\text{K/W}$$

R'_b, *Resistance of prime or unfinned surface:*

$$R'_b = \frac{1}{hA'_b} = \frac{1}{100\ \text{W/m}^2\cdot\text{K}\times0.5\times2\pi(0.07\ \text{m})} = 454.7\times10^{-4}\ \text{m}\cdot\text{K/W}$$

R'_f, *Resistance of fins:* The fin resistance may be determined from

$$R'_f = \frac{T_b - T_\infty}{q'_f} = \frac{1}{\eta_f hA'_f}$$

The fin efficiency may be obtained from Fig. 3.19,

$$r_{2c} = r_o + t/2 = 0.096\ \text{m} \qquad L_c = L + t/2 = 0.026\ \text{m}$$

Continued …..

$$A_p = L_c t = 5.2\times10^{-5}\ m^2 \qquad r_{2c}/r_1 = 1.45 \qquad L_c^{3/2}\left(h/kA_p\right)^{1/2} = 0.375$$

Fig. 3.19 $\rightarrow \eta_f \approx 0.88$.

The total fin surface area per meter length

$$A'_f = 250\left[\pi\left(r_o^2 - r_b^2\right)\times 2\right] = 250\ m^{-1}\left[2\pi\left(0.096^2 - 0.07^2\right)\right]m^2 = 6.78\ m.$$

Hence

$$R'_f = \left[0.88\times100\ W/m^2\cdot K\times6.78\ m\right]^{-1} = 16.8\times10^{-4}\ m\cdot K/W$$

$$1/R'_{equiv} = \left(1/16.8\times10^{-4} + 1/454.7\times10^{-4}\right)W/m\cdot K = 617.2\ W/m\cdot K$$

$$R'_{equiv} = 16.2\times10^{-4}\ m\cdot K/W.$$

Neglecting the *contact resistance,*

$$R'_{tot} = (3.034 + 0.390 + 16.2)10^{-4}\ m\cdot K/W = 19.6\times10^{-4}\ m\cdot K/W$$

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times19.6\times10^{-4}\ m\cdot K/W + 320\ K = 393.9\ K$$ <

$$T_1 = T_i - q'R'_w = 393.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 382.5\ K$$ <

$$T_b = T_1 - q'R'_b = 382.5\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 381.0\ K.$$ <

Including the *contact resistance,*

$$R'_{tot} = \left(19.6\times10^{-4} + 2.411\times10^{-4}\right)m\cdot K/W = 22.0\times10^{-4}\ m\cdot K/W$$

$$T_i = 37,700\ W/m\times22.0\times10^{-4}\ m\cdot K/W + 320\ K = 402.9\ K$$ <

$$T_{1,i} = 402.9\ K - 37,700\ W/m\times3.034\times10^{-4}\ m\cdot K/W = 391.5\ K$$ <

$$T_{1,o} = 391.5\ K - 37,700\ W/m\times2.411\times10^{-4}\ m\cdot K/W = 382.4\ K$$ <

$$T_b = 382.4\ K - 37,700\ W/m\times3.902\times10^{-5}\ m\cdot K/W = 380.9\ K.$$ <

COMMENTS: (1) The effect of the contact resistance is small.

(2) The effect of including the aluminum fins may be determined by computing T_i without the fins. In this case $R'_{tot} = R'_c + R'_{conv}$, where

$$R'_{conv} = \frac{1}{h2\pi\ r_1} = \frac{1}{100\ W/m^2\cdot K\ 2\pi(0.066\ m)} = 241.1\times10^{-4}\ m\cdot K/W.$$

Hence, $R_{tot} = 244.1\times10^{-4}\ m\cdot K/W$, and

$$T_i = q'R'_{tot} + T_\infty = 37,700\ W/m\times244.1\times10^{-4}\ m\cdot K/W + 320\ K = 1240\ K.$$

Hence, the fins have a significant effect on reducing the cylinder temperature.

(3) The overall surface efficiency is

$$\eta_o = 1 - \left(A'_f / A'_t\right)\left(1 - \eta_f\right) = 1 - 6.78\ m/7.00\ m(1 - 0.88) = 0.884.$$

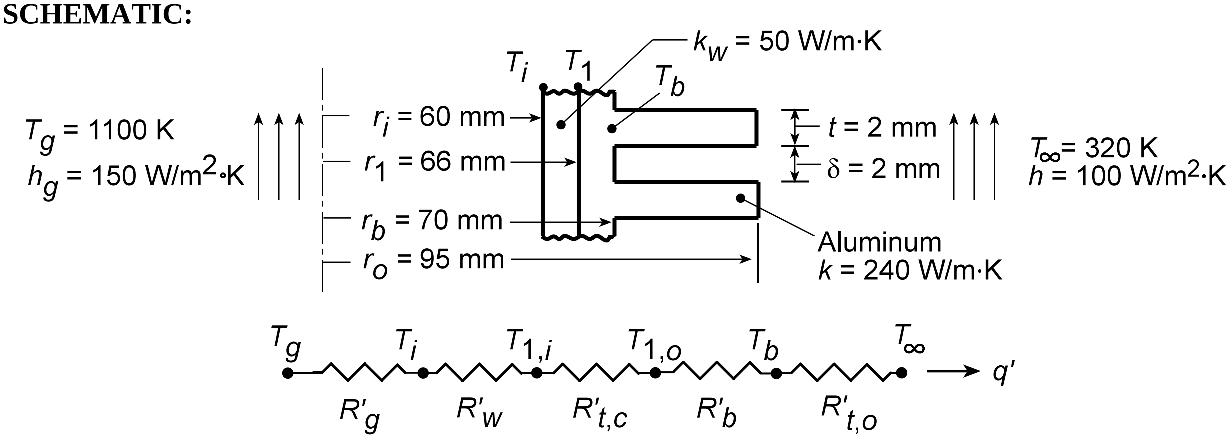
It follows that $q' = \eta_o h_o A'_t \theta_b = 37,700\ W/m$, which agrees with the prescribed value.

PROBLEM 3.148

KNOWN: Dimensions and materials of a finned (annular) cylinder wall. Combustion gas and ambient air conditions. Contact resistance.

FIND: (a) Heat rate per unit length and surface and interface temperatures, (b) Effect of increasing the fin thickness.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conditions, (2) Constant properties, (3) Uniform h over surfaces, (4) Negligible radiation.

ANALYSIS: (a) The heat rate per unit length is

$$q' = \frac{T_g - T_\infty}{R'_{tot}}$$

where $R'_{tot} = R'_g + R'_w + R'_{t,c} + R'_b + R'_{t,o}$, and

$$R'_g = \left(h_g 2\pi r_i\right)^{-1} = \left(150\,\mathrm{W/m^2\cdot K} \times 2\pi \times 0.06\mathrm{m}\right)^{-1} = 0.0177\,\mathrm{m\cdot K/W},$$

$$R'_w = \frac{\ln(r_1/r_i)}{2\pi k_w} = \frac{\ln(66/60)}{2\pi(50\,\mathrm{W/m\cdot K})} = 3.03\times10^{-4}\,\mathrm{m\cdot K/W},$$

$$R'_{t,c} = \left(R''_{t,c}/2\pi r_1\right) = 10^{-4}\,\mathrm{m^4\cdot K/W}\Big/2\pi\times0.066\mathrm{m} = 2.41\times10^{-4}\,\mathrm{m\cdot K/W}$$

$$R'_b = \frac{\ln(r_b/r_1)}{2\pi k} = \frac{\ln(70/66)}{2\pi\times240\,\mathrm{W/m\cdot K}} = 3.90\times10^{-5}\,\mathrm{m\cdot K/W},$$

$$R_{t,o} = \left(\eta_o h A'_t\right)^{-1},$$

$$\eta_o = 1 - \frac{N'A_f}{A'_t}\left(1-\eta_f\right),$$

$$A_f = 2\pi\left(r_{oc}^2 - r_b^2\right)$$

$$A'_t = N'A_f + \left(1-N't\right)2\pi r_b$$

$$\eta_f = \frac{(2r_b/m)}{\left(r_{oc}^2 - r_b^2\right)}\frac{K_1(mr_b)I_1(mr_{oc}) - I_1(mr_b)K_1(mr_{oc})}{I_0(mr_1)K_1(mr_{oc}) + K_0(mr_b)I_1(mr_{oc})}$$

$$r_{oc} = r_o + (t/2), \qquad m = (2h/kt)^{1/2}$$

Continued...

PROBLEM 3.148 (Cont.)

Once the heat rate is determined from the foregoing expressions, the desired interface temperatures may be obtained from

$$T_i = T_g - q'R'_g$$

$$T_{1,i} = T_g - q'\left(R'_g + R'_w\right)$$

$$T_{1,o} = T_g - q'\left(R'_g + R'_w + R'_{t,c}\right)$$

$$T_b = T_g - q'\left(R'_g + R'_w + R'_{t,c} + R'_b\right)$$

For the specified conditions we obtain A'_t = 7.00 m, η_f = 0.902, η_o = 0.906 and $R'_{t,o}$ = 0.00158 m·K/W. It follows that

$$q' = 39,300\,\text{W/m}$$ <

$$T_i = 405\text{K},\quad T_{1,i} = 393\text{K},\quad T_{1,o} = 384\text{K},\quad T_b = 382\text{K}$$ <

(b) The *Performance Calculation, Extended Surface* Model for the *Circular Fin* Array may be used to assess the effects of fin thickness and spacing. Increasing the fin thickness to t = 3 mm, with δ = 2 mm, reduces the number of fins per unit length to 200. Hence, although the fin efficiency increases (η_f = 0.930), the reduction in the total surface area (A'_t = 5.72 m) yields an increase in the resistance of the fin array ($R'_{t,o}$ = 0.00188 m·K/W), and hence a reduction in the heat rate (q' = 38,700 W/m) and an increase in the interface temperatures (T_i = 415 K, $T_{1,i}$ = 404 K, $T_{1,o}$ = 394 K, and T_b = 393 K).

COMMENTS: Because the gas convection resistance exceeds all other resistances by at least an order of magnitude, incremental changes in $R_{t,o}$ will not have a significant effect on q' or the interface temperatures.

PROBLEM 3.149

KNOWN: Dimensions of finned aluminum sleeve inserted over transistor. Contact resistance and convection conditions.

FIND: Measures for increasing heat dissipation.

SCHEMATIC: See Example 3.10.

ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from top and bottom of transistor, (3) One-dimensional radial heat transfer, (4) Constant properties, (5) Negligible radiation.

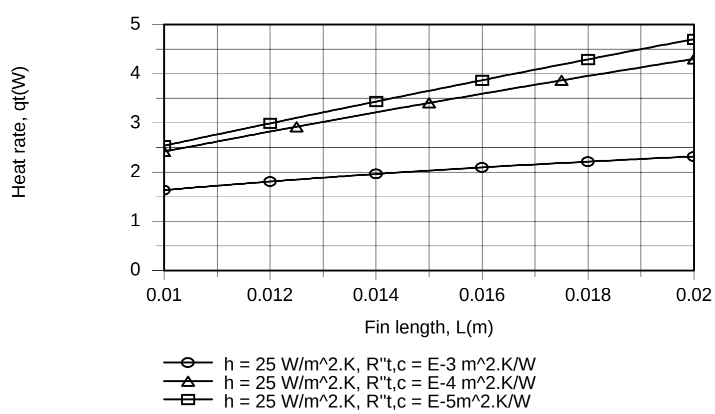
ANALYSIS: With $2\pi r_2$ = 0.0188 m and Nt = 0.0084 m, the existing gap between fins is extremely small (0.87 mm). Hence, by increasing N and/or t, it would become even more difficult to maintain satisfactory airflow between the fins, and this option is not particularly attractive.

Because the fin efficiency for the prescribed conditions is close to unity (η_f = 0.998), there is little advantage to replacing the aluminum with a material of higher thermal conductivity (e.g. Cu with k ~ 400 W/m·K). However, the large value of η_f suggests that significant benefit could be gained by increasing the fin length, $L = r_3 - r_2$.

It is also evident that the thermal contact resistance is large, and from Table 3.2, it's clear that a significant reduction could be effected by using indium foil or a conducting grease in the contact zone. Specifically, a reduction of $R''_{t,c}$ from 10^{-3} to 10^{-4} or even 10^{-5} m^2·K/W is certainly feasible.

Table 1.1 suggests that, by increasing the velocity of air flowing over the fins, a larger convection coefficient may be achieved. A value of h = 100 W/m^2·K would not be unreasonable.

As options for enhancing heat transfer, we therefore use the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array* to explore the effect of parameter variations over the ranges $10 \le L \le 20$ mm, $10^{-5} \le R''_{t,c} \le 10^{-3}$ m^2·K/W and $25 \le h \le 100$ W/m^2·K. As shown below, there is a significant enhancement in heat transfer associated with reducing $R''_{t,c}$ from 10^{-3} to 10^{-4} m^2·K/W, for which $R_{t,c}$ decreases from 13.26 to 1.326 K/W. At this value of $R''_{t,c}$, the reduction in $R_{t,o}$ from 23.45 to 12.57 K/W which accompanies an increase in L from 10 to 20 mm becomes significant, yielding a heat rate of q_t = 4.30 W for $R''_{t,c} = 10^{-4}$ m^2·K/W and L = 20 mm. However, since $R_{t,o} >> R_{t,c}$, little benefit is gained by further reducing $R''_{t,c}$ to 10^{-5} m^2·K/W.



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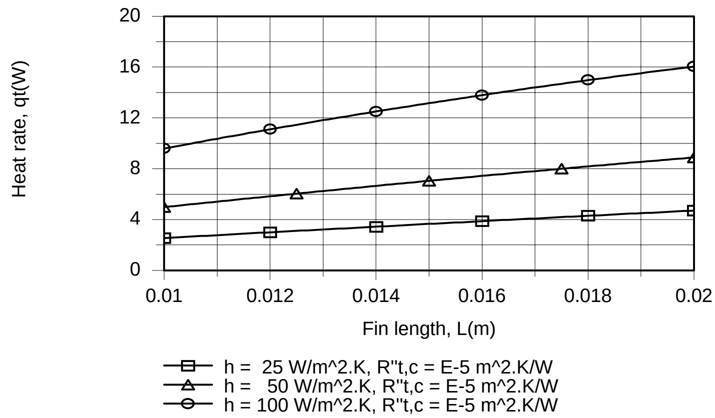
To derive benefit from a reduction in $R''_{t,c}$ to 10^{-5} $m^2 \cdot K/W$, an additional reduction in $R_{t,o}$ must be made. This can be achieved by increasing h, and for L = 20 mm and h = 100 $W/m^2 \cdot K$, $R_{t,o}$ = 3.56 K/W. With $R''_{t,c}$ = 10^{-5} $m^2 \cdot K/W$, a value of q_t = 16.04 W may be achieved.



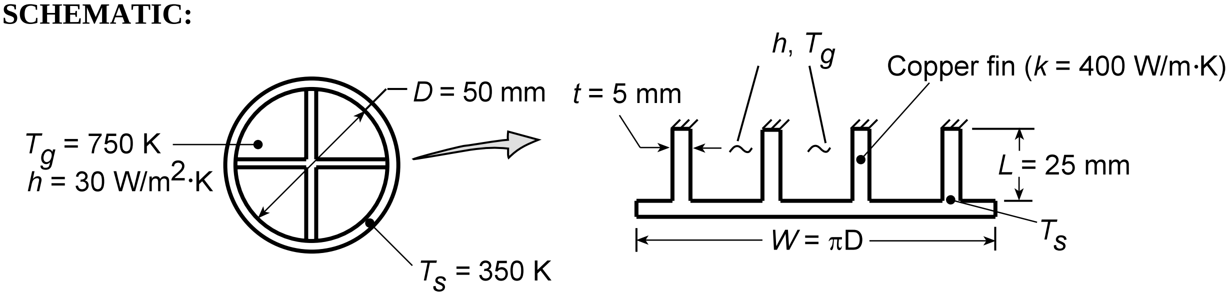
COMMENTS: In assessing options for enhancing heat transfer, the limiting (largest) resistance(s) should be identified and efforts directed at their reduction.

PROBLEM 3.150

KNOWN: Diameter and internal fin configuration of copper tubes submerged in water. Tube wall temperature and temperature and convection coefficient of gas flow through the tube.

FIND: Rate of heat transfer per tube length.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional fin conduction, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient, (6) Tube wall may be unfolded and represented as a plane wall with four straight, rectangular fins, each with an adiabatic tip.

ANALYSIS: The rate of heat transfer per unit tube length is:

$$q'_t = \eta_o h A'_t \left(T_g - T_s\right)$$

$$\eta_o = 1 - \frac{NA'_f}{A'_t}\left(1 - \eta_f\right)$$

$$NA'_f = 4 \times 2L = 8\left(0.025\text{m}\right) = 0.20\text{m}$$

$$A'_t = NA'_f + A'_b = 0.20\text{m} + \left(\pi D - 4t\right) = 0.20\text{m} + \left(\pi \times 0.05\text{m} - 4 \times 0.005\text{m}\right) = 0.337\text{m}$$

For an adiabatic fin tip,

$$\eta_f = \frac{q_f}{q_{max}} = \frac{M \tanh mL}{h\left(2L \cdot 1\right)\left(T_g - T_s\right)}$$

$$M = \left[h2\left(1\text{m} + t\right)k\left(1\text{m} \times t\right)\right]^{1/2}\left(T_g - T_s\right) \approx \left[30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)400\,\text{W/m} \cdot \text{K}\left(0.005\text{m}^2\right)\right]^{1/2}\left(400\text{K}\right) = 4382\,\text{W}$$

$$mL = \left\{\left[h2\left(1\text{m} + t\right)\right] / \left[k\left(1\text{m} \times t\right)\right]\right\}^{1/2} L \approx \left[\frac{30\,\text{W/m}^2 \cdot \text{K}\left(2\text{m}\right)}{400\,\text{W/m} \cdot \text{K}\left(0.005\text{m}^2\right)}\right]^{1/2} 0.025\text{m} = 0.137$$

Hence, tanh mL = 0.136, and

$$\eta_f = \frac{4382\text{W}\left(0.136\right)}{30\,\text{W/m}^2 \cdot \text{K}\left(0.05\text{m}^2\right)\left(400\text{K}\right)} = \frac{595\text{W}}{600\text{W}} = 0.992$$

$$\eta_o = 1 - \frac{0.20}{0.337}\left(1 - 0.992\right) = 0.995$$

$$q'_t = 0.995\left(30\,\text{W/m}^2 \cdot \text{K}\right)0.337\text{m}\left(400\text{K}\right) = 4025\,\text{W/m}$$

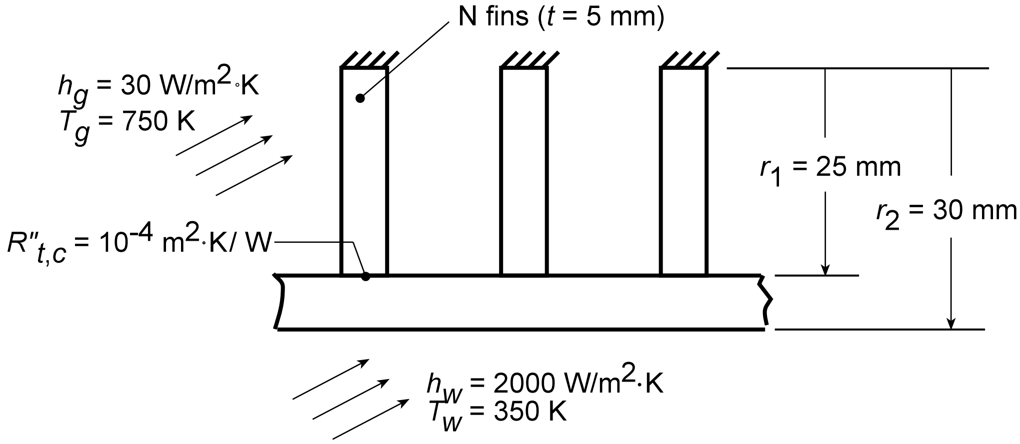
COMMENTS: Alternatively, $q'_t = 4q'_f + h\left(A'_t - A'_f\right)\left(T_g - T_s\right)$. Hence, q' = 4(595 W/m) + 30 W/m²·K (0.137 m)(400 K) = (2380 + 1644) W/m = 4024 W/m.

PROBLEM 3.151

KNOWN: Internal and external convection conditions for an internally finned tube. Fin/tube dimensions and contact resistance.

FIND: Heat rate per unit tube length and corresponding effects of the contact resistance, number of fins, and fin/tube material.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Negligible radiation, (5) Uniform convection coefficient on finned surfaces, (6) Tube wall may be unfolded and approximated as a plane surface with N straight rectangular fins.

PROPERTIES: Copper: k = 400 W/m·K; St.St.: k = 20 W/m·K.

ANALYSIS: The heat rate per unit length may be expressed as

$$q' = \frac{T_g - T_w}{R'_{t,o(c)} + R'_{cond} + R'_{conv,o}}$$

where

$$R_{t,o(c)} = \left(\eta_{o(c)} h_g A'_t\right), \quad \eta_{o(c)} = 1 - \frac{NA'_f}{A'_t}\left(1 - \frac{\eta_f}{C_1}\right), \quad C_1 = 1 + \eta_f h_g A'_f \left(R''_{t,c} / A'_{c,b}\right),$$

$$A'_t = NA'_f + \left(2\pi r_1 - Nt\right), \quad A'_f = 2r_1, \quad \eta_f = \tanh mr_1 / mr_1, \quad m = \left(2h_g / kt\right)^{1/2} \quad A'_{c,b} = t,$$

$$R'_{cond} = \frac{\ln\left(r_2/r_1\right)}{2\pi k}, \quad \text{and} \quad R'_{conv,o} = \left(2\pi r_2 h_w\right)^{-1}.$$

Using the IHT *Performance Calculation, Extended Surface Model* for the *Straight Fin Array*, the following results were obtained. For the *base case*, q' = 3857 W/m, where $R'_{t,o(c)}$ = 0.101 m·K/W, R'_{cond} = 7.25 × 10^{-5} m·K/W and $R'_{conv,o}$ = 0.00265 m·K/W. If the contact resistance is eliminated ($R''_{t,c}$ = 0), q' = 3922 W/m, where $R'_{t,o}$ = 0.0993 m·K/W. If the number of fins is increased to N = 8, q' = 5799 W/m, with $R'_{t,o(c)}$ = 0.063 m·K/W. If the material is changed to stainless steel, q' = 3591 W/m, with $R'_{t,o(c)}$ = 0.107 m·K/W and R'_{cond} = 0.00145 m·K/W.

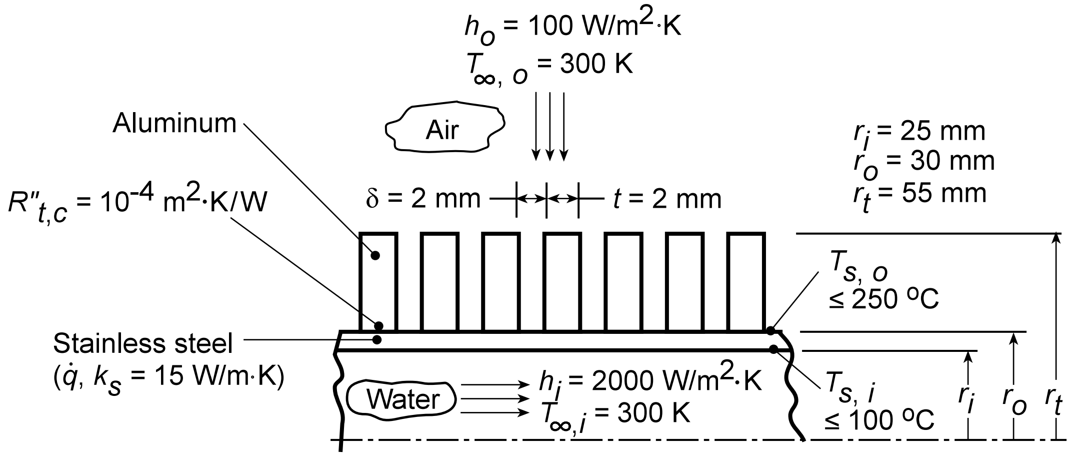
COMMENTS: The small reduction in q' associated with use of stainless steel is perhaps surprising, in view of the large reduction in k. However, because h_g is small, the reduction in k does not significantly reduce the fin efficiency (η_f changes from 0.994 to 0.891). Hence, the heat rate remains large. The influence of k would become more pronounced with increasing h_g.

PROBLEM 3.152

KNOWN: Design and operating conditions of a tubular, air/water heater.

FIND: (a) Expressions for heat rate per unit length at inner and outer surfaces, (b) Expressions for inner and outer surface temperatures, (c) Surface heat rates and temperatures as a function of volumetric heating $\dot{q}$ for prescribed conditions. Upper limit to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Constant properties, (3) One-dimensional heat transfer.

PROPERTIES: Table A-1: Aluminum, T = 300 K, k_a = 237 W/m·K.

ANALYSIS: (a) Applying Equation C.8 to the inner and outer surfaces, it follows that

$$q'(r_i) = \dot{q}\pi r_i^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]$$ <

$$q'(r_o) = \dot{q}\pi r_o^2 - \frac{2\pi k_s}{\ln(r_o/r_i)}\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]$$ <

(b) From Equations C.16 and C.17, energy balances at the inner and outer surfaces are of the form

$$h_i\left(T_{\infty,i}-T_{s,i}\right) = \frac{\dot{q}r_i}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]}{r_i\ln(r_o/r_i)}$$ <

$$U_o\left(T_{s,o}-T_{\infty,o}\right) = \frac{\dot{q}r_o}{2} - \frac{k_s\left[\frac{\dot{q}r_o^2}{4k_s}\left(1-\frac{r_i^2}{r_o^2}\right)+\left(T_{s,o}-T_{s,i}\right)\right]}{r_o\ln(r_o/r_i)}$$ <

Accounting for the fin array and the contact resistance, Equation 3.104 may be used to cast the overall heat transfer coefficient U_o in the form

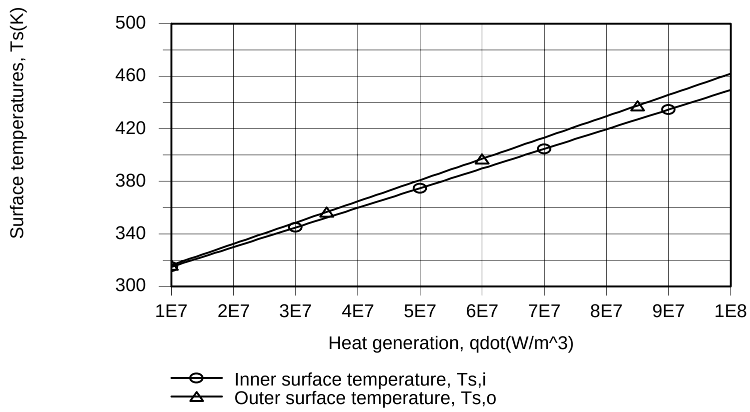
$$U_o = \frac{q'(r_o)}{A'_w\left(T_{s,o}-T_{\infty,o}\right)} = \frac{1}{A'_w R'_{t,o(c)}} = \frac{A'_t}{A'_w}\eta_{o(c)}h_o$$

where $\eta_{o(c)}$ is determined from Equations 3.105a,b and $A'_w = 2\pi r_o$.

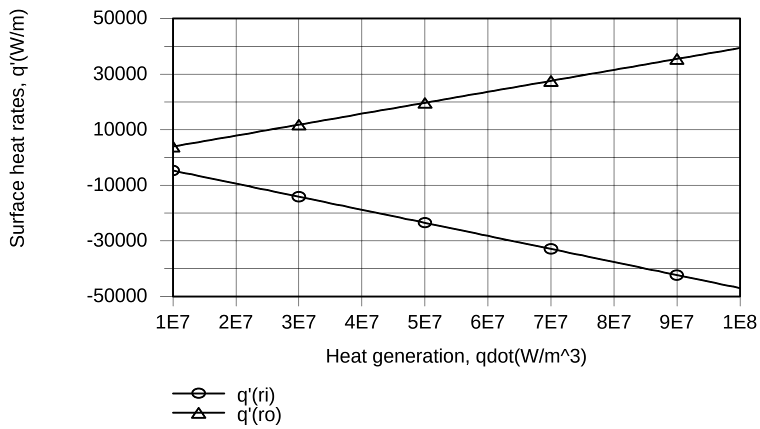
Continued...

PROBLEM 3.152 (Cont.)

(c) For the prescribed conditions and a representative range of $10^7 \le \dot{q} \le 10^8$ W/m^3, use of the relations of part (b) with the capabilities of the IHT *Performance Calculation Extended Surface Model* for a *Circular Fin Array* yields the following graphical results.



It is in this range that the upper limit of $T_{s,i}$ = 373 K is exceeded for $\dot{q}$ = 4.9×10^7 W/m^3, while the corresponding value of $T_{s,o}$ = 379 K is well below the prescribed upper limit. The expressions of part (a) yield the following results for the surface heat rates, where heat transfer in the negative r direction corresponds to $q'(r_i) < 0$.



For $\dot{q}$ = 4.9×10^7 W/m^3, $q'(r_i)$ = -2.30×10^4 W/m and $q'(r_o)$ = 1.93×10^4 W/m.

COMMENTS: The foregoing design provides for comparable heat transfer to the air and water streams. This result is a consequence of the nearly equivalent thermal resistances associated with heat transfer from the inner and outer surfaces. Specifically, $R'_{conv,i} = (h_i 2\pi r_i)^{-1}$ = 0.00318 m·K/W is slightly smaller than $R'_{t,o(c)}$ = 0.00411 m·K/W, in which case $|q'(r_i)|$ is slightly larger than $q'(r_o)$, while $T_{s,i}$ is slightly smaller than $T_{s,o}$. Note that the solution must satisfy the energy conservation requirement, $\pi\left(r_o^2 - r_i^2\right)\dot{q} = |q'(r_i)| + q'(r_o)$.